Constructing Membership Functions using Kinetic Data

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ABSTRACT—Comminution is a critical multi-criteria decision-making problem that significantly impacts the efficiency of grinding mills. The use of dispersants can substantially enhance comminution performance. In this study, three indicators of quantity and quality were employed to construct a fuzzy comprehensive evaluation system. Specifically, the effects of 10 different dispersing agents on the comminution of potassium feldspar powders, focusing on particles larger than 10 μm (y_{+10}), were analyzed. Based on comminution kinetics, a novel membership function was constructed and compared with the cubic parabolic membership function. The method for generating the novel membership function involved six steps: plotting a scatter diagram of comminution, transposing the coordinates, normalizing the dependent variables, fitting the mathematical model, truncating the segment of the mathematical function, and completing the truncated part of the mathematical function. The consistency between the change rate of y_{+10} and the degree of membership of y_{+10} demonstrated the suitability and advantages of the proposed method. The method applied to other fields was described in the end.

Index Terms—Construction; Membership function; Kinetic equation; Fuzzy comprehensive evaluation

I. INTRODUCTION

TO reduce the particle size in mineral processing, comminution is extensively employed. Comminution is a considerably complex process and a multi-criteria decision-making problem. Previous studies [1]-[4] have shown that pulp rheology plays a significant role in wet comminution processes, affecting comminuting efficiency. Dispersants can greatly improve the efficiency due to their effects on the slurry rheology [5][6]. Therefore, research on the selection of effective dispersants is essential.

The fuzzy comprehensive evaluation method has been widely employed for the assessment of teaching quality [7], sustainability risks [8], grinding machining quality [9], the classification performance of air classifiers [10], and the enhancement of grinding machine design [11]. Nevertheless, this method has seldom been employed in the domain of comminution. In this paper, three indicators reflecting product quality and quantity were selected to construct a simple and practical fuzzy comprehensive evaluation method for the comminution of ultra-fine powder [12]. In accordance with the tenets of fuzzy set theory, the degree of membership can be indicated by a candidate object that has a membership ranging between 0 and 1 [13]. The methods employed to ascertain membership functions encompass fuzzy statistics [14], dualistic contrast compositors [15], set-valued statistics [16], interpolation theory [17], and incremental methods [18]. Additionally, distinct membership functions yield disparate outcomes. For instance, Hameed [19] proposed a more reliable system of student assessment based on Gaussian membership functions, which provided varying scores and ranking orders, as an alternative to traditional triangular membership functions.

The process of determining the membership function is inherently subjective, rendering the selection process susceptible to bias. A pivotal step in the application of fuzzy mathematical evaluation is the establishment of suitable and realistic membership functions [20]. When calculating the degree of membership, a membership function selected based on the nature of the indicators still exhibits a high level of subjectivity [21]. This paper proposes a novel membership function based on parameter feedback.

Manuscript received May 13, 2024; revised Dec 10, 2024. This research was funded by the Scientific Development Foundation of Dongying, China (DJB2022017).

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II. EXPERIMENT

A. Experimental materials and equipment

Potassium feldspar powder samples were collected in Jiangsu Province. The non-agglomerated raw powders were subjected to sonication and dispersion (HK-5200B, Huier Instruments & Equipment Co., Hangzhou, China) in deionized water for a period of 10 minutes. Subsequently, the powder grain size was determined via particle size analyzer (Mastersizer 2000, Malvern Panalytical Ltd., UK). The particle size distribution is illustrated in Figure 1. Ten dispersants (Chengxin Chemical Material Supply Station, Hangzhou, China) were employed in the milling experiments. These included sodium hexametaphosphate ($(NaPO_3)_6$, purity: 99%), sodium pyrosulfate ($Na_2S_2O_7$, purity: 99%), sodium oxalate ($Na_2C_2O_4$, purity: 99.96%), polycarboxylate (code: CP-40, purity: 99%), polycarboxylate (code: HT-5050, purity: 99%), sodium polyacrylate (code: NP-45, purity: 99%), sodium polyacrylate (code: SF-3000, purity: 99%), ammonium salt of polycarboxylate (code: SF-3000N, purity:

99%), polycarboxylate (code: DL-884, purity: 99%), and polycarboxylate (code: WD-40, purity: 99%). The polymeric dispersants of the same type exhibited a range of molecular weights. Unless otherwise stated, the solvents and reagents were purchased and used without any purification.

