Composite Hierarchical Anti-disturbance Fuzzy Control for Nonlinear Interconnected Systems Via Disturbance Observer

Shangchun Mao, Cheng Qian, Zhilian Yan, Taiping Jiang

***Abstract*—This paper is devoted to the composite hierarchical anti-disturbance fuzzy control for nonlinear interconnected systems (NISs) subject to multiple disturbances, including a norm-bounded disturbance and an unknown disturbance generated by an exogenous system. $\mathcal{H}_\infty$ control and the composite disturbance-observer-based control consisting of a feedforward compensation term and a state-feedback sampled-data control are employed to attenuate and compensate these two types of disturbances in the T-S fuzzy framework, respectively. To start with, NISs, exogenous disturbance, disturbance observer, and composite controller are all modeled using T-S fuzzy model technology. By building a time-dependent function, a sufficient condition is then established to ensure the exponential stability of the estimation error system and closed-loop NISs with a prescribed $\mathcal{H}_\infty$ performance level. Following this, the joint design of the desired observer and composite disturbance-observer-based fuzzy controller is developed. Finally, a numerical simulation is performed to demonstrate the effectiveness of the presented composite hierarchical control scheme.**

***Index Terms*—Interconnected system, T-S fuzzy system, Disturbance observer, Composite controller**

I. Introduction

NOWADAYS, nonlinear systems have received sustained attention in the control field resulting from their outstanding ability to model actual systems [1]. Nonlinear interconnected systems (NISs) are a typical type of nonlinear systems composed of multiple interacting subsystems [2] and have potential applications in various branches of engineering fields, including but not limited to communication systems, power systems, aerospace systems, and robotic arm systems [3, 4]. Due to the effects of nonlinear interconnection among subsystems, the traditional linear system theory cannot be directly applied to NISs. In this case, the T-S fuzzy model is one of the most efficient techniques because it can provide a localized linear representation of nonlinear systems through fuzzy sets and IF-THEN fuzzy rules [5–8].

To date, numerous papers and applications have witnessed the fusion of T-S fuzzy model technology and diverse NISs [9–11]. For example, a decentralized adaptive T-S fuzzy controller was designed for a type of state-constrained NISs based on a nonlinear state-dependent barrier function in [10]. And the T-S fuzzy model technology was used to stochastic NISs with nontriangular structural dynamic uncertainties in [11].

In real life, disturbances are widely present in various control systems, which have a significant impact on the stability and performance of the control system [12]. As exogenous disturbances are often unmeasurable, many control techniques have been developed to reduce the adverse effects of exogenous disturbance [13–16]. Disturbance observer (DO) has become one of the most popular methods for dealing with exogenous disturbance because of their simplicity and effectiveness in estimating the equivalent disturbance and compensating it back into the system [17–19]. In [20], a terminal sliding mode control approach based on DO was employed to stabilize and control a fractional-order arch micromotor system. In [21], a robust active controller and DO were used to improve the comfort of the car driver. In [22, 23], when studying T-S fuzzy nonlinear systems, for the convenience of research, they also chose to perform T-S fuzzification on the DO.

In recent years, sampled-data systems have garnered significant attention because they can more accurately describe the basic characteristics of actual engineering than continuous- or discrete-time systems [24]. Since sampled-data control only requires the system's state information at sampling instances, it can greatly reduce the amount of information transmitted, greatly improving the control efficiency of the system [25, 26]. Sampled-data control has been widely used in various systems of research. In [27], a sampled-data fuzzy controller incorporating both current and delayed state information was used in a T-S fuzzy system with a time lag to study the sampled-data stabilization problem. In [28], a dissipative analysis and quantized sampled-data control were designed to study T-S fuzzy network control systems under random network attacks, and the effectiveness of the proposed method was verified by experiments. Recently, sampled-data control has also been shown to be an effective method for designing DO. In [29], a sampled-data adaptive output feedback controller was used to deal with nonlinear systems with unmeasurable states, uncertain dynamics, and unknown time-varying exogenous disturbances. The unknown time-varying exogenous disturbances were estimated by designing DO. A discrete-time DO was developed in [30] to ensure robust stability in uncertain sampled-data control systems.

In summary, this paper studies the design of composite hierarchical anti-disturbance fuzzy controller for NISs with

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multiple disturbances based on DO. The overall structure of this paper is as follows: Section II uses T-S fuzzy model technology to describe NISs, exogenous disturbance, DO, and composite controller, respectively. Constructing the estimation error system and closed-loop NISs. Section III proposes a composite controller design. The design is implemented in the form of linear matrix inequalities to ensure that the estimation error system and closed-loop NISs are exponentially stable (ES) under zero disturbances and have an $\mathcal{H}_\infty$ performance level under zero initial conditions. Section IV gives a simulation example to verify the effectiveness of the proposed method. Section V summarizes the paper.

II. PRELIMINARIES

Unless explicitly indicated, the notations used throughout are the same as those provided in Refs. [31, 32].

Consider a NIS with J subsystems modeled using T-S fuzzy modeling technology. The l IF-THEN rules of the j subsystem are described as follows:

$$\begin{aligned}\mathfrak{Z}_j^l: &IF\ z_{j1}(t) is\ \Gamma_{j1}^l\ and\ \ldots\ and\ z_{jp}(t)\ is\ \Gamma_{jp}^l,\\ \dot{x}_j(t) =&A_j^l x_j(t) + B_j^l(u_j(t)+d_j(t)) + C_j^l\zeta_j(t)\\ &+\sum_{n=1,n\neq j}^{J} D_{nj}^l x_n(t),\end{aligned} \tag{1}$$

where $x_j(t)\in\mathbb{R}^{i_j}$ is the state vector of the jth subsystem, $u_j(t)\in\mathbb{R}^{i_j}$ is the control input of the jth subsystem, $d_j(t)\in\mathbb{R}^{i_j}$ is the unknown disturbance generated by the exogenous system of the jth subsystem, $\zeta_j(t)\in\mathbb{R}^{m_j}$ is the norm-bounded external disturbance vector of the jth subsystem, and $D_{nj}^l\in\mathbb{R}^{i_j\times i_j}$ is the interconnection term between the jth subsystem and the nth subsystem of the interconnected system. $A_j^l\in\mathbb{R}^{i_j\times i_j}$, $B_j^l\in\mathbb{R}^{i_j\times i_j}$, and $C_j^l\in\mathbb{R}^{i_j}$ are all known parameters of the system.

Using single-point fuzzification, product reasoning, and average weighted defuzzification methods to system (1), the model of the jth fuzzy subsystem is obtained as follows:

$$\begin{aligned}\dot{x}_j(t) =&\sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\{A_j^l x_j(t) + B_j^l(u_j(t)+d_j(t))\\ &+C_j^l\zeta_j(t) + \sum_{n=1,n\neq j}^{J} D_{nj}^l x_n(t)\},\end{aligned} \tag{2}$$

where

$$\begin{aligned}g_j^l(\upsilon_j(t)) &= \frac{\theta_j^l(\upsilon_j(t))}{\sum_{\varrho=1}^{r_j}\theta_j^\varrho(\upsilon_j(t))},\\ \theta_j^l(\upsilon_j(t)) &= \prod_{q=1}^{p} \mathit{\Gamma}_{jq}^l(\upsilon_{jq}(t)),\end{aligned}$$

in which $\mathit{\Gamma}_{jq}^l(\upsilon_{jq}(t)):\mathcal{W}_{\upsilon_{jq}}\subset\mathbb{R}\to\mathbb{R}_{[0,1]}$ is the membership function of $\upsilon_{jq}(t)$ on the set $\mathcal{W}_{\upsilon_{jq}}$ and we can get the following properties of $g_j^l(\upsilon_j(t)): g_j^l(\upsilon_j(t))\in[0,1]$ and $\sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))=1$. Assume the disturbance is produced by an exogenous system:

$$\begin{cases}\dot{\xi}_j(t) = F_j\xi_j(t),\\ d_j(t) = E_j\xi_j(t).\end{cases} \tag{3}$$

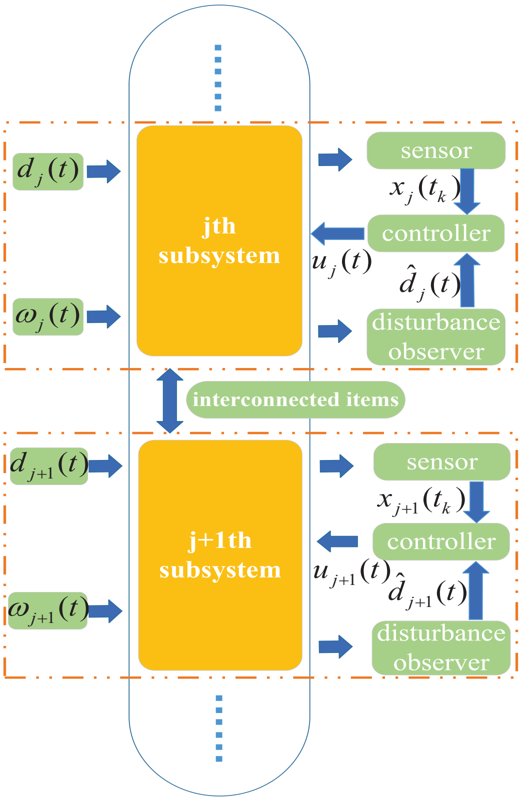

Fig. 1. Composite control structure diagram of NISs.

