A SMNIR Cyber Rumour Propagation Prediction Model Based on Evolutionary Game

Qiujuan Tong, Shengqi Yue, Jianke Zhang, and Luwen Wang

Abstract—**In the current online environment, the prevalence of rumours can cause significant damage to the stability and clarity of the platform. Consequently, it has become a top priority to explore the relevant factors and take effective measures. This paper considers the influence of marketing accounts and netizens in the context of rumour spreading. A two-party evolutionary game model of marketing accounts-netizens is constructed in order to identify the equilibrium point and to discuss the related stability. This model is combined with the infectious disease model to construct the SMNIR model. The analysis of the impact of relevant parameters on the evolutionarily stable state is conducted, and the rationality of the model is verified through examples, which shows good agreement with actual data and a small deviation.**

Index Terms—**evolutionary game; equilibrium point; stable strategy; rumour prediction**

I. INTRODUCTION

IN our current digital age, the flow of information on social media platforms has liberated users from the limitations of time and space, providing them with a boundless information landscape. The surge in internet users and the expanding reach of the internet have compounded the efforts required to safeguard the online environment. Marketing accounts and netizens are pivotal players in shaping online discourse. Marketing accounts, with their focus on promoting various offerings and ideologies, employ strategic content to swiftly and extensively reach a wide audience. Meanwhile, netizens contribute to a vibrant online culture by sharing their perspectives and engaging in dynamic exchanges through social media platforms.

In the digital realm, where the veracity of information is frequently uncertain [1], entities such as marketing accounts and netizens may unknowingly propagate misinformation. Rumours in the online sphere [2] are defined as falsehoods disseminated via the internet that lack a factual foundation and are typically malicious and targeted. Crafting a robust model to decipher the interplay between these two groups in rumour propagation is both theoretically compelling and practically significant.

Manuscript received October 5, 2024; revised January 23, 2025. This research was funded by the Key R & D Program Project of Shanxi Province, China (Grant No. 2024NC-YBXM-217), the Science and Technology Plan Project of Xi'an, China (Grant No. 22GXFW0124) and Xi'an University of Posts and Telecommunications Graduate Student Innovation Fund (Grant No. CXJJYL2024010).

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The body of research that specifically examines the dynamics between marketing accounts and netizens in the context of online rumor propagation is quite limited. Traditional approaches often rely on game theory and infectious disease models to explore the interactions between these two entities. In contrast, this study introduces a tailored evolutionary game model that reflects the relationship between marketing accounts and netizens, aligning with and complementing the epidemic model. Through simulation experiments and empirical verifications, it assesses the influence of various factors on system stability and provides insights into the management of online rumour dissemination.

The layout of this paper unfolds in the following manner: Section II is a literature review. Section III delves into the formulation of the two-party game model. Subsequently, Section IV showcases the simulation and analytical findings pertaining to the model. Section V offers an exposition of the SMNIR model. Conclusively, Section VI encapsulates our final reflections and conclusions.

II. LITERATURE REVIEW

A historical overview of the study of online public opinion evolution shows that the initial SIR model [3], an infectious disease framework, was later complemented by the innovative DK model [4] in the 1960s. This model highlighted the analogous patterns between the spread of online discourse and that of infectious diseases. Building upon this, Maki and Thomson provided a detailed mathematical dissection of the DK model, culminating in the introduction of the MT model [5]. Since then, the contagion model has become a cornerstone in the analysis of online opinion trends. Academics have further enhanced the SIR model by incorporating a variety of contexts, reflecting the intricate nature of information spread. Examples include the SIHR model, which accounts for the dynamics of forgetting and recalling [6], and the microblogging opinion evolution model, which explores the nuances of anti-following behaviors [7], the role of opinion leaders [8], and additional significant elements.

The spread of online public opinion is a complex interplay involving numerous parties and their interests. Public opinion, including rumors, is inherently connected to these stakeholders. In this context, evolutionary game theory [9], [10] has become a pivotal area of research within the domain of online public sentiment. The most frequently utilized models are those involving two or three interacting parties. Li et al. [11] and Xiao et al. [12], in their individual studies, dissected the factors influencing online opinion and the processes through which it spreads, using evolutionary game theory as a framework. Xie et al. [13] created a three-party game model to examine the dynamics between medical professionals, government entities, and the general public, focusing on the key drivers of participant behavior. Yang et al. [14] proposed a model for "situational response" based on evolutionary game theory, exemplified by the "8*12 Tianjin Port Explosion," to study the dynamics of public sentiment and the mechanisms of online opinion diffusion. Jiang et al. [15] simulated interactions between the government and netizens in the realm of public opinion and information exchange, while Wen et al. [16], focusing on the educational sector, established a three-way game model among media, students, and university administrators.

Inspired by the analytical depth of game theory, researchers have proposed diverse frameworks and models for further exploration. Hou et al. [17] merged the concepts of evolutionary game theory with complex network analysis to create a model for the networked spread of competing narratives in public opinion. Song et al. [18] utilized complex network analysis to scrutinize how the choice of initial dissemination nodes affects the propagation of public opinion and the evolutionary game-theoretic basis for individual decisions to engage in dissemination. Chen et al. [19] crafted a stochastic three-party evolutionary game model for the "media-netizen-government" system during high-profile communication events, integrating a comprehensive set of parameters reflecting government actions. Zhang et al. [20] enhanced the "media-government" and "netizen-government" models by embedding the fair utility function. Wang et al. [21] proposed a nuanced three-party evolutionary game model that encompasses both affirmative and negative public sentiments, concentrating on the equilibrium of strategic behaviors among stakeholders.

Evolutionary game theory provides a lens through which to analyze rational decision-making in scenarios characterized by limited information. Players in these games continually adapt their strategies in pursuit of maximizing benefits. The core tenets of this theory encompass the ideas of stable evolutionary strategies and replication processes [22]. In the contemporary online sphere, various groups, driven by commercial objectives, engage in the release and spread of information that shapes public opinion. Marketing influencers exemplify this, using their large audiences to amplify their impact. Jia et al. [23] incorporated marketing influencers into an evolutionary game model to examine the dynamics of strategic choices and interactions. The propagation of online rumors is often the result of strategic interest games played by involved parties.

The main innovations of this paper are as follows:

1) A two-party evolutionary game model of marketing accounts and netizens is constructed to analyse the effects of good vibe plus and additional revenue on the decision-making of marketing accounts and netizens in spreading rumours.
2) The SMNIR model was proposed by combining the epidemic model with evolutionary game theory.

III. A Two-party Game Model

A. Setting of Game Subjects

The model is based on the following assumptions:

- The process of online public opinion dissemination is inherently connected to the interest-driven needs of specific individuals and the costs incurred in spreading information. Rumor creation is also influenced by these dynamics. In this study, we concentrate on netizens and marketing accounts as our subjects of interest, with all game participants demonstrating a level of rationality that is finite in nature. Our aim is to explore the stability of the game participants over time and to analyze how shifts in pertinent parameters can affect their strategic choices.
- The two subjects of the game, netizen and marketing account, are denoted by N and M respectively. Netizens are generally sensitive to information, and they may spread rumors under the influence of mixed online public opinion and external factors. Two choices of netizens: to spread or not to spread. The strategy of disseminating rumours is designated as SR, while the strategy of no spreading of rumours is designated as SP. The collective strategy can be represented by $Ns = \{SR, SP\}$. In order to gain insight into social benefits and traffic factors, marketing numbers frequently publish audio related to public opinion. However, the veracity of the information contained within these audio recordings is uncertain, which has led to the publication of rumours by marketing accounts. Similarly, marketing account employ the same two strategies as netizen, which can be defined as follows: $Ms = \{SR, SP\}$.
- The probability of an Internet user selecting strategy SR is denoted by x, the probability of selecting strategy SP is $1-x$, and the probability of a marketing account choosing strategy SR is y, the probability of choosing strategy SP is $1-y$. Both x and y satisfy the following conditions: $0 \leq x \leq 1, 0 \leq y \leq 1$.