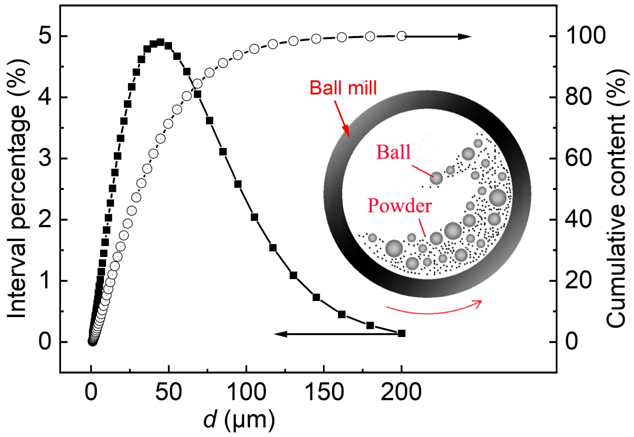

Fig. 1. Comminution by ball mill

A ball mill, developed by Tsinghua University in China, was employed for the purpose of comminution. The operational principle of the ball mill is illustrated in Figure 1. In particular, the internal diameter of the ceramic ball mill tank was 250 mm, with a volume of 13.5 L. The diameter of the stainless steel balls was available in two specifications, measuring 12 mm or 17 mm.

B. Experimental methods

A series of single-factor experiments were conducted with the objective of identifying the optimal dispersants. The experimental conditions are detailed in Table I.

TABLE I
EXPERIMENTAL CONDITIONS

Speed Ratio (%)	95
Specific surface area of media (m^2/kg)	0.2
Concentration (%)	55
Grinding time (h)	3
Amount of the powder (kg)	3.66
Amount of dispersant (g)	3.66

The quantity of dispersant employed was 0.1%, calculated in accordance with the mass of the dry potassium feldspar. The grinding process was conducted in accordance with the same conditions. The grinding experiments were conducted, and the mill was terminated after a period of three hours. Subsequently, an adequate quantity of the sample was removed in order to ascertain the particle size distribution. The measurements were repeated on each sample three times in parallel, and the mean value was calculated. Furthermore, a suspension devoid of any dispersant was subjected to grinding for the purpose of comparison.

C. Characterization of properties

y_{+10} represents the proportion of powder particles exceeding 10 μm in diameter. d_{97} denotes the particle size corresponding to 97% of the cumulative particle size distribution of the sample. The fractal dimension of the particle size distribution can be expressed by the following equation [22].

$$S(d) = k\left(\frac{d}{d_{\max}}\right)^{3-D} \tag{1}$$

where d is the particle size (μm), $S(d)$ is the function of the cumulative particle size distribution of the sample (%), k is a constant, D is the fractal dimension of the particle size distribution and is dimensionless, and d_{max} is the particle size corresponding to 100% of the cumulative particle size distribution of the sample (μm) [23]. A reduction in the D value will result in a narrowing of the particle size distribution. In this paper, the term 'particle size' refers to the particle size of an equal volume, as measured using an analytical instrument.

The indices y_{+10}, D, and d_{97} were translated into the degrees of membership μ_1, μ_2, and μ_3, respectively, in accordance with the pertinent membership function $f(y_{+10})$, $f(D)$, and $f(d_{97})$. The comprehensive evaluation index C was calculated according to the degree of membership and the assigned weights (w_1, w_2, and w_3). The weights were calculated using the analytic hierarchy process (AHP) [24] and yielded values of 0.69, 0.15 and 0.16. The calculation process is illustrated in a flowchart in Figure 2 [25].

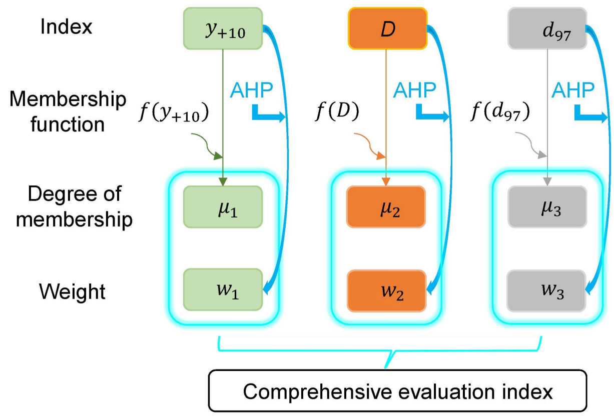

Fig. 2. Fuzzy comprehensive evaluation flowchart

III. RESULTS AND DISCUSSION

In consideration of the limitations in available space, the sole indicator y_{+10} was employed for the purpose of identifying the optimal dispersant. This indicator is employed to elucidate the methodology and the superiority of the novel membership function.