Using the same T-S fuzzy modeling and processing method as (1) and (2) to (3), we can get the fuzzy perturbation exogenous system

$$\mathfrak{Z}_j^l: IF\ z_{j1}(t)\ is\ \Gamma_{j1}^l\ and\ \ldots\ and\ z_{jp}(t)\ is\ \Gamma_{jp}^l,$$

$$\begin{cases}\dot{\xi}_j(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))F_j^l\xi_j(t),\\ d_j(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))E_j^l\xi_j(t),\end{cases} \tag{4}$$

where $F_j^l\in\mathbb{R}^{i_j\times i_j}$ and $E_j^l\in\mathbb{R}^{i_j\times i_j}$ are known parameters. To estimate the unknown disturbance $d_j(t)$, a T-S fuzzy DO is designed as follows:

$$\mathfrak{Z}_j^l: IF\ z_{j1}(t)\ is\ \Gamma_{j1}^l\ and\ \ldots\ and\ z_{jp}(t)\ is\ \Gamma_{jp}^l,$$

$$\begin{cases}\dot{Z}_j(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\{F_j^l\hat{\xi}_j(t) + L_j^l B_j^l E_j^l\hat{\xi}_j(t)\\ \qquad + L_j^l(A_j^l x_j(t) + B_j^l u_j(t)\\ \qquad + \sum_{n=1,n\neq j}^{J} D_{nj}^l x_n(t))\},\\ \hat{\xi}_j(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\{Z_j(t) - L_j^l x_j(t)\},\\ \hat{d}_j(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))E_j^l\hat{\xi}_j(t).\end{cases} \tag{5}$$

Based on the sampling mechanism, $h\geqslant h_k = t_{k+1}-t_k$, $t_k>0$ is the sampling interval, $k\in\mathbb{Z}_{\geqslant 0}$. Then the fuzzy controller can be designed as

$$\mathfrak{Z}_j^\varrho: IF\ z_{j1}(t)\ is\ \Gamma_{j1}^\varrho\ and\ \ldots\ and\ z_{jp}(t)\ is\ \Gamma_{jp}^\varrho,$$

$$u_j(t) = \sum_{\varrho=1}^{r_j} g_j^{\varrho}(z_j(t_k))\{K_j^{\varrho} x_j(t_k) - \hat{d}_j(t)\}, \tag{6}$$

where $x_j(t_k)$ represents the system state of the jth subsystem at sampling time t_k.

Remark 1. In (1), we choose to use the interconnection term in the form of $\sum_{n=1,n\neq j}^{J} D_{nj}^l x_n(t)$. It expresses the interconnection relationship between different subsystems through a certain parameter D_{nj}^l, and applies the state $x_n(t)$ of other subsystems to the target subsystem directly. This form not only intuitively reflects the interaction between subsystems but also can adapt to different NIS by adjusting the structure of the parameter D_{nj}^l.

Remark 2. In this paper, the NIS (2), exogenous system (4), DO (5), and controller (6) are all modeled using T-S fuzzy modeling. The main purpose of this treatment is to ensure that the exogenous disturbance-related parts share a unified modeling framework with the NIS to maintain overall consistency. This unified modeling method can significantly simplify the analysis process of the system, especially based on linear matrix inequality (LMI), making the design steps more systematic and efficient.

Now, the disturbance estimation error is defined as follows:

$$e_j(t) = \xi_j(t) - \hat{\xi}_j(t). \tag{7}$$

According to (2), (4), and (5), we can get

$$\begin{aligned} \dot{e}_j(t) =& \dot{\xi}_j(t) - \dot{\hat{\xi}}_j(t) \\ =& \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\{F_j^l \xi_j(t) - (\dot{Z}_j(t) - L_j^l \dot{x}_j(t))\} \\ =& \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\{(F_j^l + L_j^l B_j^l E_j^l) e_j(t) \\ &+ L_j^l C_j^l \zeta_j(t)\}. \end{aligned} \tag{8}$$

To enhance readability, we provide a concise description of the following form:

$$\tau_j^l(t) = \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t))\tau_j^l.$$

According to the (2) and (6), the closed-loop NIS is as follows:

$$\begin{aligned} \dot{x}_j(t) =& \sum_{l=1}^{r_j} g_j^l(\upsilon_j(t)) \sum_{\varrho=1}^{r_j} g_j^{\varrho}(z_j(t_k))\{A_j^l x_j(t) \\ &+ B_j^l(K_j^{\varrho} x_j(t_k) - \hat{d}_j(t)) \\ &+ B_j^l d_j(t) + C_j^l \zeta_j(t) + \sum_{n=1,n\neq j}^{J} D_{nj}^l x_n(t)\}, \\ =& A_j^l(t) x_j(t) + B_j^l(t) K_j^{\varrho}(t_k) x_j(t_k) \\ &+ B_j^l(t) E_j^l(t) e_j(t) + C_j^l(t)\zeta_j(t) \\ &+ \sum_{n=1,n\neq j}^{J} D_{nj}^l(t) x_n(t). \end{aligned} \tag{9}$$

Define the reference output as follows:

$$Z_j(t) = G_{1j}^l(t) x_j(t) + G_{2j}^l(t) e_j(t). \tag{10}$$

Lemma 1. [33] When there are matrices $\mathcal{X}$ and $\mathcal{Y}$ of any suitable dimensions, the following relationship is satisfied:

$$2\mathcal{X}^T\mathcal{Y} \leqslant \sigma^{-1}\mathcal{X}^T\mathcal{X} + \sigma\mathcal{Y}^T\mathcal{Y},$$

where σ is a positive scalar.

Lemma 2. The following statements are true:

1) [34](Continuous Jensen Inequality) For any positive definite matrix $\mathcal{N} \in \mathbb{R}^{n\times n}$, scalars a and b, vector function $\epsilon : [a, b] \to \mathbb{R}^n$, the following inequality holds:

$$\mathcal{X}^T\mathcal{N}\mathcal{X} \leq (b-a)\int_a^b \epsilon^T(\sigma)\mathcal{N}\epsilon(\sigma)\, d\sigma,$$

where $\mathcal{X} = \int_a^b \epsilon(\sigma)\, d\sigma$.

2) [35] (Discrete Jensen inequality): For any positive matrix $\mathcal{N} \in \mathbb{R}^{n\times n}, \mathcal{N}^T = \mathcal{N} > 0$, two positive integers a and b satisfy $b \geqslant a \geqslant 1$, then the following inequality holds:

$$\left(\sum_{t=a}^{b} \mathcal{X}(t)\right)^T \mathcal{N} \left(\sum_{t=a}^{b} \mathcal{X}(t)\right) \leqslant \hat{d} \sum_{t=a}^{b} \mathcal{X}^T(t)\mathcal{N}\mathcal{X}(t),$$

where $\hat{d} = b - a + 1$.

Lemma 3. [36] For a given matrix

$$\mathcal{Z} = \begin{bmatrix} \mathcal{Z}_{11} & \mathcal{Z}_{12} \\ \mathcal{Z}_{21} & \mathcal{Z}_{22} \end{bmatrix},$$

the following three conditions are equivalent:

1) $\mathcal{Z} < 0$;
2) $\mathcal{Z}_{11} < 0, \mathcal{Z}_{22} - \mathcal{Z}_{21}\mathcal{Z}_{11}^{-1}\mathcal{Z}_{12} < 0$;
3) $\mathcal{Z}_{22} < 0, \mathcal{Z}_{11} - \mathcal{Z}_{12}\mathcal{Z}_{22}^{-1}\mathcal{Z}_{21} < 0$.

To enable further analysis, it is necessary to propose the following definition.

Definition 1. [37]The estimation error system (7) and closed-loop NIS (9) are said to be ES with a prescribed $\mathcal{H}_\infty$ performance level γ, if

1) Estimation error system (7) and closed-loop NIS (9) are ES when $\zeta_j(t) = 0$.