B. Definition of Game Subjects Gains and Losses and Related Parameters

In order to present the benefit matrix for each participant in the game, an extensive examination of pertinent scholarly works was undertaken. This process unveiled the respective advantages and disadvantages of each participant, as well as the pertinent parameters for their settings. Further elucidation of these findings is presented in the subsequent sections.

1) Netizen

The primary drivers of individual decision-making behaviour are psychological satisfaction or material benefits [21]. For netizens, the benefits of participating in online opinion communication are reflected in the satisfaction of personal social belonging. The satisfaction of emotional values such as curiosity through the communication and discussion of public opinion is an important motivation for netizens to participate in the process of public opinion communication. It can be observed that rumors are discussed more frequently than facts, and that distorted facts tend to attract more attention. Therefore, the gains from netizens choosing to spread rumours and not spreading rumours (normal participation in public opinion dissemination) can be defined as R_{11} and R_{12} respectively. Irrespective of the chosen strategy, the process of exploring, paying attention to and disseminating the rumor inevitably requires a certain amount of time and energy. The cost of energy and time for SR is C_1, while the cost of energy and time for SP is C_2.

As the saying goes, "Rumour-mongering is a mouthful, but rumour-fighting is a legful." This implies that the cost of rumour-mongering is very low, so $C_1 < C_2$. Furthermore, when netizens select the strategy SR, they will also be subject to monitoring by the platform, which may result in the blocking or deletion of comments. The penalty is denoted as αP_1, the parameter α $(0 \le \alpha \le 1)$ for the coefficient of regulation of Internet users by the platform.

2) Marketing Account

The primary objective of marketing accounts has historically been to generate revenue through the creation and dissemination of opinion topics. This is achieved by leveraging the power of rumors, which are more likely to attract attention and gain advertising revenue and other commercial interests. The circulation of rumours is often more effective than the dissemination of facts, and marketing accounts may potentially become enablers of rumors. The benefits of posting a rumour and not posting a rumour (normal publication of public opinion information) by a marketing account are R_{21} and R_{22} respectively $(R_{21} > R_{22})$, the corresponding costs are as follows: C_3 and C_4 $(C_3 < C_4)$. In addition, when marketing accounts choose strategy SR, they will also be penalized by the platform, which may result in being required to make rectifications or have their accounts suspended. This penalty is recorded as βP_3. β is the regulatory coefficient of the platform for marketing account.

3) Interaction Between Netizen and Marketing Account

Netizens and marketing accounts will select their respective game strategies. When both choose the strategy SR, this will result in the prevalence of rumours and other undesirable information. At this time, netizens will be subjected to additional losses P_2 caused by the poor platform atmosphere, while the marketing accounts will benefit from the rumors, increasing traffic and revenue A_2. When netizen selects the strategy SP, the marketing account is attributable to the loss of attention and the loss of comments resulting from the decline of ΔC. When both select the strategy SP, the netizen is subject to the positive atmosphere plus A_1.

4) The Pay-off Matrix of Game Model

Based on the above parameter settings, the two-party game payoff matrix of netizen, marketing account can be obtained.

TABLE I
PAY-OFF MATRIX OF MARKETING ACCOUNT AND NETIZEN.

S(GS)	N	M
$N(SR), M(SR)$	$R_{11} - C_1 - \alpha P_1 - P_2$	$R_{21} - C_3 - \beta P_3 + A_2$
$N(SP), M(SR)$	$R_{12} - C_2$	$R_{21} - C_3 - \beta P_3 - \Delta C$
$N(SR), M(SP)$	$R_{11} - C_1 - \alpha P_1$	$R_{22} - C_4$
$N(SP), M(SP)$	$R_{12} - C_2 + A_1$	$R_{22} - C_4 - \Delta C$

C. Expectation of Game Strategy and Proof of Stability

1) The Strategic Expectation of the two-party and Replicated Dynamic Equation

In conjunction with the benefit matrix presented in Table I, the anticipated benefits for both parties can be quantified.

$\overline{E}$ denotes the average return of Netizens, E_1 represents the expected return of Internet users choosing strategy SR, E_2 represents the expected return of Internet users choosing strategy SP.

$$\begin{aligned} E_1 &= y\left[R_{11} - C_1 - \alpha P_1 - P_2\right] + (1-y)\left[R_{11} - C_1 - \alpha P_1\right] \\ &= R_{11} - C_1 - \alpha P_1 - yP_2 \\ E_2 &= y\left[R_{12} - C_2\right] + (1-y)\left[R_{12} - C_2 + A_1\right] \\ &= R_{12} - C_2 + (1-y)A_1 \\ \overline{E} &= xE_1 + (1-x)E_2 \end{aligned}$$

The equation for the replication dynamics of Netizen can be expressed as follows:

$$\begin{aligned} f(x) &= \frac{dx}{dt} = x\left[E_1 - \overline{E}\right] \\ &= x(1-x)(E_1 - E_2) \\ &= x(1-x)(R_{11} - C_1 - \alpha P_1 - yP_2 - R_{12} \\ &\quad + C_2 - (1-y)A_1) \end{aligned} \tag{1}$$

$\overline{G}$ denotes the average revenue of marketing accounts. G_1 denotes the expected revenue of marketing accounts that have chosen strategy SR, and G_2 denotes the expected revenue of marketing accounts who have chosen strategy SP.

$$\begin{aligned} G_1 &= x\left[R_{21} - C_3 - \beta P_3 + A_2\right] + (1-x)\left[R_{21} - C_3 \right. \\ &\quad \left. - \beta P_3 - \Delta C\right] \\ &= R_{21} - C_3 - \beta P_3 - \Delta C + x(\Delta C + A_2) \\ G_2 &= x\left[R_{22} - C_4\right] + (1-x)\left[R_{22} - C_4 - \Delta C\right] \\ &= R_{22} - C_4 - (1-x)\Delta C \\ \overline{G} &= yG_1 + (1-y)G_2 \end{aligned}$$

The equation for the replication dynamics of the marketing account can be expressed as:

$$\begin{aligned} f(y) &= \frac{dy}{dt} = y\left[G_1 - \overline{G}\right] \\ &= y(1-y)(G_1 - G_2) \\ &= y(1-y)(R_{21} - C_3 - \beta P_3 + xA_2 \\ &\quad - R_{22} + C_4) \end{aligned} \tag{2}$$

2) Stability Analysis of Evolutionary Strategy

Let $f(x) = 0$, we can obtain $x_1 = 0, x_2 = 1$, and $y^* = \frac{R_{11} - R_{12} - C_1 + C_2 - \alpha P_1 - A_1}{P_2 - A_1}$. In accordance with the stability theorem of reproducible dynamic differential equation and evolutionary stability strategy, If $f(x) = 0$ and $\frac{\partial f(x)}{\partial x}|_x < 0$, then at this point x is evolutionary stability strategy.