A. Results from the conventional approach

The rate of comminution was observed to be high during the initial stages of milling and relatively low during the subsequent post-milling stages [26]. The proportion of raw powder with a diameter greater than 10 μm (84.76%) initially declined rapidly, followed by a slower rate of decrease. If y_{+10} is equal to 84.76%, it can be concluded that the grinding process has no effect and that the degree of membership has a minimum value of 0. Conversely, if y_{+10} is equal to 0, it can be concluded that the grinding process is perfect and that the degree of membership has a maximum value of 1. In order to calculate the indicators' membership in accordance with the variation characteristics of y_{+10}, the cubic parabolic membership function was selected, as in (2).

The control experiment did not include the use of a dispersant, whereas the 10 different dispersants were employed in the contrast experiments. The results of the experiment and the degree of membership (μ_1) are presented in Table II.

$$f(y_{+10}) = \begin{cases} 1, y_{+10} \le 0 \\ (\frac{84.76 - y_{+10}}{84.76})^3, 0 < y_{+10} < 84.76 \\ 0, y_{+10} \ge 84.76 \end{cases} \quad (2)$$

TABLE II
y_{+10} OF THE EXPERIMENTAL RESULTS

Indicator and Membership	y_{+10} (%)	μ_1
No dispersant	22.77	0.39
$(NaPO_3)_6$	28.30	0.30
$Na_2S_2O_7$	26.22	0.33
$Na_2C_2O_4$	21.75	0.41
CP-40	21.71	0.41
HT-5050	19.47	0.46
NP-45	23.09	0.39
SF-3000	22.51	0.40
SF-3000N	24.23	0.36
DL-884	25.29	0.35
WD-40	20.15	0.44

The maximum degree of membership was 0.46 and the most effective dispersant was identified as HT-5050 based on the index y_{+10}. The results were analyzed in accordance with the efficiency factors set forth in (3) [27].

$$E_\Psi = \left| \frac{\Psi_b - \Psi_a}{\Psi_a} \right| \quad (3)$$

where E_Ψ is the efficiency factor, Ψ is the index y_{+10} or μ_1, Ψ_b is the y_{+10} value or μ_1 value of the ground particles after 3 hours with a dispersant, Ψ_a is the y_{+10} value or μ_1 value of the ground particles after 3 hours without a dispersant.

It is evident that μ_1 underwent a significant increase when HT-5050 was employed, resulting in an efficiency factor of 17.95%. However, the proportion of particles larger than 10 μm declined to 14.49%, representing a 3.46% difference.

B. Results from the novel approach

Establishment of the novel membership function

During the process of ultra-fine comminution, the grain size indicator undergoes a functional change [28]. As the duration of grinding increases, the grain size value decreases, accompanied by the formation of sieve residue from the ground potassium feldspar powder. Moreover, several studies have demonstrated that variations in grain size during comminution align with the n-order crushing kinetic model [29]-[31]. The revised kinetic equation proposed by Aliavden [32] was employed to describe the grinding process of materials, as illustrated by the following equation.

$$y(t) = y_0 \exp(-kt^n) \quad (4)$$

where y_0 represents the initial sieve residue of the ground material with a certain particle size, k denotes the grinding rate constant, and n indicates the time index, which is determined by the properties of the ground material and the grinding conditions.

In order to ensure that y_{+10} is as close to 0 as possible, the comminution time was set to 9 hours. The initial five hours of grinding experiments were conducted at 30-minute intervals. At 30-minute intervals, the mill was halted, and a sufficient quantity of the sample was removed to ascertain the particle size distribution without disrupting the pulp: solid ratio. Thereafter, the grinding process was resumed. The final product was obtained after nine hours of comminution. Particle size distribution measurements were carried out on the samples obtained after grinding. When the potassium feldspar powders were ground for nine hours without the use of a dispersant, the y_{+10} value was found to be 3.92%. The y_{+10} values of the samples are presented in Figure 3.