2) Under the zero initial condition, the following inequality is satisfied:

$$\sum_{j=1}^{J} \int_0^\infty z_j^T(s) z_j(s)\, ds \leqslant \gamma^2 \sum_{j=1}^{J} \int_0^\infty \zeta_j^T(s)\zeta_j(s)\, ds.$$

III. MAIN RESULTS

Now construct a Lyapunov function:

$$V(t) = \sum_{j=1}^{J} V_j(t), \tag{11}$$

where

$$V_j(t) = \sum_{r=1}^{3} V_{rj}(t),$$

$$\begin{aligned} V_{1j}(t) &= x_j^T(t) P_{1j} x_j(t) + e_j^T(t) P_{2j} e_j(t), \\ V_{2j}(t) &= (t_{k+1} - t)\int_{t_k}^{t} e^{2\delta(s-t)} \dot{x}_j^T(s) P_{3j} \dot{x}_j(s)\, ds, \\ V_{3j}(t) &= (t_{k+1} - t)\Delta_j^T(t) H_j \Delta_j(t), \end{aligned}$$

with $P_{\varsigma j} > 0$ with $\varsigma \in \ell_3$, $\Delta_j(t) = col\{x_j(t), x_j(t_k)\}$, and

$$H_j = \begin{bmatrix} He\{\frac{X_j}{2}\} & -X_j + Y_j \\ * & He\{-Y_j + \frac{X_j}{2}\} \end{bmatrix}.$$

Since $0 \leqslant h_k \leqslant h$, it follows that $h_k = \vartheta h, 0 \leqslant \vartheta \leqslant 1$. Thus

$$\begin{aligned} V_{1j}(t) + V_{3j}(t) = &\frac{t - t_k}{h_k}\Delta_j^T(t)\begin{bmatrix} P_{1j} & 0 \\ 0 & 0 \end{bmatrix}\Delta_j(t) \\ &+ \frac{t_{k+1} - t}{h_k}\vartheta\Delta_j^T(t)\Psi_j\Delta_j(t) \\ &+ \frac{t_{k+1} - t}{h_k}(1 - \vartheta)\Delta_j^T(t)\begin{bmatrix} P_{1j} & 0 \\ 0 & 0 \end{bmatrix}\Delta_j(t) \\ &+ e_j^T(t)P_{2j}e_j(t), \end{aligned}$$

where

$$\Psi_j = \begin{bmatrix} P_{1j} + hHe\{\frac{X_j}{2}\} & h(-X_j + Y_j) \\ * & hHe\{-Y_j + \frac{X_j}{2}\} \end{bmatrix} > 0. \quad (12)$$

For $t \in [t_k, t_{k+1})$, from the above analysis and inequality (12), we can know that $V_{1j}(t) + V_{3j}(t)$ is positive definite. According to $P_{\varsigma j} > 0$, we can determine that $V_{2j}(t)$ is positive definite. Therefore, we can conclude that $V_j(t) = \sum_{r=1}^{3} V_{rj}(t)$ is positive definite.
Hence,

$$V_j(t) \geqslant V_{1j}(t) + V_{3j}(t).$$

Let

$$V_{1j}(t) + V_{3j}(t) = \Delta_j^T(t)\bar{\Psi}_j\Delta_j(t) + e_j^T(t)P_{2j}e_j(t).$$

Then, there exists

$$V_{1j}(t) + V_{3j}(t) \geq \lambda_{min}(\bar{\Psi}_j)\left|\Delta_j(t)\right|^2 + \lambda_{min}\{P_{2j}\}\left|e_j(t)\right|^2,$$

and there scalars positive scalar

$$\begin{aligned} \epsilon_{0j} &= \lambda_{min}\{P_{2j}\}, \\ \epsilon_j &= \lambda_{min}(\bar{\Psi}_j) = min\left\{\lambda_{min}(P_{1j}), \lambda_{min}(\Psi_j)\right\}. \end{aligned}$$

Therefore,

$$\begin{aligned} V_j(t) &\geqslant V_{1j}(t) + V_{3j}(t) \\ &\geq \epsilon_j\left|\Delta_j(t)\right|^2 + \epsilon_{0j}\left|e_j(t)\right|^2 \\ &= \epsilon_j\left(\left|\varepsilon_j(t)\right|^2 + \left|\varepsilon_j(t_k)\right|^2\right) + \epsilon_{0j}\left|e_j(t)\right|^2, \end{aligned}$$

such that

$$V_j(t) \geqslant \epsilon_j\left|\varepsilon_j(t)\right|^2. \quad (13)$$

A. Stability and $\mathcal{H}_\infty$ Performance Analysis

In this section, we use the Lyapunov function to conduct $\mathcal{H}_\infty$ stability analysis on the estimation error system (7) and closed-loop NIS (9) and derive the conditions under which the estimation error system (7) and closed-loop NIS (9) achieve ES under the $\mathcal{H}_\infty$ performance index γ.

Theorem 1. For given scalars $\delta > 0$, $\gamma > 0$, if there exist scalar $\sigma > 0$, and matrices $P_{\varsigma j} > 0$ with $\varsigma \in \ell_3$, X_j, Y_j, M_{1j}, M_{2j}, T_j, R_{1j}, R_{2j}, K_j^{ϱ}, L_j^l such that the following inequalities and (12) hold, then the estimation error system (7) and closed-loop NISs (9) are ES with $\mathcal{H}_\infty$ performance index γ:

$$\Psi_{0j}^{l\varrho} = \begin{bmatrix} \Theta_{11j}^l & \Theta_{12j}^l & \Theta_{13j}^l & \Theta_{14j}^{l\varrho} & \Theta_{16j}^l \\ * & \Theta_{22j} & \Theta_{23j}^l & \Theta_{24j}^{l\varrho} & \Theta_{26j}^l \\ * & * & \Theta_{33j}^l & 0 & \Theta_{36j}^l \\ * & * & * & \Theta_{44j} & 0 \\ * & * & * & * & \Theta_{66j} \end{bmatrix} < 0, \quad (14)$$

$$\Psi_{1j}^{l\varrho} = \begin{bmatrix} \bar{\Theta}_{11j}^l & \bar{\Theta}_{12j}^l & \Theta_{13j}^l & \bar{\Theta}_{14j}^{l\varrho} & \Theta_{16j}^l \\ * & \bar{\Theta}_{22j} & \Theta_{23j}^l & \bar{\Theta}_{24j}^{l\varrho} & \Theta_{26j}^l \\ * & * & \Theta_{33j}^l & 0 & \Theta_{36j}^l \\ * & * & * & \bar{\Theta}_{44j} & 0 \\ * & * & * & * & \Theta_{66j} \end{bmatrix} < 0, \quad (15)$$

$$\Psi_{2j}^{l\varrho} = \begin{bmatrix} \Theta_{11j}^l & \Theta_{12j}^l & \Theta_{13j}^l & \Theta_{14j}^{l\varrho} & \Theta_{15j} & \Theta_{16j}^l \\ * & \Theta_{22j} & \Theta_{23j}^l & \Theta_{24j}^{l\varrho} & \Theta_{25j} & \Theta_{26j}^l \\ * & * & \Theta_{33j}^l & 0 & 0 & \Theta_{36j}^l \\ * & * & * & \Theta_{44j} & \Theta_{45j} & 0 \\ * & * & * & * & \Theta_{55j} & 0 \\ * & * & * & * & * & \Theta_{66j}^l \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned} \Theta_{11j}^l =& He\{-\frac{X_j}{2} - M_{1j} + R_{1j}^T A_j^l\} + 2\delta P_{1j} \\ &+ 2\sigma^{-1}(J-1)\sum_{n=1, n\neq j}^{J}(D_{jn}^l)^T D_{jn}^l \\ &+ \sigma R_{1j}^T R_{1j} + (G_{1j}^l)^T G_{1j}^l \\ \Theta_{12j}^l =& P_{1j} - M_{2j} - R_{1j}^T + (R_{2j}^T A_j^l)^T, \\ \Theta_{13j}^l =& R_{1j}^T B_j^l E_j^l + (G_{1j}^l)^T G_{2j}^l, \\ \Theta_{14j}^{l\varrho} =& X_j - Y_j + M_{1j}^T - T_j + R_{1j}^T B_j^l K_j^{\varrho}, \\ \Theta_{15j} =& hM_{1j}^T, \Theta_{16j}^l = R_{1j}^T C_j^l, \\ \Theta_{22j} =& -He\{R_{2j}^T\} + \sigma R_{2j}^T R_{2j}, \\ \Theta_{23j}^l =& R_{2j}^T B_j^l E_j^l, \Theta_{24j}^{l\varrho} = M_{2j}^T + R_{2j}^T B_j^l K_j^{\varrho}, \\ \Theta_{25j} =& hM_{2j}^T, \Theta_{26j}^l = R_{2j}^T C_j^l, \\ \Theta_{33j}^l =& He\{P_{2j}F_j^l + P_{2j}L_j^l B_j^l E_j^l\} + 2\delta P_{2j} \\ &+ (G_{2j}^l)^T G_{2j}^l, \\ \Theta_{36j}^l =& P_{2j}L_j^l C_j^l, \Theta_{44j} = He\{Y_j - \frac{X_j}{2} + T_j\}, \\ \Theta_{45j} =& hT_j^T, \Theta_{55j} = -he^{-2\delta h}P_{3j}, \Theta_{66j} = -\gamma^2 I, \\ \bar{\Theta}_{11j}^l =& He\{(h\delta - \frac{1}{2})X_j - M_{1j} + R_{1j}^T A_j^l\} \\ &+ 2\sigma^{-1}(J-1)\sum_{n=1, n\neq j}^{J}(D_{jn}^l)^T D_{jn}^l \\ &+ \sigma R_{1j}^T R_{1j} + (G_{1j}^l)^T G_{1j}^l + 2\delta P_{1j}, \\ \bar{\Theta}_{12j}^l =& P_{1j} - M_{2j} - R_{1j}^T + (R_{2j}^T A_j^l)^T + hHe\{\frac{X_j}{2}\}, \\ \bar{\Theta}_{14j}^{l\varrho} =& (1 - 2h\delta)(X_j - Y_j) + M_{1j}^T - T_j + R_{1j}^T B_j^l K_j^{\varrho}, \\ \bar{\Theta}_{22j} =& -He\{R_{2j}^T\} + \sigma R_{2j}^T R_{2j} + hP_{3j}, \\ \bar{\Theta}_{24j}^{l\varrho} =& M_{2j}^T + R_{2j}^T B_j^l K_j^{\varrho} + h(-X_j + Y_j), \\ \bar{\Theta}_{44j} =& He\{(1 - 2h\delta)(Y_j - \frac{X_j}{2}) + T_j\}. \end{aligned}$$