If $y^* = \frac{R_{11} - R_{12} - C_1 + C_2 - \alpha P_1 - A_1}{P_2 - A_1}$, then $f(x) = 0$. It can be demonstrated that, regardless of the value of x, the two strategies SR and SP employed by the netizen will reach stable state. Furthermore, the probability of the netizen selecting a strategy will remain constant over time.

If $y > \frac{R_{11} - R_{12} - C_1 + C_2 - \alpha P_1 - A_1}{P_2 - A_1}$, there exist $x_1 = 0$ or $x_2 = 1$, and $f(x) = 0$, this is a steady state. While if $\frac{\partial f(x)}{\partial x}|_{x_1=0} < 0$, $\frac{\partial f(x)}{\partial x}|_{x_2=1} > 0$, the equilibrium solution is designated by $x = 0$, at this juncture, the strategy SP is a stable strategy for internet users. Similarly if $y < \frac{R_{11} - R_{12} - C_1 + C_2 - \alpha P_1 - A_1}{P_2 - A_1}$, when $\frac{\partial f(x)}{\partial x}|_{x_1=0} > 0$, $\frac{\partial f(x)}{\partial x}|_{x_2=1} < 0$, $x = 1$ is the equilibrium solution, and the strategy SR is a stable strategy for the netizen.

Let $f(y) = 0$, we can obtain $y_1 = 0, y_2 = 1$, and $x^* = \frac{R_{21} - R_{22} - C_3 + C_4 - \beta P_3}{-A_2}$. Similarly when $f(y) = 0$ and $\frac{\partial f(y)}{\partial y}|_y < 0$ are satisfied, y is evolutionary stability strategy.

If $x^* = \frac{R_{21}-R_{22}-C_3+C_4-\beta P_3}{-A_2}$, then $f(y) = 0$, at this point, no matter what value of y is taken, the two strategies SR and SP of the marketing account can reach stable state, and the strategy selection probability of the marketing account will not change over time.

If $x^* > \frac{R_{21}-R_{22}-C_3+C_4-\beta P_3}{-A_2}$, there exist $y_1 = 0$ or $y_2 = 1$, and $f(x) = 0$, this is a steady state. When $\frac{\partial f(y)}{\partial y}|_{y_1=0} > 0$, $\frac{\partial f(y)}{\partial y}|_{y_2=1} < 0$, $y = 1$ is the equilibrium solution, the strategy SR is a stable strategy for marketing account in this moment. Conversely if $x^* < \frac{R_{21}-R_{22}-C_3+C_4-\beta P_3}{-A_2}$, while if $\frac{\partial f(y)}{\partial y}|_{y_1=0} < 0$, $\frac{\partial f(y)}{\partial y}|_{y_2=1} > 0$, $y = 0$ is the equilibrium solution, the strategy SP is a stable strategy of marketing account for now.

The aforementioned analysis yielded a total of five equilibrium points, namely the pure strategy equilibrium $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ and the mixed strategy equilibrium point (x^*, y^*).

The equilibrium point stability analysis method proposed by Friedman [24] defines a point as an evolutionary equilibrium point if $\lim_{t\to\infty}(x(t), y(t)) = (x_0, y_0)$, the stability of its equilibrium point can be judged by the eigenvalues of the Jacobi matrix of that system. If all the eigenvalues of the matrix corresponding to the equilibrium point are negative real numbers, the equilibrium point can be designated as the local asymptotically stable equilibrium point of the dynamic system. The corresponding Jacobi matrix of the system is as follows:

$$J(x,y) = \begin{bmatrix} \frac{\partial f(x)}{\partial x} & \frac{\partial f(x)}{\partial y} \\ \frac{\partial f(y)}{\partial x} & \frac{\partial f(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$\begin{aligned} K_{11} =& (1-2x)\left[R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - A_1 \right. \\ & \left. + y(A_1 - P_2)\right] \\ K_{12} =& x(1-x)(A_1 - P_2) \\ K_{21} =& y(1-y)A_2 \\ K_{22} =& (1-2y)\left[R_{21} - C_3 - R_{22} + C_4 - \beta P_3 + xA_2\right] \end{aligned} \tag{3}$$

Substituting the equilibrium point $(0,0)$, the Jacobi matrix of the system at the equilibrium point $(0,0)$ is found to be:

$$J(x,y) = \begin{bmatrix} \frac{\partial f(x)}{\partial x} & \frac{\partial f(x)}{\partial y} \\ \frac{\partial f(y)}{\partial x} & \frac{\partial f(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix}$$

$$K_{11} = R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - A_1$$
$$K_{22} = R_{21} - C_3 - R_{22} + C_4 - \beta P_3$$

In order for the eigenvalues of the matrix to be all negative, the following conditions must be satisfied:

$$R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - A_1 < 0$$
$$R_{21} - C_3 - R_{22} + C_4 - \beta P_3 < 0$$

Similarly, the stability determinations for the remaining equilibrium points and the corresponding conditions are presented in this following. The stabilization conditions for the equilibrium points $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$ are given in order from Eq. 4 to Eq. 7.

$$\begin{cases} R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - A_1 < 0 \\ R_{21} - C_3 - R_{22} + C_4 - \beta P_3 < 0 \end{cases} \tag{4}$$

$$\begin{cases} R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - P_2 < 0 \\ R_{21} - C_3 - R_{22} + C_4 - \beta P_3 > 0 \end{cases} \tag{5}$$

$$\begin{cases} R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - A_1 > 0 \\ R_{21} - C_3 - R_{22} + C_4 - \beta P_3 + A_2 < 0 \end{cases} \tag{6}$$

$$\begin{cases} R_{11} - C_1 - \alpha P_1 - R_{12} + C_2 - P_2 > 0 \\ R_{21} - C_3 - R_{22} + C_4 - \beta P_3 + A_2 > 0 \end{cases} \tag{7}$$

As for (x^*, y^*), In certain instances, the eigenvalues of the Jacobi matrix are not negative, so this equilibrium point is unstable.

IV. Simulation and Analysis of Game Model

A. Equilibrium Point of Game Model

The evolutionary game model established in the preceding section was utilized to simulate the evolutionary trajectories and to dissect the equilibrium states of netizens and marketing accounts via computational tools.

The initial value of the two-party strategy selection probability is set to $[x_0, y_0]$=$[0.5, 0.5]$, and the parameter settings are defined as follows, in accordance with the definitions and constraints of the relevant parameters presented in section III.

$$\begin{aligned} & R_{11} = 25, R_{12} = 20, C_1 = 10, C_2 = 15 \\ & R_{21} = 28, R_{12} = 26, C_3 = 14, C_2 = 17 \\ & A_1 = 8, A_2 = 5, P_1 = 14, P_2 = 15 \\ & P_3 = 22, \alpha = 0.5, \beta = 0.5 \end{aligned} \tag{8}$$

1) Equilibrium Point $(0,0)$

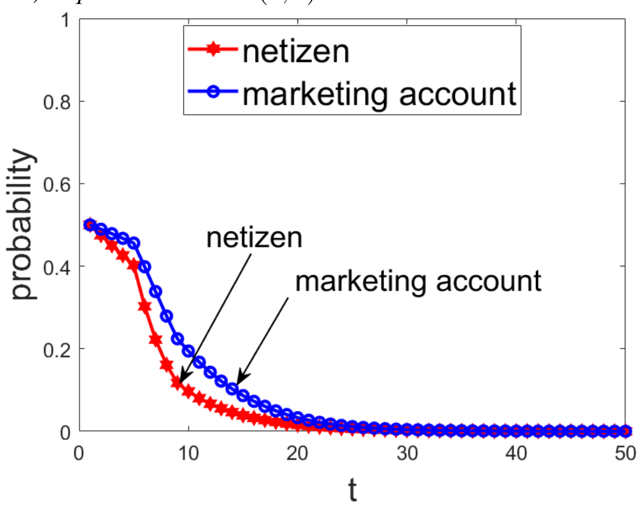

Fig. 1. The trend in the evolution of the two-party game strategy (ultimately stabilised at $(0,0)$).