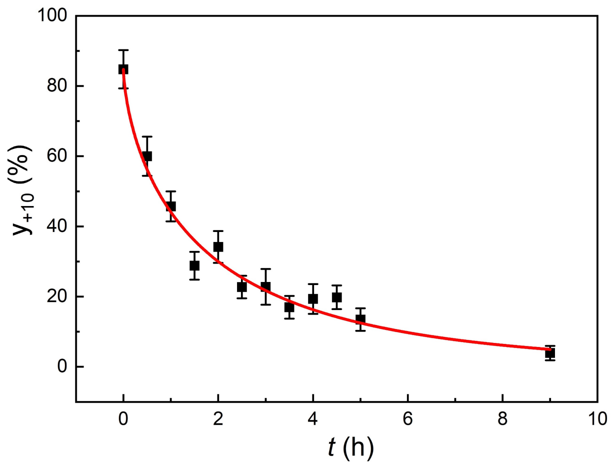

Fig. 3. Variation of y_{+10}

The Levenberg–Marquardt algorithm [33] was employed to fit the data points. The resulting R^2 value of 0.97 indicated a satisfactory fit. The comminution kinetics equation for potassium shale can be expressed as follows:

$$y_{+10}(t) = 84.76e^{-0.64t^{0.66}} \quad (5)$$

The novel membership function of y_{+10} can be obtained based on kinetic data, the process of which is outlined in the following section.

(a) A scatter diagram of the index is constructed, wherein t represents the grinding time and y_{+10} denotes the proportion of powder exceeding 10 μm in diameter.

(b) The scatter diagram is transposed, with the independent variable transformed into the dependent variable.

(c) The novel dependent variable, t', is normalized.

(d) The model is fitted, with an R^2 value of 0.94.

$$t' = -0.88\ln(0.22\ln(y_{+10})) \quad (6)$$

(e) The segment of the mathematical function falling outside the range of 0 to 1 is truncated.

(f) The value range of the function domain and range is then determined, and a novel membership function is subsequently obtained:

$$f_1(y_{+10}) = \begin{cases} 1, y_{+10} \le 4.35 \\ -0.88\ln(0.22\ln(y_{+10})), 4.35 < y_{+10} < 84.76 \\ 0, y_{+10} \ge 84.76 \end{cases} \quad (7)$$

The construction of a novel membership function based on comminution kinetics requires a total of six steps, as illustrated in the subfigures of Figure 4.

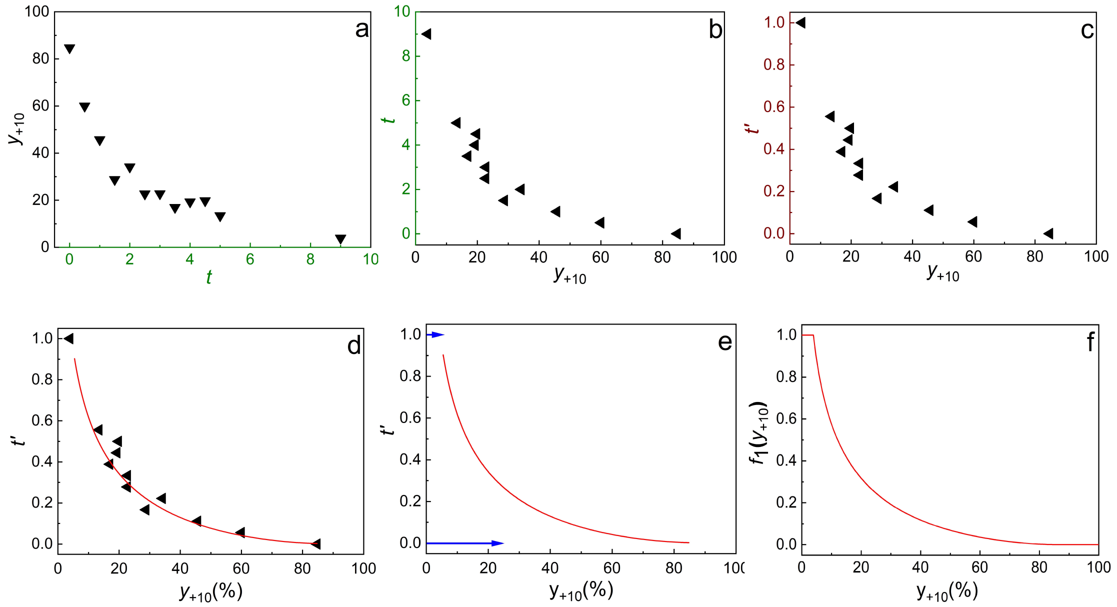

Fig. 4. Six steps to construct the novel membership function

Validation process

The degrees of membership μ'_1 of y_{+10} with different dispersants, calculated by the novel membership function, are presented in Table III. The maximum degree of membership was 0.37, and the efficiency factor was 15.63%, representing a difference of 1.14%.