Proof: Calculating $\dot{V}_{rj}(t)$, $(r=1,2,3)$, we can get

$$\begin{aligned}\dot{V}_{1j}(t) =&2x_j^T(t)P_{1j}\dot{x}_j(t)+2e_j^T(t)P_{2j}L_j^l(t)C_j^l(t)\zeta_j(t)\\&+2e_j^T(t)(P_{2j}(F_j^l(t)+L_j^l(t)B_j^l(t)E_j^l(t)))e_j(t),\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{V}_{2j}(t) =&-\int_{t_k}^{t}e^{2(\delta+2\eta)(s-t)}\dot{x}_j^T(s)P_{3j}\dot{x}_j(s)\,ds\\&+(t_{k+1}-t)\dot{x}_j^T(t)P_{3j}\dot{x}_j(t)-2(\delta+2\eta)V_{2j}(t),\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{V}_{4j}(t) =&-\Big[x_j^T(t)He\{\frac{X_j}{2}\}x_j(t)+x_j^T(t)(-X_j+Y_j)x_j(t_k)\\&+x_j^T(t_k)(-X_j^T+Y_j^T)x_j(t)+x_j^T(t_k)He\{-Y_j\\&+\frac{X_j}{2}\}x_j(t_k)\Big]+2(t_{k+1}-t)\Big[x_j^T(t_k)(-X_j^T\\&+Y_j^T)\dot{x}_j(t)+x_j^T(t)He\{\frac{X_j}{2}\}\dot{x}_j(t)\Big].\end{aligned} \tag{19}$$

Let

$$\mathcal{J}_j(t)=\frac{1}{t-t_k}\int_{t_k}^{t}\dot{x}_j(s)\,ds. \tag{20}$$

By using the continuous Jensen inequality of Lemma 2, the (18) can be transformed into

$$\begin{aligned}\dot{V}_{2j}(t)\le&-e^{-2\delta h}(t-t_k)\mathcal{J}_j^T(t)P_{3j}\mathcal{J}_j(t)\\&+(t_{k+1}-t)\dot{x}_j^T(t)P_{3j}\dot{x}_j(t)-2\delta V_{2j}(t).\end{aligned} \tag{21}$$

Calculating $2\delta V_{rj}(t)$, $(r=1,2,3)$, we can get

$$2\delta V_{1j}(t)=2\delta x_j^T(t)P_{1j}x_j(t)+2\delta e_j^T(t)P_{2j}e_j(t), \tag{22}$$

$$\begin{aligned}2\delta V_{2j}(t)=&2\delta(t_{k+1}-t)\\&\times\int_{t_k}^{t}e^{2(\delta+2\eta)(s-t)}\dot{x}_j^T(s)P_{3j}\dot{x}_j(s)\,ds,\end{aligned} \tag{23}$$

$$\begin{aligned}2\delta V_{3j}(t)=&2\delta(t_{k+1}-t)\Big[x_j^T(t)He\{\frac{X_j}{2}\}x_j(t)\\&+x_j^T(t_k)(-X_j^T+Y_j^T)x_j(t)\\&+x_j^T(t)(-X_j+Y_j)x_j(t_k)\\&+x_j^T(t_k)He\{-Y_j+\frac{X_j}{2}\}x_j(t_k)\Big].\end{aligned} \tag{24}$$

In addition, for matrices $M_{1j}, M_{2j}, T_j, R_{1j}, R_{2j}$ of a certain dimension, the following equations hold:

$$\begin{aligned}0=&2\sum_{j=1}^{J}[x_j^T(t)M_{1j}^T+\dot{x}_j^T(t)M_{2j}^T+x_j^T(t_k)T_j^T][-x_j(t)\\&+x_j(t_k)+(t-t_k)\mathcal{J}_j(t)],\end{aligned} \tag{25}$$

$$\begin{aligned}0=&2\sum_{j=1}^{J}[x_j^T(t)R_{1j}^T+\dot{x}_j^T(t)R_{2j}^T][-\dot{x}_j(t)+A_j^l(t)x_j(t)\\&+B_j^l(t)K_j^{\varrho}(t_k)x_j(t_k)+B_j^l(t)E_j^l(t)e_j(t)\\&+C_j^l(t)\zeta_j(t)+\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)].\end{aligned} \tag{26}$$

The expression (26) can be evaluated as follows:

$$\begin{aligned}0=&2\sum_{j=1}^{J}\Big[-x_j^T(t)R_{1j}^T\dot{x}_j(t)+x_j^T(t)R_{1j}^TA_j^l(t)x_j(t)\\&+x_j^T(t)R_{1j}^TB_j^l(t)K_j^{\varrho}(t_k)x_j(t_k)\\&+x_j^T(t)R_{1j}^TB_j^l(t)E_j^l(t)e_j(t)+x_j^T(t)R_{1j}^TC_j^l(t)\zeta_j(t)\\&+x_j^T(t)R_{1j}^T\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)-\dot{x}_j^T(t)R_{2j}^T\dot{x}_j(t)\\&+\dot{x}_j^T(t)R_{2j}^TA_j^l(t)x_j(t)+\dot{x}_j^T(t)R_{2j}^TB_j^l(t)E_j^l(t)e_j(t)\\&+\dot{x}_j^T(t)R_{2j}^TB_j^l(t)K_j^{\varrho}(t_k)x_j(t_k)+\dot{x}_j^T(t)R_{2j}^TC_j^l(t)\zeta_j(t)\\&+\dot{x}_j^T(t)R_{2j}^T\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)\Big].\end{aligned} \tag{27}$$

According to Lemma 1 and Lemma 2, the expression

$$2\sum_{j=1}^{J}x_j^T(t)R_{1j}^T\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)$$

can be transformed as follows:

$$\begin{aligned}&2\sum_{j=1}^{J}x_j^T(t)R_{1j}^T\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)\\=&2\sum_{j=1}^{J}(R_{1j}x_j(t))^T(\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t))\\\leqslant&\sum_{j=1}^{J}(\sigma(R_{1j}x_j(t))^T(R_{1j}x_j(t))\\&+\sigma^{-1}(\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t))^T(\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)))\\\leqslant&\sum_{j=1}^{J}(\sigma(R_{1j}x_j(t))^T(R_{1j}x_j(t))\\&+\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}x_n^T(t)(D_{nj}^l(t))^TD_{nj}^l(t)x_n(t))\\=&\sum_{j=1}^{J}(\sigma(R_{1j}x_j(t))^T(R_{1j}x_j(t))\\&+\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}x_j^T(t)(D_{jn}^l(t))^TD_{jn}^l(t)x_j(t)).\end{aligned}$$

By applying the same processing method and steps, we can obtain

$$\begin{aligned}&2\sum_{j=1}^{J}\dot{x}_j^T(t)R_{2j}^T\sum_{n=1,n\neq j}^{J}D_{nj}^l(t)x_n(t)\\\leqslant&\sum_{j=1}^{J}(\sigma\dot{x}_j^T(t)R_{2j}^TR_{2j}\dot{x}_j(t)\\&+\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}x_j^T(t)(D_{jn}^l(t))^TD_{jn}^l(t)x_j(t)).\end{aligned}$$

After the above processing, (27) can be converted to

$$\begin{aligned}0\leqslant&\sum_{j=1}^{J}\Big[2(-x_j^T(t)R_{1j}^T\dot{x}_j(t)+x_j^T(t)R_{1j}^TA_j^l(t)x_j(t)\\&+x_j^T(t)R_{1j}^TB_j^l(t)K_j^{\varrho}(t_k)x_j(t_k)\\&+x_j^T(t)R_{1j}^TB_j^l(t)E_j^l(t)e_j(t)+x_j^T(t)R_{1j}^TC_j^l(t)\zeta_j(t)\\&-\dot{x}_j^T(t)R_{2j}^T\dot{x}_j(t)+\dot{x}_j^T(t)R_{2j}^TA_j^l(t)x_j(t)\\&+\dot{x}_j^T(t)R_{2j}^TB_j^l(t)K_j^{\varrho}(t_k)x_j(t_k)\\&+\dot{x}_j^T(t)R_{2j}^TB_j^l(t)E_j^l(t)e_j(t)+\dot{x}_j^T(t)R_{2j}^TC_j^l(t)\zeta_j(t))\end{aligned}$$

$$+\sigma(R_{1j}x_j(t))^TR_{1j}x_j(t)+\sigma(R_{2j}\dot{x}_j(t))^TR_{2j}\dot{x}_j(t)$$
$$+2\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}x_j^T(t)(D_{jn}^l(t))^TD_{jn}^l(t)x_j(t)\Big]. \tag{28}$$