The pertinent parameter settings for the experiment were derived from the Eq. 8, with the stability conditions substituted into the calculations to ascertain their satisfaction at the equilibrium point $(0,0)$. The two probability evolution trends over time for netizens and marketing accounts are illustrated in Figure 1. It can be observed that netizens tend to choose strategy SR, which indicates a reduction in the probability of spreading rumors. Similarly, marketing accounts also tend to choose strategy SR, which also indicates a reduction in the probability of spreading rumours. In other words, both parties eventually tend to choose strategy ST, which is to say, not spreading rumours.

2) Equilibrium Points $(0,1)$ *and* $(1,0)$

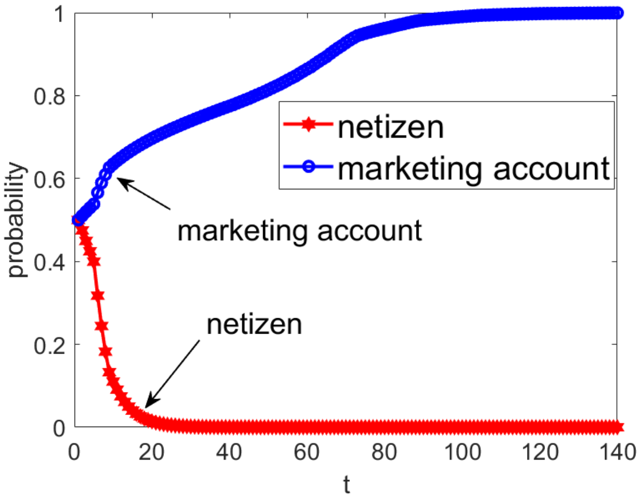

Fig. 2. The trend in the evolution of the two-party game strategy (ultimately stabilised at $(0,1)$).

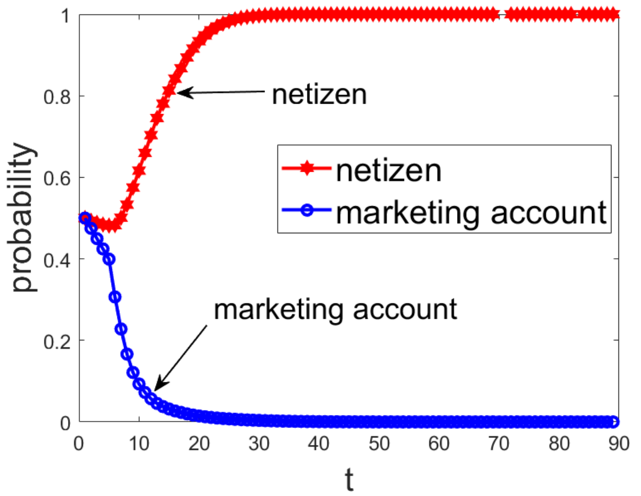

Fig. 3. The trend in the evolution of the two-party game strategy (ultimately stabilised at $(1,0)$).

By calculating $\frac{\partial f(y)}{\partial P_3} < 0$, we know that when P_3 decreases, $f(y)$ will increase; and $\frac{\partial f(y)}{\partial C_4} > 0$, we can get that when C_4 increases, $f(y)$ will increase. Consequently, the experimental results are presented in Figure 2, which only update $P_3 = 16, C_4 = 21$ based on the equation 8. Internet users choose strategy ST, where the probability of spreading rumours still tends to 0 ($x \to 0$). The evolutionary trend of the marketing account changes, choosing strategy SR, where the probability of spreading the rumour tends to 1 ($y \to 1$). The final evolutionary trend stabilises at the equilibrium point $(0,1)$.

Similarly, it can be concluded that $\frac{\partial f(x)}{\partial P_1} < 0$, $\frac{\partial f(x)}{\partial C_2} > 0$, $f(x)$ will increase as the parameter P_1 decreases, and increase as the parameter C_2 increases. Again on the basis of equation 8, when only the parameters $P_1 = 7, C_2 = 19$ are updated, the experimental results are shown in Figure 3. The marketing number chooses strategy ST, the probability of publishing rumors still tends to 0($y \to 0$), the evolutionary trend of netizens changes, and chooses strategy SR, the probability of spreading rumours tends to 1($x \to 1$), and the final evolutionary trend stabilizes at the equilibrium point $(1,0)$.

The modification of the game subject's strategy indicates that the alteration of the parameter exerts a certain degree of influence on the subject's strategy selection. Theoretically, following an increase in the cost C_4 of selecting strategy ST, the marketing accounts will tend to select a strategy with an equivalent cost. Conversely, when the platform is disregarded and the penalty for publishing rumours by the marketing accounts is reduced, the marketing accounts will tend to reconsider their strategies due to the consideration of their interests. Changes in the values of the cost C_2 of not choosing to spread rumours and the penalty P_1 for spreading rumours had impact on the netizens' strategy choices. As shown in the experiment, marketing accounts and Internet users made a shift from strategy ST to strategy SR, respectively.

3) Equilibrium Point $(1,1)$

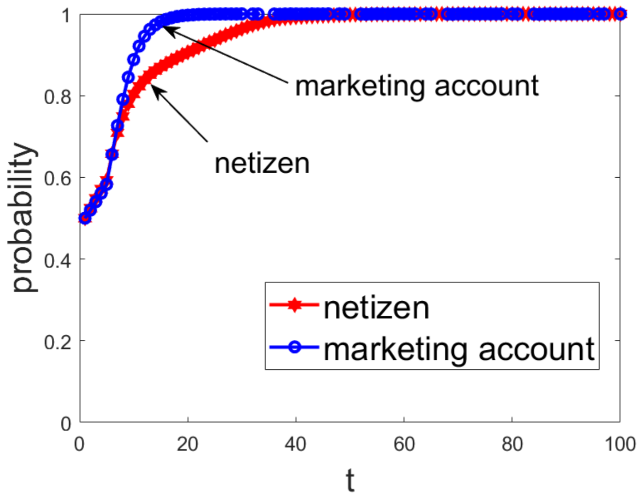

Fig. 4. The trend in the evolution of the two-party game strategy (ultimately stabilised at $(1,1)$).

When the equation 8 is updated simultaneously with the parameters of the netizen and the marketing account, both the marketing account and the netizen exhibit a shift in decision-making behaviour, as evidenced by the evolutionary trend illustrated in Figure 4. The evolutionary trends of netizens and marketing accounts are subject to change. Both choose the strategy SR, which implies that the probability of spreading or posting rumours is 1. The final evolutionary trend stabilises at the equilibrium point $(1,1)$.