TABLE III
μ'_1 OF THE EXPERIMENTAL RESULTS

Indicator and Membership	y_{+10} (%)	μ'_1
No dispersant	22.77	0.32
$(NaPO_3)_6$	28.30	0.26
$Na_2S_2O_7$	26.22	0.28
$Na_2C_2O_4$	21.75	0.34
CP-40	21.71	0.34
HT-5050	19.47	0.37
NP-45	23.09	0.32
SF-3000	22.51	0.33
SF-3000N	24.23	0.31
DL-884	25.29	0.29
WD-40	20.15	0.36

TABLE IV
y_{+10}, $f(y_{+10})$, AND $f_1(y_{+10})$ OF THE EXPERIMENTAL RESULTS

t (h)	y_{+10} (%)	μ_1	μ'_1
0	84.76	0	0
0.5	60	0.02	0.08
1	45.73	0.1	0.15
1.5	28.8	0.29	0.26
2	34.17	0.21	0.22
2.5	22.73	0.39	0.32
3	22.77	0.39	0.32
3.5	16.97	0.51	0.41
4	19.33	0.46	0.37
4.5	19.8	0.45	0.36
5	13.46	0.6	0.49
9	3.92	0.87	1

Table IV illustrates the indicator y_{+10} and its degrees of membership μ'_1 at varying grinding times, as calculated by the novel membership function.

The consistency between the change rate of the degrees of membership and the actual values was analyzed. The formula for the change rate is as in (8) and the calculation results are presented in Table V.

$$\Delta x = (x_1 - x_2) / x_1 \tag{8}$$

where x_i represents the index, $i = 1,2$.

TABLE V
Δy_{+10}, $\Delta f_1(y_{-10})$, AND $\Delta f_3(y_{-10})$ VALUES

Δy_{+10}	$\Delta\mu_1$	$\Delta\mu'_1$
0.29	-	-
0.24	-2.92	-0.72
0.37	-1.95	-0.78
-0.19	0.26	0.17
0.33	-0.84	-0.5
0	0	0
0.25	-0.31	-0.27
-0.14	0.1	0.1
-0.02	0.02	0.02
0.32	-0.32	-0.34
0.71	-0.46	-1.06
-	-	-

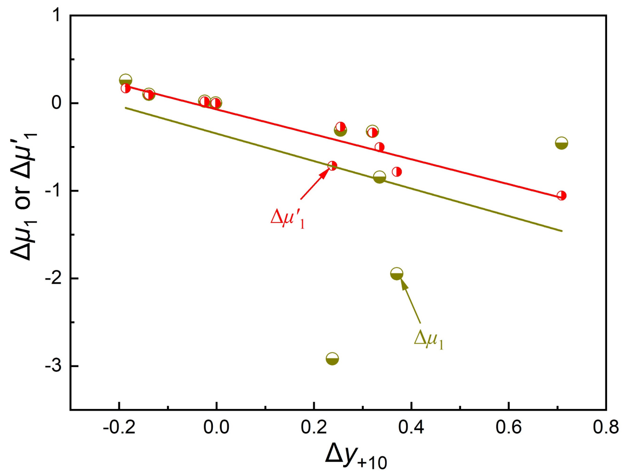

Fig. 5. Relationship between the change rate of the actual values and the two kinds of memberships

As illustrated in Figure 5, line fitting was conducted using the Origin 8.6 software, whereby $\Delta\mu_1$ and $\Delta\mu'_1$ were matched with Δy_{+10}. The R^2 value of the linear fit of $\Delta\mu'_1$ reached 0.85. In comparison, the R^2 value for the linear fit of $\Delta\mu_1$ decreased to 0.07, suggesting an enhanced consistency between $\Delta\mu'_1$ and Δy_{+10}.

C. Application case

The organic matter and nutrient contents of soil, including nitrogen (N), phosphorus and potassium, were found to have a positive correlation with crop yields within a certain range of indicator values. However, below or above this range, changes in indicator values had a minimal effect on crop yields [34]. The approximation of the nutrient supply to crop yield as an ascending semi-trapezoidal relationship proved to be somewhat inaccurate when the fuzzy integrated evaluation method was applied [35][36]. The researchers were able to derive the affiliation function based on the model equation that describes the relationship between nutrient supply and crop yield, using N fertilizer application as an illustrative example (see Figure 1 in the Ref. article [37]).