According to (10), (17), (19) - (25), and (28), we can get

$$\begin{aligned}
&\dot{V}(t)+2\delta V(t)+Z^T(t)Z(t)-\gamma^2\zeta^T(t)\zeta(t)\\
&=\sum_{j=1}^{J}(\dot{V}_j(t)+2\delta V_j(t)+Z_j^T(t)Z_j(t)-\gamma^2\zeta_j^T(t)\zeta_j(t))\\
&\leqslant\sum_{j=1}^{J}(\frac{t_{k+1}-t}{h_k}\phi_{1j}^T(t)\Psi_{1j[h_k]}(t,t_k)\phi_{1j}(t))\\
&\qquad+\frac{t-t_k}{h_k}\phi_{2j}^T(t)\Psi_{2j[h_k]}(t,t_k)\phi_{2j}(t)\\
&=\sum_{j=1}^{J}((1-\vartheta)\phi_{1j}^T(t)\Psi_{0j}(t,t_k)\phi_{1j}(t)\\
&\qquad+\frac{t_{k+1}-t}{h}\phi_{1j}^T(t)\Psi_{1j[h]}(t,t_k)\phi_{1j}(t)\\
&\qquad+\frac{t-t_k}{h}\phi_{2j}^T(t)\Psi_{2j[h]}(t,t_k)\phi_{2j}(t)),
\end{aligned}\tag{29}$$

where

$$\begin{aligned}
\phi_{1j}(t)&=\{x_j(t),\dot{x}_j(t),e_j(t),x_j(t_k),\zeta_j(t)\},\\
\phi_{2j}(t)&=\{x_j(t),\dot{x}_j(t),e_j(t),x_j(t_k),\mathcal{J}_j(t),\zeta_j(t)\},\\
\Psi_{0j}(t,t_k)&=\begin{bmatrix}\Xi_{01j}(t) & \Xi_{02j}(t,t_k)\\ * & \Xi_{03j}\end{bmatrix},\\
\Psi_{1j[h]}(t,t_k)&=\begin{bmatrix}\Xi_{11j}(t) & \Xi_{12j}(t,t_k)\\ * & \Xi_{13j}\end{bmatrix},\\
\Psi_{2j[h]}(t,t_k)&=\begin{bmatrix}\Xi_{01j}(t) & \Xi_{21j}(t,t_k)\\ * & \Xi_{22j}\end{bmatrix},\\
\Xi_{01j}(t)&=\begin{bmatrix}\Theta_{11j}(t) & \Theta_{12j}(t) & \Theta_{13j}(t)\\ * & \Theta_{22j} & \Theta_{23j}(t)\\ * & * & \Theta_{33j}(t)\end{bmatrix},\\
\Xi_{02j}(t,t_k)&=\begin{bmatrix}\Theta_{14j}(t,t_k) & \Theta_{16j}(t)\\ \Theta_{24j}(t,t_k) & \Theta_{26j}(t)\\ 0 & \Theta_{36j}(t)\end{bmatrix},\\
\Xi_{03j}&=\begin{bmatrix}\Theta_{44j} & 0\\ * & \Theta_{66j}\end{bmatrix},\\
\Xi_{11j}(t)&=\begin{bmatrix}\bar{\Theta}_{11j}(t) & \bar{\Theta}_{12j}(t) & \Theta_{13j}(t)\\ * & \bar{\Theta}_{22j} & \Theta_{23j}(t)\\ * & * & \Theta_{33j}(t)\end{bmatrix},\\
\Xi_{12j}(t,t_k)&=\begin{bmatrix}\bar{\Theta}_{14j}(t,t_k) & \Theta_{16j}(t)\\ \bar{\Theta}_{24j}(t,t_k) & \Theta_{26j}(t)\\ 0 & \Theta_{36j}(t)\end{bmatrix},\\
\Xi_{13j}&=\begin{bmatrix}\bar{\Theta}_{44j} & 0\\ * & \Theta_{66j}\end{bmatrix},\\
\Xi_{21j}(t,t_k)&=\begin{bmatrix}\Theta_{14j}(t,t_k) & \Theta_{15j} & \Theta_{16j}(t)\\ \Theta_{24j}(t,t_k) & \Theta_{25j} & \Theta_{26j}(t)\\ 0 & 0 & \Theta_{36j}(t)\end{bmatrix},\\
\Xi_{22j}&=\begin{bmatrix}\Theta_{44j} & \Theta_{45j} & 0\\ * & \Theta_{55j} & 0\\ * & * & \Theta_{66j}\end{bmatrix},\\
\Theta_{11j}(t)&=He\{-\frac{X_j}{2}-M_{1j}+R_{1j}^TA_j^l(t)\}+2\delta P_{1j}\\
&\quad+2\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}(D_{jn}^l(t))^TD_{jn}^l(t)\\
&\quad+\sigma R_{1j}^TR_{1j}+(G_{1j}^l)^T(t)G_{1j}^l(t),\\
\Theta_{12j}(t)&=P_{1j}-M_{2j}-R_{1j}^T+(R_{2j}^TA_j^l(t))^T,\\
\Theta_{13j}(t)&=R_{1j}^TB_j^l(t)E_j^l(t)+(G_{1j}^l)^T(t)G_{2j}^l(t),\\
\Theta_{14j}(t,t_k)&=X_j-Y_j+M_{1j}^T-T_j+R_{1j}^TB_j^l(t)K_j^\varrho(t_k),\\
\Theta_{15j}&=hM_{1j}^T,\Theta_{16j}(t)=R_{1j}^TC_j^l(t),\\
\Theta_{22j}&=-He\{R_{2j}^T\}+\sigma R_{2j}^TR_{2j},\\
\Theta_{23j}(t)&=R_{2j}^TB_j^l(t)E_j^l(t),\\
\Theta_{24j}(t,t_k)&=M_{2j}^T+R_{2j}^TB_j^l(t)K_j^\varrho(t_k),\\
\Theta_{25j}&=hM_{2j}^T,\Theta_{26j}(t)=R_{2j}^TC_j^l(t),\\
\Theta_{33j}(t)&=He\{P_{2j}(F_j^l(t)+L_j^l(t)B_j^l(t)E_j^l(t))\}\\
&\quad+2\delta P_{2j}+(G_{2j}^l)^T(t)G_{2j}^l(t),\\
\Theta_{36j}(t)&=P_{2j}L_j^l(t)C_j^l(t),\\
\Theta_{44j}&=He\{Y_j-\frac{X_j}{2}+T_j\},\\
\Theta_{45j}&=hT_j^T,\Theta_{55j}=-he^{-2\delta h}P_{3j},\\
\Theta_{66j}&=-\gamma^2I,\\
\bar{\Theta}_{11j}(t)&=He\{(h\delta-\frac{1}{2})X_j-M_{1j}+R_{1j}^TA_j^l(t)\}\\
&\quad+2\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J}(D_{jn}^l(t))^TD_{jn}^l(t)\\
&\quad+\sigma R_{1j}^TR_{1j}+(G_{1j}^l)^T(t)G_{1j}^l(t)+2\delta P_{1j},\\
\bar{\Theta}_{12j}(t)&=P_{1j}-M_{2j}-R_{1j}^T+(R_{2j}^TA_j^l(t))^T\\
&\quad+hHe\{\frac{X_j}{2}\},\\
\bar{\Theta}_{14j}(t,t_k)&=(1-2h\delta)(X_j-Y_j)+M_{1j}^T-T_j\\
&\quad+R_{1j}^TB_j^l(t)K_j^\varrho(t_k),\\
\bar{\Theta}_{22j}&=-He\{R_{2j}^T\}+\sigma R_{2j}^TR_{2j}+hP_{3j},\\
\bar{\Theta}_{24j}(t,t_k)&=M_{2j}^T+R_{2j}^TB_j^l(t)K_j^\varrho(t_k)+h(-X_j+Y_j),\\
\bar{\Theta}_{44j}&=He\{(1-2h\delta)(Y_j-\frac{X_j}{2})+T_j\}.
\end{aligned}$$

It is easy to see that LMIs (12), (14) - (16) can ensure that

$$\dot{V}_j(t)+2\delta V_j(t)+Z_j^T(t)Z_j(t)-\gamma^2\zeta_j^T(t)\zeta_j(t)\leqslant 0. \tag{30}$$

Now, we consider the case when $\zeta_j(t)=0$. Under this condition, we derive from inequality (30) that

$$\dot{V}_j(t)+2\delta V_j(t)\leqslant 0. \tag{31}$$

From (11) and (31), it follows that for any $t\in[t_k,t_{k+1})$

$$\begin{aligned}
V_j(t)&\leqslant e^{-2\delta(t-t_k)}V_j(t_k)\\
&\leqslant e^{-2\delta(t-t_{k-1})}V_j(t_{k-1})\\
&\vdots\\
&\leqslant e^{-2\delta t}V_j(0).
\end{aligned}$$

Hence, it can be concluded from the above inequality and (13) that

$$|\varepsilon_j(t)|\leqslant\sqrt{\frac{V_j(0)}{\epsilon_j}}e^{-\delta t},$$

which means that the estimation error system (7) and closed-loop NISs (9) are ES with a decay rate δ under the situation where $\zeta_j(t)=0$.