B. The Effect of Relevant Parameters on Evolutionary Trend of the Game

The equilibrium point serves as a pivotal reference point for determining the optimal strategic approach for each participant within a given scenario. As the relevant parameters undergo alteration, the strategies selected are correspondingly adjusted. Furthermore, the distinct values of these parameters play a crucial role in shaping the evolutionary stabilisation strategy. In this section, the impact of these relevant parameters on the evolutionary trajectory of the game will be explored in depth through the careful regulation of the relevant variables, and the corresponding suggested strategies will be given.

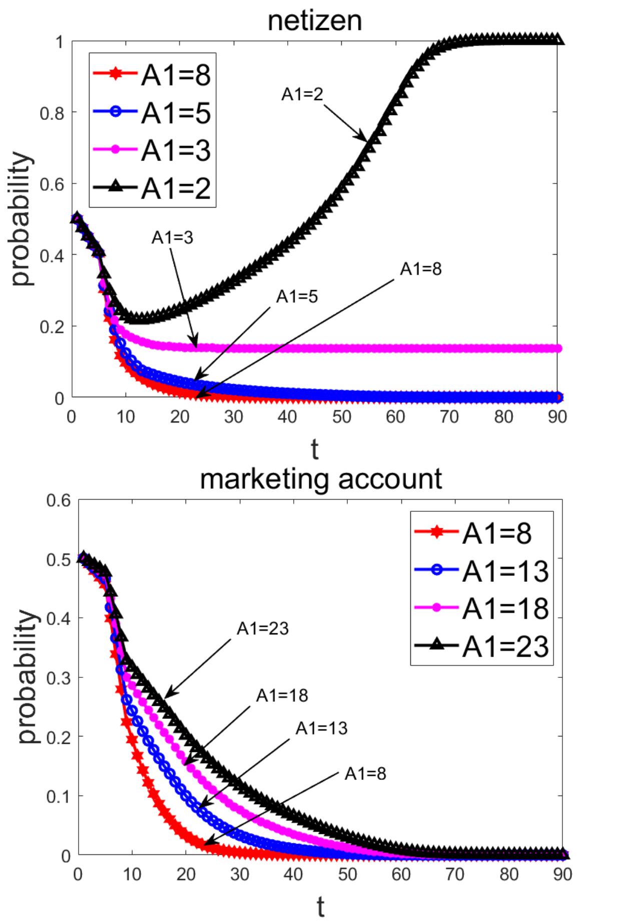

Fig. 5. Impact of the netizens' positive atmosphere plus A_1 (based on $(0,0)$).

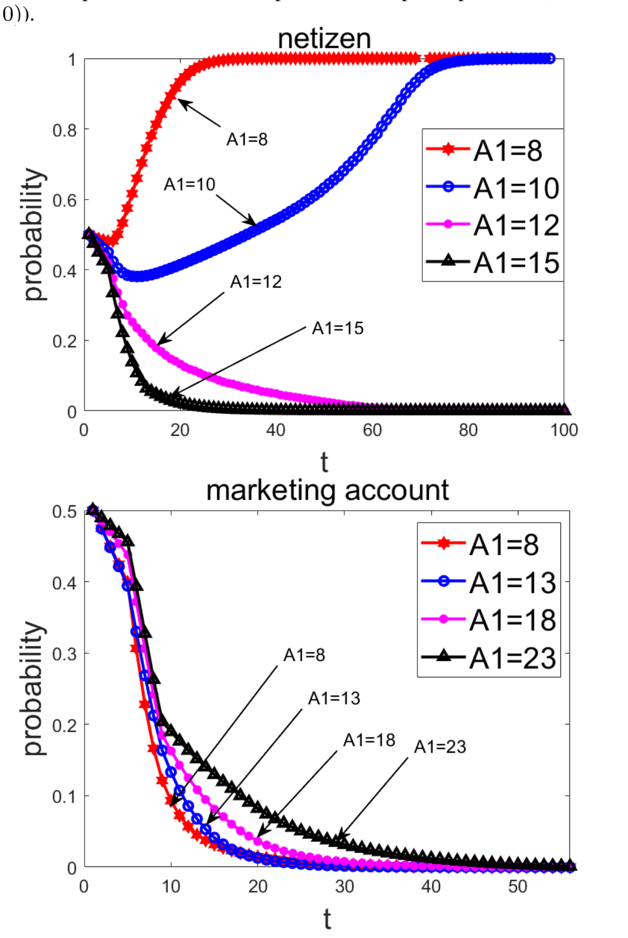

Fig. 6. Impact of the netizens' positive atmosphere plus A_1 (based on $(1,0)$).

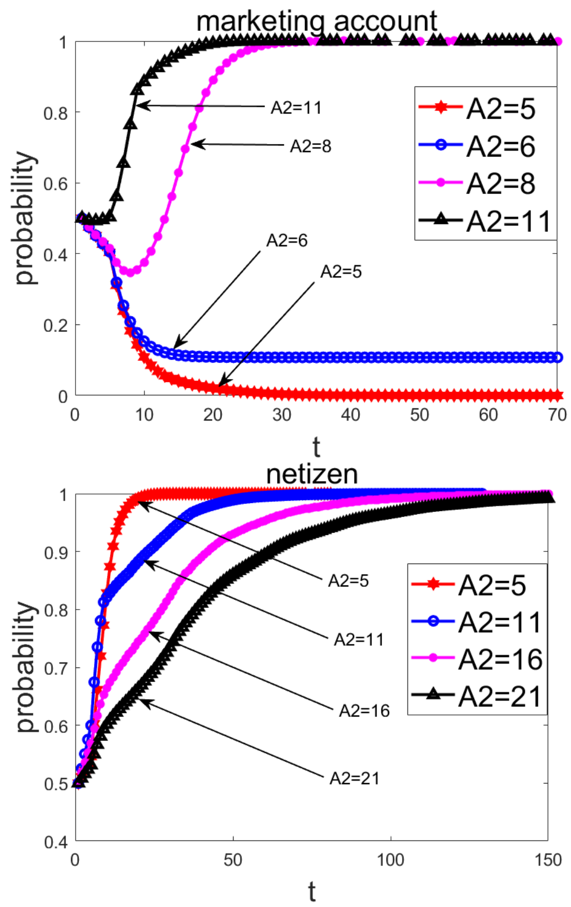

Fig. 7. Impact of the Extra revenue A_2 from marketing accounts (based on $(1,0)$).