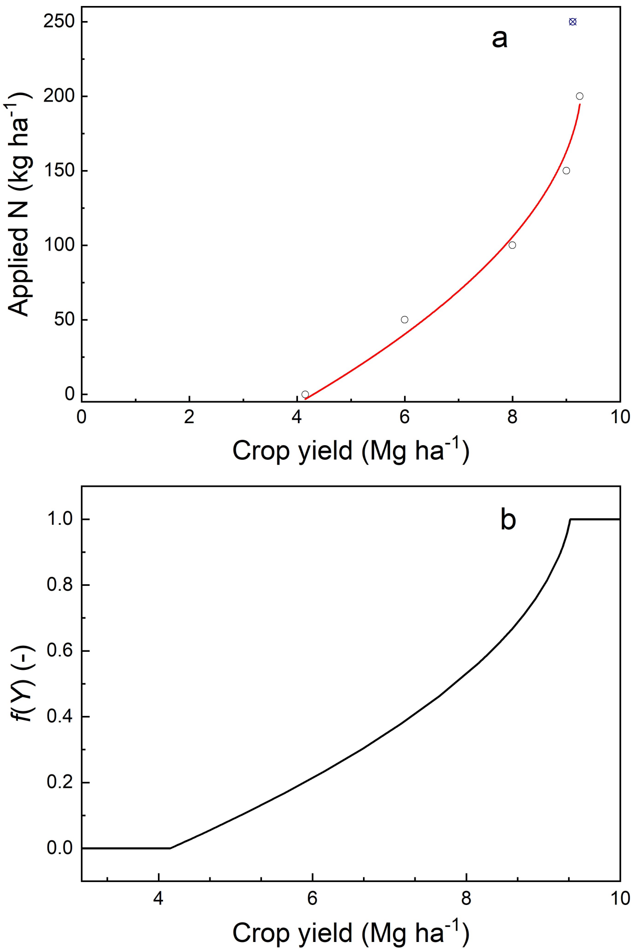

Fig. 6. Example of data from the field trials showing the observed (*circles*) and modeled (*line*) yields at different levels of N application

Given the impossibility of having two affiliations for a given indicator value, the data situated above the inflection point (Crop yield = 9.12 Mg ha^{-1}, Applied N = 250 kg ha^{-1}) was initially excluded (Figure 6a). This was followed by the computation of the inverse function based on the transformed data, as in (9).

$$N = 218.18 - \sqrt{85330.58 - 9090.91Y} \quad (9)$$

where Y is crop yield (Mg ha^{-1}), N is fertilizer N applied (kg ha^{-1}). $Y \in [4.15, 9.25]$.

The domain of definition and the range were subsequently determined (Figure 6b), culminating in the derivation of the novel membership function, as in (10).

$$f(Y) = \begin{cases} 1, Y \geq 9.25 \\ \dfrac{218.18 - \sqrt{85330.58 - 9090.91Y}}{200}, 4.15 < Y < 9.25 \\ 0, Y \leq 4.15 \end{cases} \quad (10)$$

IV. Conclusions

(1) It had been demonstrated that dispersants did not exhibit consistent aid-grinding efficiency when used on potassium shale, even when the same type of polymeric dispersant was employed, but with varying molecular weights. Polycarboxylate HT-5050 was found to be the most effective dispersant.

(2) The methodology for generating the novel membership function, based on comminution kinetics, comprised six stages: the construction of a scatter diagram of comminution, the transposition of the coordinates, the normalization of the dependent variables, the fitting of the model, the truncation the 0-1 domain portion of the function, and the definition of the range of the domain. The method has the potential to be applied not only in the field of ultrafine comminution, but also in other fields.

(3) The discrepancy between $E_{y_{+10}}$ and E_{μ_1} was 3.46%; however, the discrepancy between $E_{y_{+10}}$ and $E_{\mu'_1}$ was 1.14%. The R^2 value indicated that the novel membership function exhibited robust performance (R^2 = 0.85), whereas the cubic parabolic membership function was unreliable (R^2 = 0.07). These findings point to the conclusion that the μ'_1 calculated by the novel function is more dependable.

Acknowledgment

ZHU Zaisheng thanks Prof. LUO Zhenfu for guiding him into the field of fuzzy mathematics.

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