Next, we will discuss the $\mathcal{H}_\infty$ performance of the estimation error system (7) and closed-loop NISs (9). Based on the inequality (30) and two proven parts, $V_j(t)>0$ and $\delta>0$, we can get the following conclusion:

$$\dot{V}_j(t)+Z_j^T(t)Z_j(t)-\gamma^2\zeta_j^T(t)\zeta_j(t)\leqslant 0. \tag{32}$$

For a given $\hat{Q}\gg 1$, integrating (32) from 0 to $t_{\hat{Q}}$ yields that

$$V_j(t_{\hat{Q}})-V_j(t_{\hat{Q}-1})+V_j(t^-_{\hat{Q}-1})-...-V_j(0)+\int_0^{t_{\hat{Q}}}\left(Z_j^T(s)Z_j(s)-\gamma^2\zeta_j^T(s)\zeta_j(s)\right)ds\leqslant 0.$$

Since $V_j(t_{\hat{Q}})\geqslant 0, V_j(0)=0$ and $V_j(t^-_{K-1})-V_j(t_{K-1})=0$, $K=2,3,...,\hat{Q}$, so we can get

$$\int_0^{t_{\hat{Q}}}Z_j^T(s)Z_j(s)\,ds\leqslant\gamma^2\int_0^{t_{\hat{Q}}}\zeta_j^T(s)\zeta_j(s)\,ds.$$

When $t_{\hat{Q}}\to\infty$, it follows that

$$\int_0^{\infty}Z_j^T(s)Z_j(s)\,ds\leqslant\gamma^2\int_0^{\infty}\zeta_j^T(s)\zeta_j(s)\,ds.$$

Therefore, we have

$$\sum_{j=1}^{J}\int_0^{\infty}Z_j^T(s)Z_j(s)\,ds\leqslant\gamma^2\sum_{j=1}^{J}\int_0^{\infty}\zeta_j^T(s)\zeta_j(s)\,ds,$$

which means that the estimation error system (7) and closed-loop NISs (9) have an $\mathcal{H}_\infty$ performance level γ. ■

B. Controller Synthesis

Theorem 2. For given scalars $\delta>0$, $\gamma>0$, $\varpi>0$, the estimation error system (7) and closed-loop NIS (9) are ES with an $\mathcal{H}_\infty$ performance index γ if there exist scalar $\sigma>0$, and matrices $P_{\varsigma j}>0$ with $\varsigma\in\ell_3$, X_j, Y_j, M_{1j}, M_{2j}, T_j, R_{1j}, R_{2j}, S_j^{ϱ}, V_j^l such that the following inequalities holds:

$$\varPsi_j=\begin{bmatrix}\bar{P}_{1j}+hHe\{\frac{\bar{X}_j}{2}\} & h(-\bar{X}_j+\bar{Y}_j)\\ * & hHe\{-\bar{Y}_j+\frac{\bar{X}_j}{2}\}\end{bmatrix}>0, \tag{33}$$

$$\varPsi_{0j}^{l\varrho}=\begin{bmatrix}\Xi_{01j}^{l\varrho} & \Xi_{02j}^{l} & \Xi_{03j}^{l}\\ * & \Xi_{04j} & \Xi_{05j}\\ * & * & \Xi_{06j}\end{bmatrix}<0, \tag{34}$$

$$\varPsi_{1j}^{l\varrho}=\begin{bmatrix}\Xi_{11j}^{l\varrho} & \Xi_{02j}^{l} & \Xi_{03j}^{l}\\ * & \Xi_{04j} & \Xi_{05j}\\ * & * & \Xi_{06j}\end{bmatrix}<0, \tag{35}$$

$$\varPsi_{2j}^{l\varrho}=\begin{bmatrix}\Xi_{01j}^{l\varrho} & \Xi_{21j} & \Xi_{02j}^{l} & \Xi_{03j}^{l}\\ * & \Xi_{22j} & \Xi_{23j} & \Xi_{24j}\\ * & * & \Xi_{04j} & \Xi_{05j}\\ * & * & * & \Xi_{06j}\end{bmatrix}<0, \tag{36}$$

where

$$\Xi_{01j}^{l\varrho}=\begin{bmatrix}\Phi_{11j}^{l} & \Phi_{12j}^{l} & \Phi_{13j}^{l} & \Phi_{14j}^{l\varrho}\\ * & \Phi_{22j} & \Phi_{23j}^{l} & \Phi_{24j}^{l\varrho}\\ * & * & \Phi_{33j}^{l} & 0\\ * & * & * & \Phi_{44j}\end{bmatrix},$$

$$\Xi_{02j}^{l}=\begin{bmatrix}\Phi_{16j}^{l} & \Phi_{17j}\\ \Phi_{26j}^{l} & 0\\ \Phi_{36j}^{l} & 0\\ 0 & 0\end{bmatrix},$$

$$\Xi_{03j}^{l}=\begin{bmatrix}D_{j1}^{l}\bar{R}_{1j} & 0 & 0 & 0\\ D_{j2}^{l}\bar{R}_{1j} & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots\\ D_{jJ}^{l}\bar{R}_{1j} & 0 & 0 & 0\end{bmatrix}^T_{(J-1)\times 4},$$

$$\Xi_{04j}=\begin{bmatrix}-\gamma^2 I & 0\\ * & -I\end{bmatrix},$$

$$\Xi_{05j}=\begin{bmatrix}0 & 0 & \dots & 0\\ 0 & 0 & \dots & 0\end{bmatrix}_{2\times(J-1)},$$

$$\Xi_{06j}=diag\{-\frac{\sigma}{2(J-1)}I,\dots,-\frac{\sigma}{2(J-1)}I\}_{(J-1)\times(J-1)},$$

$$\Xi_{11j}^{l\varrho}=\begin{bmatrix}\hat{\Phi}_{11j}^{l} & \hat{\Phi}_{12j}^{l} & \Phi_{13j}^{l} & \hat{\Phi}_{14j}^{l\varrho}\\ * & \hat{\Phi}_{22j} & \Phi_{23j}^{l} & \hat{\Phi}_{24j}^{l\varrho}\\ * & * & \Phi_{33j}^{l} & 0\\ * & * & * & \hat{\Phi}_{44j}\end{bmatrix},$$

$$\Xi_{21j}=\begin{bmatrix}\Phi_{15j}\\ \Phi_{25j}\\ 0\\ \Phi_{45j}\end{bmatrix},\Xi_{22j}=\Phi_{55j},\Xi_{23j}=\begin{bmatrix}0 & 0\end{bmatrix},$$

$$\Xi_{24j}=\begin{bmatrix}0 & 0 & \dots & 0\end{bmatrix}_{1\times(J-1)},$$

$$\begin{aligned}
\bar{R}_{1j}&=R_{1j}^{-1},\Phi_{11j}^{l}=2\delta\bar{P}_{1j}+He\{-\frac{\bar{X}_j}{2}-\bar{M}_{1j}^T\\
&\quad+\bar{R}_{1j}^T(A_j^l)^T\}+\sigma I,\Phi_{12j}^{l}=\bar{P}_{1j}-\bar{M}_{2j}-\bar{R}_{1j}\\
&\quad+\varpi\bar{R}_{1j}^T(A_j^l)^T,\\
\Phi_{13j}^{l}&=B_j^lE_j^l+\bar{R}_{1j}^T(G_{1j}^l)^TG_{2j}^l,\\
\Phi_{14j}^{l\varrho}&=\bar{X}_j-\bar{Y}_j+\bar{M}_{1j}^T-\bar{T}_j+B_j^lS_j^{\varrho},\\
\Phi_{15j}&=h\bar{M}_{1j}^T,\Phi_{16j}^{l}=C_j^l,\Phi_{17j}=\bar{R}_{1j}^T(G_{1j}^l)^T,\\
\Phi_{22j}&=\varpi(-He\{\bar{R}_{1j}\}+\sigma\varpi I),\Phi_{23j}^{l}=\varpi B_j^lE_j^l,\\
\Phi_{24j}^{l\varrho}&=\bar{M}_{2j}^T+\varpi B_j^lS_j^{\varrho},\Phi_{25j}=h\bar{M}_{2j}^T,\Phi_{26j}^{l}=\varpi C_j^l,\\
\Phi_{33j}^{l}&=He\{P_{2j}F_j^l+V_j^lB_j^lE_j^l\}+2\delta P_{2j}+(G_{2j}^l)^TG_{2j}^l,\\
\Phi_{36j}^{l}&=V_j^lC_j^l,\Phi_{44j}=He\{\bar{Y}_j-\frac{\bar{X}_j}{2}+\bar{T}_j\},\\
\Phi_{45j}&=h\bar{T}_j^T,\Phi_{55j}=-he^{-2\delta h}\bar{P}_{3j},\\
\hat{\Phi}_{11j}^{l}&=2\delta\bar{P}_{1j}+He\{(h\delta-\frac{1}{2})\bar{X}_j-\bar{M}_{1j}^T+\bar{R}_{1j}^T(A_j^l)^T\}\\
&\quad+\sigma I,\\
\hat{\Phi}_{12j}^{l}&=\bar{P}_{1j}-\bar{M}_{2j}-\bar{R}_{1j}+\varpi\bar{R}_{1j}^T(A_j^l)^T+hHe\{\frac{\bar{X}_j}{2}\},\\
\hat{\Phi}_{14j}^{l\varrho}&=\bar{X}_j-\bar{Y}_j+\bar{M}_{1j}^T-\bar{T}_j+B_j^lS_j^{\varrho}+2h\delta(-\bar{X}_j+\bar{Y}_j),\\
\hat{\Phi}_{22j}&=-\varpi He\{\bar{R}_{1j}\}+\sigma\varpi^2 I+h\bar{P}_{3j},\\
\hat{\Phi}_{24j}^{l\varrho}&=\bar{M}_{2j}^T+\varpi B_j^lS_j^{\varrho}+h(-\bar{X}_j+\bar{Y}_j),\\
\hat{\Phi}_{44j}&=He\{\bar{Y}_j-\frac{\bar{X}_j}{2}+\bar{T}_j\}+2\delta hHe\{-\bar{Y}_j+\frac{\bar{X}_j}{2}\}.
\end{aligned}$$