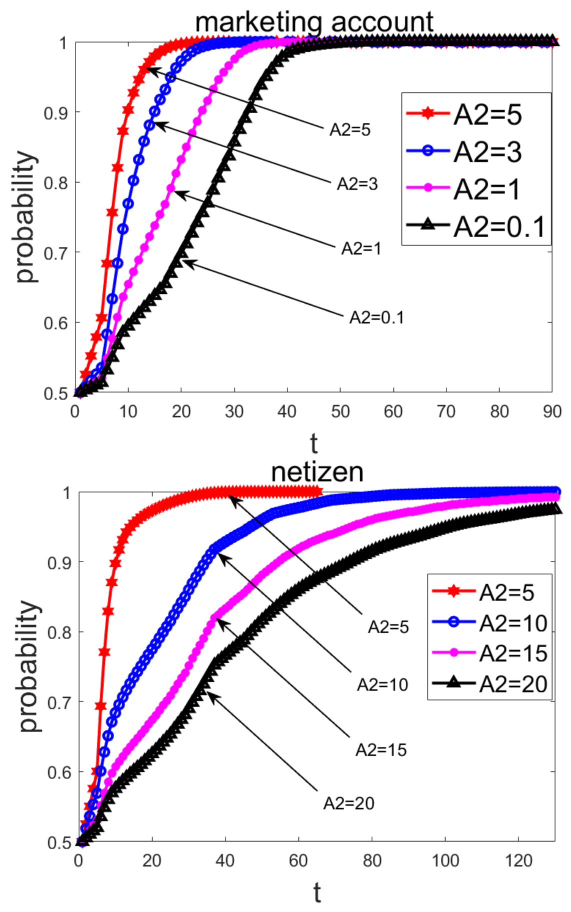

Fig. 8. Impact of the Extra revenue A_2 from marketing accounts (based on $(1,1)$).

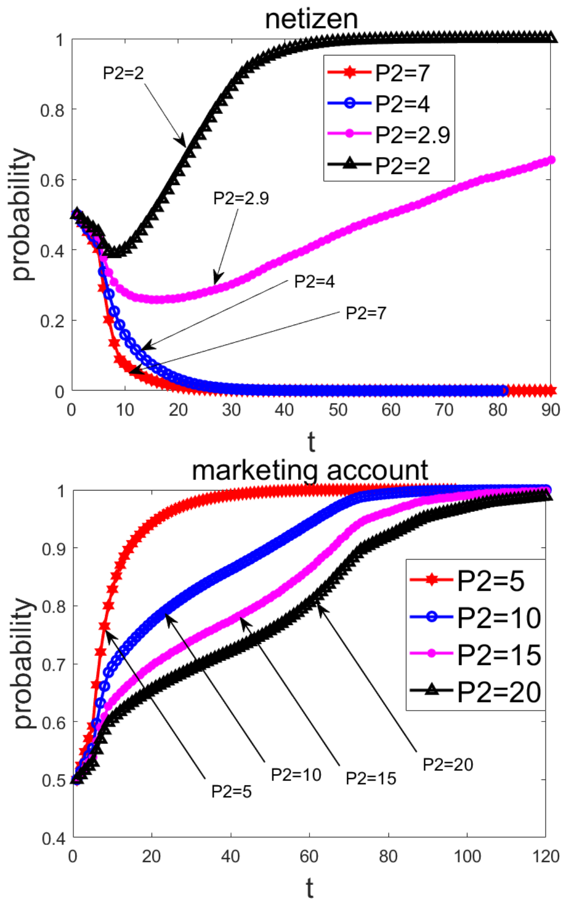

Fig. 9. Impact of additional loss P_2 for netizens (based on $(0, 1)$).

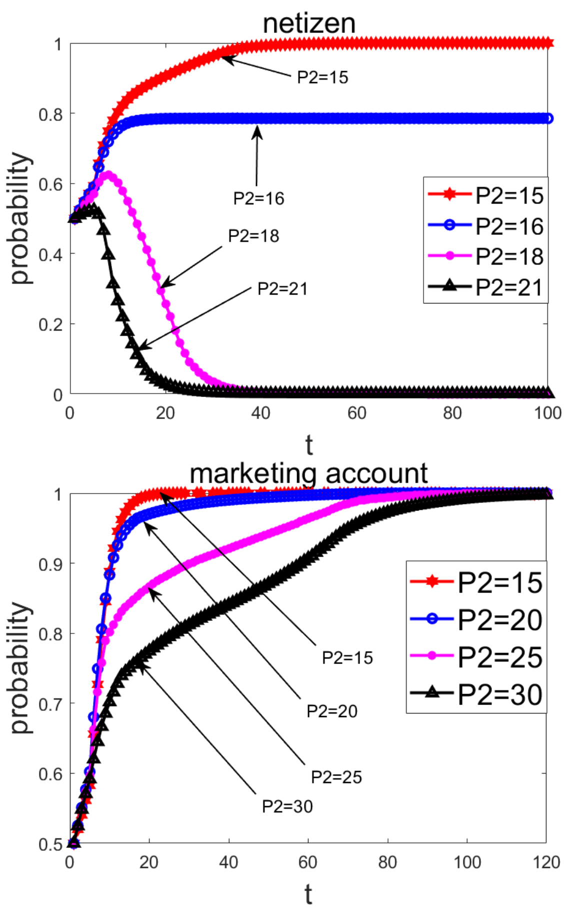

Fig. 10. Impact of additional loss P_2 from netizens (based on $(1, 1)$).

1) Impact Analysis of Parameter A_1

In accordance with the stability conditions of the equilibrium points as presented in Table II, the two equilibrium points related to the parameter A_1 are $(0, 0)$ and $(1, 0)$. With the updated parameters A_1 based on the parameters of the equilibrium points $(0, 0)$ and $(1, 0)$, netizens and marketing accounts will reconsider their decision-making choices when the netizens are subjected to a decrease in the good atmosphere bonus, as shown in Figure 5 and Figure 6. As the value of A_1 declines, the evolution of netizens' strategies towards 0 will decelerate. When the value of A_1 reaches a certain threshold, the evolution of netizens will eventually converge to 1, while the evolution of marketing numbers will accelerate to 0.

It can be posited that a positive atmosphere plays a role in influencing the decision-making processes of netizens to a certain extent. A positive social platform atmosphere encourages netizens to engage in more critical thinking and discernment of opinion information. Conversely, if the atmosphere of public opinion deteriorates, netizens will be affected by public opinion information and become the spreaders of rumours. The addition of a favourable atmosphere will prompt marketing professionals to exercise greater care when selecting their message release strategies, particularly in the event of a significant loss of traffic.

2) Impact Analysis of Parameter A_2

While marketing accounts benefit from a high volume of traffic, they may also receive additional benefits such as subsidies from certain platforms. These supplementary benefits can influence the decision-making processes of marketing accounts. The parameter A_2 are updated based on the equilibrium points $(1, 0)$ and $(1, 1)$. Extra revenue for marketing accounts impacts on strategy evolution trends for both marketing accounts and netizens. As illustrated in Figures 7 and 8, the evolution of marketing accounts gradually shifts towards 1 as additional traffic and exposure increase. Once the value of A_2 reaches a certain level, the marketing account finally adopt the strategy SR. The evolution curve of netizens gradually slows down.

The extra revenue gained by the marketing account makes it easier for the marketing number's decision-making choices to be tilted towards decisions with higher returns. When the additional revenue is excessive, the marketing account opts for the decision of disseminating rumors, which has a high revenue potential. This will result in an increase in the number of rumors. Consequently, the regulation of the supplementary revenue of marketing accounts represents a key factor in the prevention of the proliferation of rumors.

3) Impact Analysis of Parameter P_2

Based on the parameters of the equilibrium points $(0, 1)$ and $(1, 1)$, the effect of the netizen's extra loss on the strategy evolution trend of the marketing account and the netizen is explored by updating the parameter P_2 . Figures 9 and 10 illustrate the evolution trend of netizens when the extra loss of netizens due to the poor platform atmosphere decreases. As shown in these figures, the evolution trend of netizens gradually turns to 1, which indicates that the strategy SR is chosen. The value of P_2 will determine whether the evolution curve of the marketing account will be flat or steep. When the evolution curve of the marketing account decreases, it tends to be steeper and accelerates towards 1.

The dissemination of rumor can result in additional loss for Internet users, including misinformation, psychological distress, and emotional distress. The extent of these losses can influence their decision-making process. Therefore, implementing measures to reduce the impact of these additional loss on Internet users can contribute to the control of rumor.

V. The SMNIR Model Based on Game Model

The epidemic model, as a classic public opinion research model, has been widely employed in studies pertaining to public opinion communication. Building upon the game model concerning netizens and marketing accounts, the SMNIR model was developed by integrating the infectious disease model.

A. Introduction of The SMNIR Model

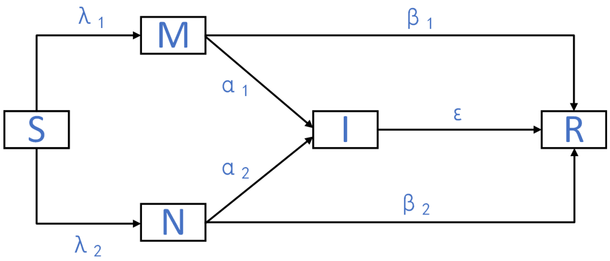

Fig. 11. The basic scheme of the SMNIR model.

- The node states S, I, and R in the model are essentially analogous to those defined in the conventional public opinion SIR model. The node state of S represents the susceptible population, which is assumed to be universally susceptible in the model due to the pervasive flow of information on the Internet and the multitude of channels through which people obtain information. Two additional node states, M and N, are introduced between S and I. M represents the group of individuals who engage in the discussion of public opinion information released by the marketing accounts, while N represents the spontaneous exchange of public opinion among netizens. I represents the collective of individuals who disseminate the rumour when public opinion evolves into a rumour, comprising both the marketing accounts and netizens. R represents the group of individuals who gradually cease their involvement in the dissemination of public opinion information, effectively exiting the system and unlikely to return.
- Subsequently, the state transfer rules of each node are presented. Two transfer paths exist for the susceptible individuals. Upon exposure to the public opinion information released by marketing accounts and observing the public opinion discussions of netizens, individuals in state S will have the probability of participating in them, respectively. This will result in becoming members of the state M with a probability of λ_1, or becoming members of the state N with a probability of λ_2. There are two transfer paths for both nodes M and N. Both groups will have individuals who choose to disseminate rumors or withdraw from the system because they are indifferent to public opinion. The transfer probabilities from M to I, N to I, M to R, and N to R are denoted by α_1, α_2, β_1, and β_2, respectively. The transfer probability from node I to R is ε, indicating that as time passes, the heat recedes and people gradually stop spreading and discussing the rumours and exit the system. The state transfer diagram of the overall SMNIR model is shown in Figure 11.

The derivation of the state transfer differential equations for the SMNIR model is based on the node definitions in the aforementioned assumptions and the transfer rules.

$$\begin{cases} \dfrac{dS}{dt} = -\lambda_1 SM - \lambda_2 SN \\ \dfrac{dM}{dt} = \lambda_1 SM - \alpha_1 M - \beta_1 M \\ \dfrac{dN}{dt} = \lambda_2 SN - \alpha_2 N - \beta_2 N \\ \dfrac{dI}{dt} = \alpha_1 M + \alpha_2 N - \varepsilon I \\ \dfrac{dR}{dt} = \beta_1 M + \beta_2 N + \varepsilon I \end{cases} \tag{9}$$

B. Related Simulations and Discussions

It is well established that social network platforms play a pivotal role in shaping public opinion. Consequently, it is imperative to conduct experiments on suitable social network structures. The degree distribution of the BA scale-free network [25] exhibits the structural properties of a power law distribution, as illustrated in Figure 12. The majority of nodes in the network have a low in-degree, indicating that the information access and acquisition available to Internet users is limited. Conversely, a small number of nodes have a high in-degree, suggesting that the marketing numbers, though small in number, have a strong appeal. Consequently, the experiment was conducted on the BA scale-free network.

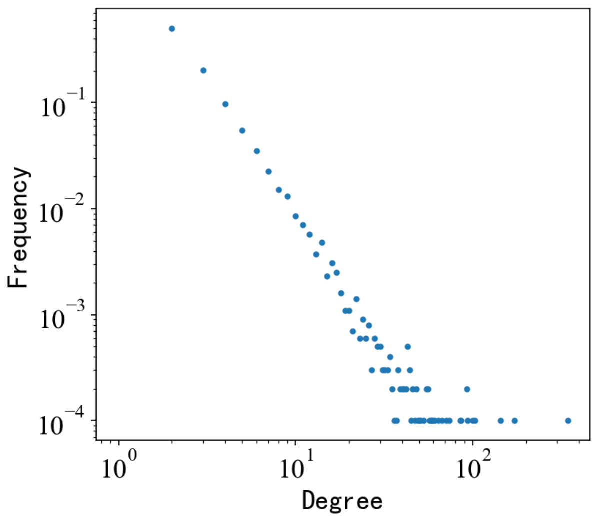

Fig. 12. The degree distributionBA of scale-free network.

In the SMNIR model, the two transfer paths M to I and N to I correspond to the marketing account and the netizen choice decision SR. Similarly, the two transfer paths M to R and N to R correspond to the marketing account and the netizen choice decision ST. As discussed in subsection 4.2, the relevant parameters affect the decision choices of the netizen and the marketing account. Accordingly, in the SMNIR model, the impact of the pertinent parameters is reflected in the fact that the density of node I changes when the transfer probability of the corresponding path changes. An increase indicates a facilitation of the choice decision SR, a decrease indicates a suppression of the choice decision SR, and vice versa, a facilitation of the choice decision ST.

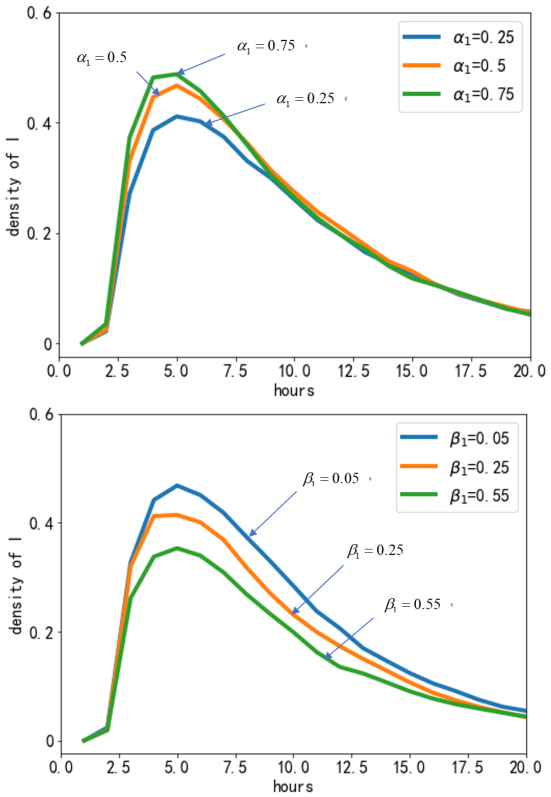

Fig. 13. Impact of α_1 and β_1 in the SMNIR model.