Therefore, the DO gain and controller gain are obtained by

$$L_j^l=P_{2j}^{-1}V_j^l,K_j^{\varrho}=S_j^{\varrho}\bar{R}_{1j}^{-1}. \tag{37}$$

Proof: Denote

$$R_{2j}=\varpi R_{1j},$$

and block diagonal matrices

$$X_o = diag\{R_{1j} \quad R_{1j}\},$$
$$X_a = diag\{R_{1j} \quad R_{1j} \quad I \quad R_{1j} \quad I\},$$
$$X_b = diag\{R_{1j} \quad R_{1j} \quad I \quad R_{1j} \quad R_{1j} \quad I\}.$$

According to Lemma 3, (34), (35), and (36) are respectively equivalent to

$$\hat{\Psi}^{l\varrho}_{0j} = \begin{bmatrix} \bar{\Phi}^l_{11j} & \Phi^l_{12j} & \Phi^l_{13j} & \Phi^{l\varrho}_{14j} & \Phi^l_{16j} \\ * & \Phi_{22j} & \Phi^l_{23j} & \Phi^{l\varrho}_{24j} & \Phi^l_{26j} \\ * & * & \Phi^l_{33j} & 0 & \Phi^l_{36j} \\ * & * & * & \Phi_{44j} & 0 \\ * & * & * & * & \Phi_{66j} \end{bmatrix} < 0, \quad (38)$$

$$\hat{\Psi}^{l\varrho}_{1j} = \begin{bmatrix} \breve{\Phi}^l_{11j} & \hat{\Phi}^l_{12j} & \Phi^l_{13j} & \hat{\Phi}^{l\varrho}_{14j} & \Phi^l_{16j} \\ * & \hat{\Phi}_{22j} & \Phi^l_{23j} & \hat{\Phi}^{l\varrho}_{24j} & \Phi^l_{26j} \\ * & * & \hat{\Phi}^l_{33j} & 0 & \Phi^l_{36j} \\ * & * & * & \hat{\Phi}_{44j} & 0 \\ * & * & * & * & \Phi_{66j} \end{bmatrix} < 0, \quad (39)$$

$$\hat{\Psi}^{l\varrho}_{2j} = \begin{bmatrix} \vec{\Xi}^{l\varrho}_{21j} & \vec{\Xi}^l_{22j} \\ * & \vec{\Xi}_{23j} \end{bmatrix} < 0, \quad (40)$$

where

$$\vec{\Xi}^{l\varrho}_{21j} = \begin{bmatrix} \bar{\Phi}^l_{11j} & \Phi^l_{12j} & \Phi^l_{13j} & \Phi^{l\varrho}_{14j} \\ * & \Phi_{22j} & \Phi^l_{23j} & \Phi^{l\varrho}_{24j} \\ * & * & \Phi^l_{33j} & 0 \\ * & * & * & \Phi_{44j} \end{bmatrix},$$

$$\vec{\Xi}^l_{22j} = \begin{bmatrix} \Phi_{15j} & \Phi^l_{16j} \\ \Phi_{25j} & \Phi^l_{26j} \\ 0 & \Phi^l_{36j} \\ \Phi_{45j} & 0 \end{bmatrix},$$

$$\vec{\Xi}_{23j} = \begin{bmatrix} \Phi_{55j} & 0 \\ * & \Phi_{66j} \end{bmatrix},$$

$$\begin{aligned} \bar{\Phi}^l_{11j} = & 2\delta\bar{P}_{1j} + He\{-\frac{\bar{X}_j}{2} - \bar{M}^T_{1j} + \bar{R}^T_{1j}(A^l_j)^T\} + \sigma I \\ & + 2\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J} \bar{R}^T_{1j}(D^l_{jn})^T(D^l_{jn})\bar{R}_{1j} \\ & + \bar{R}^T_{1j}(G^l_{1j})^T(G^l_{1j})\bar{R}_{1j}, \\ \breve{\Phi}^l_{11j} = & 2\delta\bar{P}_{1j} + He\{(h\delta - \frac{1}{2})\bar{X}_j - \bar{M}^T_{1j} + \bar{R}^T_{1j}(A^l_j)^T\} \\ & + 2\sigma^{-1}(J-1)\sum_{n=1,n\neq j}^{J} \bar{R}^T_{1j}(D^l_{jn})^T(D^l_{jn})\bar{R}_{1j} \\ & + \bar{R}^T_{1j}(G^l_{1j})^T(G^l_{1j})\bar{R}_{1j} + \sigma I. \end{aligned}$$

Pre-multiply and post-multiply (33) by X_o^T and X_o, (38) and (39) by X_a^T and X_a, (40) by X_b^T and X_b, respectively, and together with the change of matrix variables defined by

$$\begin{aligned} & X_j = R^T_{1j}\bar{X}_jR_{1j}, P_{1j} = R^T_{1j}\bar{P}_{1j}R_{1j}, M_{1j} = R^T_{1j}\bar{M}_{1j}R_{1j}, \\ & M_{2j} = R^T_{1j}\bar{M}_{2j}R_{1j}, Y_j = R^T_{1j}\bar{Y}_jR_{1j}, T_j = R^T_{1j}\bar{T}_jR_{1j}, \\ & V^l_j = P_{2j}L^l_j, S^\varrho_j = K^\varrho_jR^{-1}_{1j}. \end{aligned}$$

We can obtain (12), (14), (15), and (16) in Theorem 1. Thus, the proof is complete. ∎

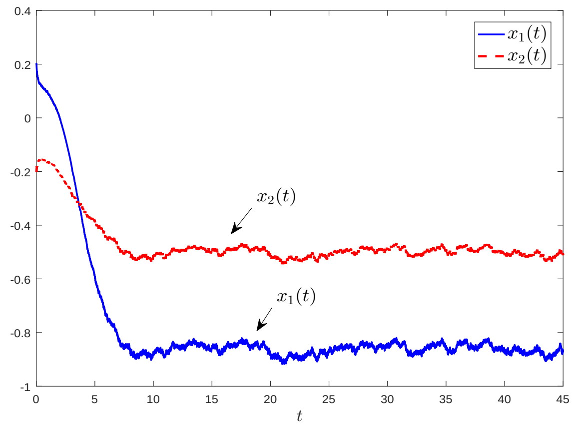

Fig. 2. State trajectories of subsystems $x_1(t)$ and $x_2(t)$ without disturbance and controller.

IV. NUMERICAL SIMULATION

In this section, we provide an example to demonstrate the effectiveness of the proposed composite controller. Consider the NIS (2) with the following parameter values:

$$\begin{aligned} \dot{x}_j(t) = & \sum_{l=1}^{2} g^l_j(\upsilon_j(t))\{A^l_jx_j(t) + B^l_j(u_j(t) + d_j(t)) \\ & + C^l_j\zeta_j(t) + \sum_{n=1,n\neq j}^{J} D^l_{nj}x_n(t)\}, \end{aligned}$$

where $x_j(t) = col\{x_{j_1}(t), x_{j_2}(t)\}$. $\dot{x}_{j_1}(t) = x_{j_2}(t)$, $\upsilon_j(t) = (x_{j_1}(t))^2$. $g^1_j(\upsilon_j(t)) = 1 - \upsilon_j(t)$, $g^2_j(\upsilon_j(t)) = 1 - g^1_j(\upsilon_j(t))$, $j, l, \varrho \in \ell_2$, and

$$A^1_1 = \begin{bmatrix} -3 & 8 \\ 2 & -5 \end{bmatrix}, A^2_1 = \begin{bmatrix} -4 & 5 \\ 3 & -3 \end{bmatrix},$$
$$A^1_2 = \begin{bmatrix} -2.25 & 6 \\ 1.5 & -3.75 \end{bmatrix}, A^2_2 = \begin{bmatrix} -3.75 & 4.5 \\ 1.5 & -1.5 \end{bmatrix},$$
$$B^1_1 = B^2_1 = B^1_2 = B^2_2 = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 0.5 \end{bmatrix},$$
$$C^1_1 = C^2_1 = C^1_2 = C^2_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$D^1_{11} = \begin{bmatrix} 1.31 & -1 \\ -1 & 2.2 \end{bmatrix}, D^2_{11} = \begin{bmatrix} 0.22 & -2 \\ -1 & 0.1 \end{bmatrix},$$
$$D^1_{21} = \begin{bmatrix} 1.1 & -1 \\ -1 & 3.1 \end{bmatrix}, D^2_{21} = \begin{bmatrix} 0.51 & -2 \\ -1 & 0.2 \end{bmatrix},$$
$$F^1_1 = F^2_1 = F^1_2 = F^2_2 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix},$$
$$\zeta_1(t) = 9e^{-0.15t}sin(0.3t), \zeta_2(t) = 15e^{-0.15t}cos(0.5t).$$