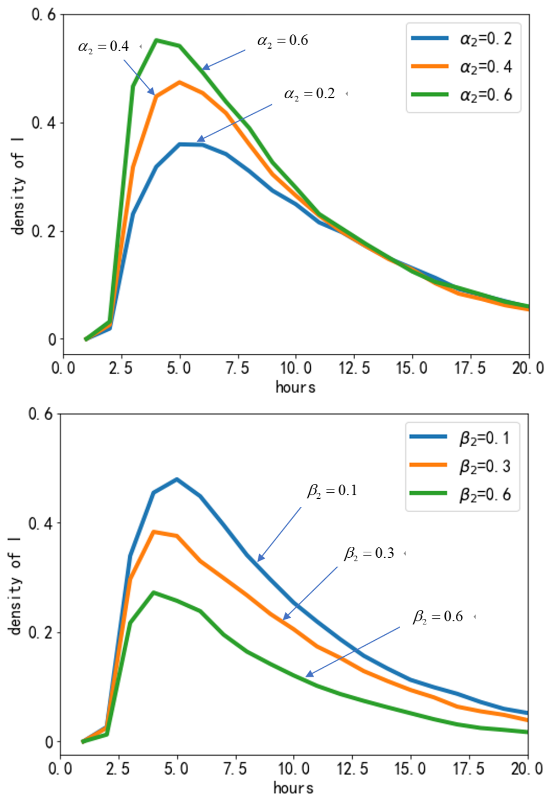

Fig. 14. Impact of α_2 and β_2 in the SMNIR model.

After constructing the BA scale-free network with 2000 nodes, the initial marketing account nodes M is pre-set to 50, the netizen nodes N is 100, and the rest of the nodes are susceptible nodes. Figures 5.3 and 5.4 show the change in the number of nodes I corresponding to the change in path transfer probability, respectively. As illustrated in Figure 13, when the transfer probability α_1 from M to I increases and the transfer probability α_2 from M to R decreases, In accordance with the observed increase in the number of peak in node I, which corresponds to an increase in the relevant parameter A_2 in the game model, it can be concluded that the additional traffic to the marketing number has indeed promoted the selection of the SR strategy. As illustrated in Figure 14, an increase in the transfer probability α_2 from N to I and a decrease in the transfer probability β_2 from N to R, which corresponds to a decrease in the relevant parameter A_1 in the game model or an increase in parameter P_2, results in an increase in the number of peak at node I. This suggests that when netizens experience a decrease in positive atmosphere plus or a increase in additional loss, it acts as a facilitator in choosing the strategy of SR.

C. Experimental Prediction of Real Internet Rumor

A genuine simulation experiment is conducted with the objective of validating the efficacy of the SMNIR model by collating authentic online public opinion data. The data presented in this paper is derived from the online rumour, "A student was poisoned to death after stealing takeaway from Xiangtan University," which was disseminated on social network platforms. The temporal scope of the study encompasses the period from 11:00 on April 19th to 6:00 on April 21st, 2024.

The data obtained was used as a basis for the construction of a BA scale-free network, with the total number of nodes assumed to be 500. Of these, 68 were designated as M nodes, 62 as N nodes, and the remaining nodes were designated as susceptible nodes. The prediction is based on the actual curves and is carried out by parameter tuning. Furthermore, simulated prediction experiments were conducted using the SIR and SMNIR models, and the result prediction outcomes

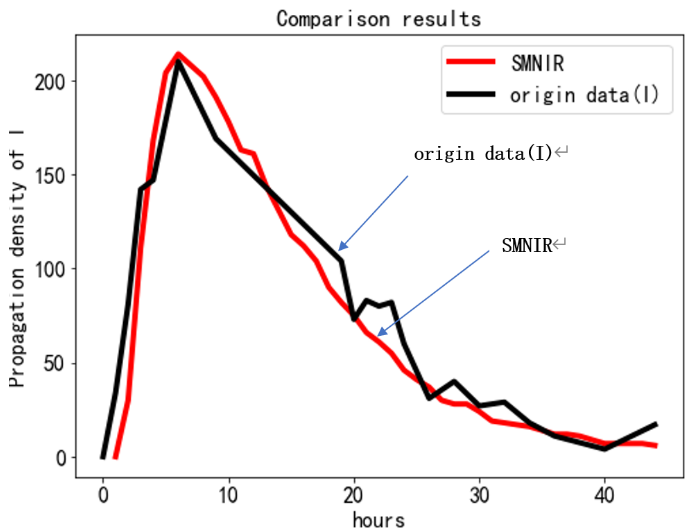

Fig. 15. The SMNIR model prediction error analysis.

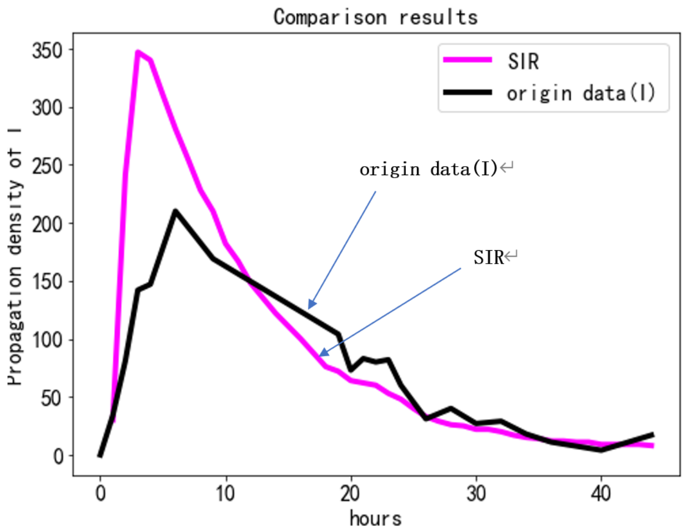

Fig. 16. The SIR model prediction error analysis.

TABLE II
MODEL COMPARING ERROR RESULTS.

Model	SIR	SMNIR
MAE	49.7619	17.8571
RMSE	89.3082	23.3860

The data prediction errors of the two models were calculated separately. The result was shown in Tab. II. It can be observed that the SMNIR model proposed in this paper can more accurately approximate the peak value of the actual rumour heat as well as the evolutionary trend, with a smaller error.

VI. CONCLUSION

This paper considers the pivotal role of marketing accounts and netizens in the propagation of rumors. To this end, a two-party evolutionary game model of marketing accounts and netizens is constructed, the equilibrium points are discussed, and related stabilisation strategies are presented. Furthermore, numerical simulation experiments are conducted. The SMNIR model, which combines the evolutionary game and infectious disease model, is then constructed. The following conclusions are finally obtained.

1) The creation of a positive atmosphere can influence the decision-making processes of online users, prompting them to be more discerning about the information they consume and to avoid becoming unwitting propagators of misinformation. It is recommended that platforms implement measures to encourage netizens to engage in the debunking of rumours and the timely dissemination of updated information, in order to maintain a positive public opinion environment.
2) The generation of additional revenue from marketing activities can influence the decision-making processes of marketing personnel. An increase in this revenue can lead to a tendency for marketing personnel to disseminate rumours. It is recommended that platforms implement controls to regulate this type of revenue and encourage marketing numbers to assume a more constructive role, rather than becoming purveyors of rumors in the pursuit of high revenue.
3) The SMNIR model has been subjected to numerical simulation and example validation, demonstrating its predictive and effective capacity in the context of rumors.

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