In addition to the above parameters,

$$E^l_j = G^l_{1j} = G^l_{2j} = I_{2\times 2}.$$

In numerical simulation, we assume $\varpi = 0.15$, $h = 0.1$, $\delta = 0.3$, $\gamma = 2$. By solving (12) and (34), (35) and (36) in Theorem 2, the following controller gain and DO gain can be obtained

$$K^1_1 = \begin{bmatrix} -3.2065 & -4.1855 \\ -2.5877 & -7.0272 \end{bmatrix},$$
$$K^2_1 = \begin{bmatrix} -3.0342 & -3.9412 \\ -2.3111 & -6.8979 \end{bmatrix},$$

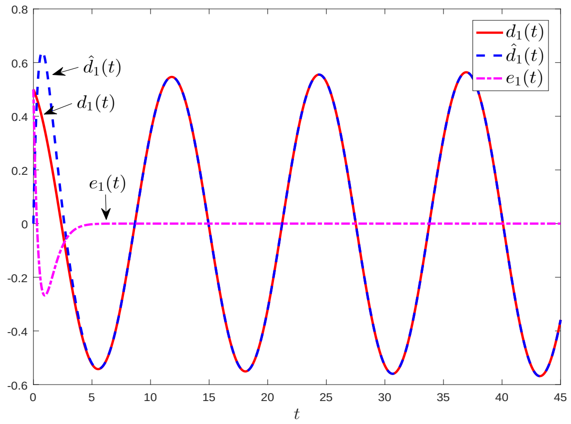

Fig. 3. State trajectories of the exogenous disturbance subsystem $d_1(t)$, the DO subsystem $\hat{d}_1(t)$, and the disturbance estimation error subsystem $e_1(t)$.

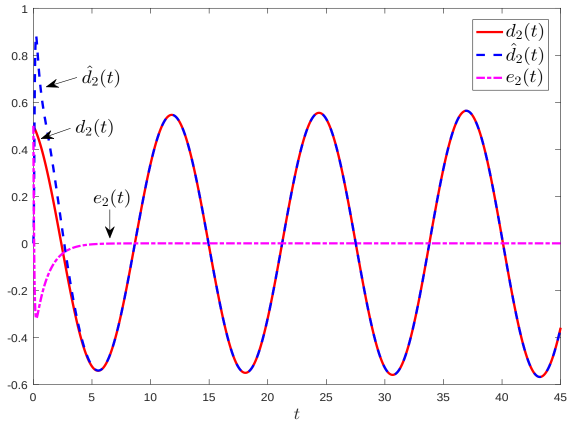

Fig. 4. State trajectories of the exogenous disturbance subsystem $d_2(t)$, the DO subsystem $\hat{d}_2(t)$, and the disturbance estimation error subsystem $e_2(t)$.

$$K_2^1 = \begin{bmatrix} -3.8582 & -2.4496 \\ 0.1524 & -10.6619 \end{bmatrix},$$

$$K_2^2 = \begin{bmatrix} -3.8582 & -2.4496 \\ 0.1524 & -10.6619 \end{bmatrix},$$

$$L_1^1 = \begin{bmatrix} -1.3588 & 2.5824 \\ 1.1943 & -5.0900 \end{bmatrix},$$

$$L_1^2 = \begin{bmatrix} -1.1716 & 2.6936 \\ 1.1094 & -5.4012 \end{bmatrix},$$

$$L_2^1 = \begin{bmatrix} -1.3723 & 2.5838 \\ 1.2269 & -5.1299 \end{bmatrix},$$

$$L_2^2 = \begin{bmatrix} -1.1581 & 2.7203 \\ 1.1209 & -5.4423 \end{bmatrix}.$$

Fig. 2 illustrates the subsystem state of the NIS in the absence of disturbances and a controller. As shown, the system state fails to achieve stabilization in the absence of a controller. Figs. 3 and 4 depict the behavior of the estimation error $e_j(t)$, which gradually converges to zero. This indicates that the estimated disturbance value becomes asymptotically close to the actual disturbance. Figs. 3 and 4 reflect the effectiveness of the DO. Fig. 5 illustrates the state trajectories of the subsystems $x_1(t)$ and $x_2(t)$, under the influence of a controller without an integrated disturbance observer. Compared to the state trajectory shown in Fig. 2, the unstable state of the subsystem in Fig. 5 is significantly

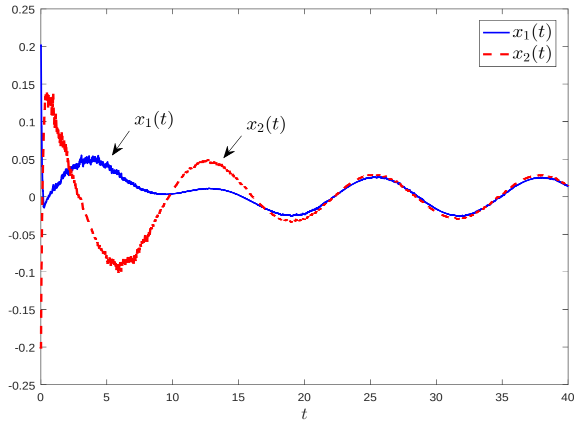

Fig. 5. Without the disturbance observer, the controller controls the state trajectory of the subsystems $x_1(t)$ and $x_2(t)$.

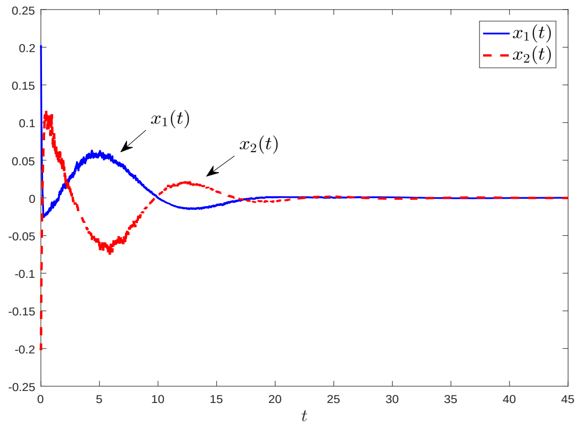

Fig. 6. State trajectories of subsystems $x_1(t)$ and $x_2(t)$ after adding the controller.

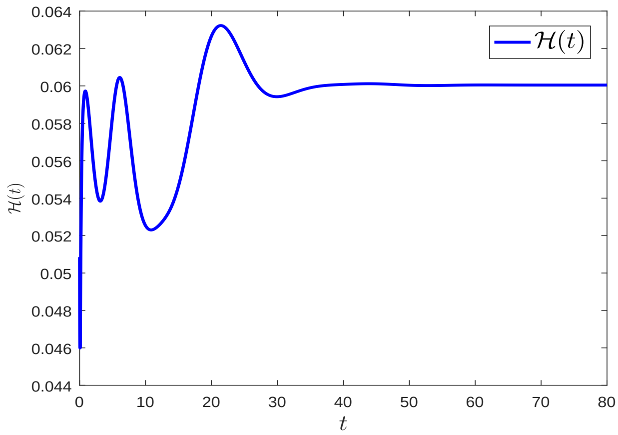

Fig. 7. The trajectory of $\mathcal{H}(t)$.

mitigated. However, disturbance observer, the states $x_1(t)$ and $x_2(t)$ continue to oscillate around zero. By comparing the states of the exogenous disturbance system in Figs. 3 and 4, it is obvious that Fig. 5 cannot suppress the exogenous disturbance. This is because the controller lacks a disturbance observer. Fig. 6 is a subsystem state diagram after adding disturbance and composite controller. The subsystem state gradually tends to zero, reflecting the composite controller's effectiveness.

Motivated by [38], we define

$$\mathcal{H}(t) = \sqrt{\frac{\sum_{j=1}^{2} \int_0^\infty Z_j^T(s) Z_j(s)\, ds}{\sum_{j=1}^{2} \int_0^\infty \zeta_j^T(s) \zeta_j(s)\, ds}}. \tag{41}$$

Equation (41) illustrates the anti-disturbance performance of $\mathcal{H}_\infty$. According to Fig. 7, we can see that as time goes by, $\mathcal{H}(t)$ finally stabilizes at γ^*=0.0599 and is much smaller than γ_{min}=0.1442. This outcome demonstrates the efficacy of the proposed composite controller design based on DO.

V. CONCLUSION

In this paper, the composite hierarchical anti-disturbance fuzzy control of NISs with multiple disturbance was addressed based on the DO. A fuzzy DO (5) was designed to estimate the unknown disturbance generated by an exogenous system. Subsequently, a composite disturbance-observer-based control scheme (6) was developed by integrating a feedforward compensation term based on the fuzzy DO with a state-feedback sampled-data control law, while $\mathcal{H}_\infty$ control was adopted to attenuate another norm-bounded disturbance. By building a time-dependent function $V(t)$, a criterion was derived in Theorem 1 to ensure the exponential stability of the estimation error system and closed-loop NISs with a prescribed $\mathcal{H}_\infty$ performance level. The joint design of the desired DO and composite disturbance-observer-based fuzzy controller was proposed in Theorem 2. Finally, a numerical simulation was used to validate the presented composite hierarchical fuzzy control scheme. As a higher number of fuzzy rules increases the computational burden and reduces the real-time performance of the system, we will optimize the number of fuzzy rules in future work.

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