Adaptive Prescribed Performance Tracking Control for a Class of Nonlinear Uncertain Pure-feedback Systems

Zhenyan Wang, Yan Zhao*, Jiangbo Yu and Chengdong Li

Abstract—**An adaptive tracking control strategy is investigated for a class of nonlinear uncertain pure-feedback systems with prescribed performance constraints. Remarkably, the considered systems allow for unknown constant parameters and non-affine control input. First, by utilizing backstepping method along with barrier Lyapunov function (BLF), we introduce an output tracking control strategy. This method ensures that the predefined performance is maintained and the output asymptotically tracks a specified reference signal. Additionally, adaptive estimate and integrator are applied in this procedure to overcome the challenges posed by parametric uncertainty and non-affine input burden. The results indicate that under the presented adaptive tracking controller, the tracking error stays within the prescribed performance bounds (PPB), and all signals in the closed-loop system remain bounded. To conclude, the effectiveness of the BLF-based tracking control method is confirmed through simulations.**

Index Terms—**Uncertain pure-feedback systems; integrator; adaptive estimate; prescribed performance; barrier Lyapunov function (BLF)**

I. Introduction

TRACKING control is a key issue in the domain of nonlinear systems control. For instance, it can be applied to various scenarios, including motion trajectory control of flexible joint manipulators [1], and path tracking control of autonomous underwater vehicles [2], etc. This problem of asymptotic tracking has a long-standing history and has been extensively investigated over the last thirty years, as demonstrated in studies such as [3]-[6] and the associated references. It is important to note that traditional approaches mainly focus on the tracking control without ensuring the specified performance constraints. In practice, performance constraints are essential for practical systems. For example, as written in [7], maintaining contact and preventing excessive forces are critical in a robot's contact task. The end effector may sustain damage, and the surrounding environment could be at risk if contact is lost, leading to uncontrollable consequences. Therefore, it is undoubtedly meaningful to consider asymptotic tracking control with specified performance requirements.

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Zhenyan Wang is a postgraduate student of the School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan, Shandong, 250101, China. (e-mail: wzyz2011@163.com).

Yan Zhao is an associate professor of the School of Science, Shandong Jianzhu University, Jinan, Shandong, 250101, China. (corresponding author, e-mail: zhaoyan@sdjzu.edu.cn).

Jiangbo Yu is an associate professor and research fellow of the School of Science, Shandong Jianzhu University, Jinan, Shandong, 250101, China. (e-mail: jbyu@sdjzu.edu.cn).

Chengdong Li is a professor of the School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan, Shandong, 250101, China. (e-mail: lichengdong@sdjzu.edu.cn).

Many interesting results had been generated in this direction over the past ten years. In particular, Bechlioulis and Rovithakis in [8] introduced a new output error transformation that ensured the tracking error remained within PPB. In [9], an event-triggered control method with predefined performance constraints was achieved by integrating velocity transformation into the BLF and introducing an intermediate variable in the system. To guarantee the tracking error stayed within PPB, the findings in [10] proposed a control design method that included a simplified-order extended state observer, an input-to-state stable supply rate variation technique, a tan-type BLF. In contrast to these developments, the literature also presents important findings on pure-feedback systems, which are considered more applicable in practical systems than strict-feedback systems. For example, the study in [11] used fuzzy control methods to approximate unknown functions and estimate unmeasured states, thereby relaxing the constraints on partial derivatives in controller design. Additionally, the research in [12] significantly reduced the complexity of controller design and achieved the system's desired performance by incorporating active disturbance rejection control into the controller design process. Work in [13] tackled the problem of random disturbances by decoupling non-parametric components and redesigning controller with predetermined performance.

Despite the aforementioned achievements in prescribed performance control, many open problems remain for nonlinear systems characterized by uncertainty, nonlinearity, and non-affine structure. Based on the authors' knowledge, limited results are available that focus on tracking error with prescribed performance for general nonlinear pure-feedback systems with parameter uncertainty. The related works in [14] and [15] proposed adaptive control strategies for tracking in nonlinear pure-feedback systems, but did not involve prescribed performance constraints. Building on existing research, we propose a adaptive tracking controller for uncertain pure-feedback systems with predefined performance. The controller guarantees that the reference trajectory is asymptotically tracked within PPB. This finding generalizes the category of nonlinear uncertain systems subject to the predefined performance constraints and further improves the existing results such as [8][12][15]-[17].

Compared to the previous results, the innovations can be outlined in three key points:

(1) Differing from the strict-feedback systems discussed in [8]-[10] as well as the pure-feedback systems considered in [11][12][16][17], this work focuses on the uncertain pure-feedback systems involving both unknown constant parameters and non-affine form, which further broadens the category of nonlinear uncertain systems.

(2) By constructing a BLF and applying state transformation, the challenging tracking control with predefined performance problem turns into a stabilization problem in a regional case. Different from the existing control schemes that use adaptive fuzzy strategies[11][18] and neural network strategies[19][20], we employ the technique of adaptive tuning function to address parameter uncertainty, thereby reducing the complexity of the control algorithm.

(3) The developed controller guarantees that all signals within the closed-loop system remain bounded as well as the tracking error tends towards a prescribed compact range within PPB. Moreover, if the bounding constants are carefully set when $\varphi^- + \varphi^+ = 0$, the tracking error can be driven towards zero, while also enhancing the system's performance in steady-state conditions.

II. Problem Statement

We consider the following uncertain pure-feedback systems

$$\left\{\begin{array}{rcl} \dot{x}_i &=& \theta^{\mathrm{T}} f_i(\bar{x}_i) + g_i(\bar{x}_i, x_{i+1}), i = 1, \cdots, n-1 \\ \dot{x}_n &=& \theta^{\mathrm{T}} f_n(\bar{x}_n) + g_n(\bar{x}_n, u) \\ y &=& x_1, \end{array}\right. \tag{1}$$

where $x = (x_1 \cdots x_n) \in R^n$ represents the system state, with u and y as the control input and output, respectively; $\theta \in R^r$ is the unknown constant parameter, while $f_1 \cdots, f_n$ and $g_1, \cdots, g_n$ are known smooth functions. It should be noted that $g_n(\bar{x}_n, u)$ involves the control input u, which suggests it does not necessarily exhibit an affine structure.

Here we provide some assumptions and lemmas on the considered systems (1).

Assumption 1. There is a positive constant ε, so that

$$\left|\frac{\partial g_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right| \geq \varepsilon, \, i = 1, \cdots, n, \tag{2}$$

where $x_{n+1} = u$.

Assumption 2. It is assumed that the desired trajectory y_d and its $(n+1)$-order time derivatives $y_d^{(i)}(i = 1, 2, \cdots, n+1)$ are both bounded as well as known.

Remark 1. The systems (1) investigated in this article do not have an affine form, which represent a broader range of nonlinear uncertain systems compared to strict-feedback systems. In reality, many practical systems can be represented by a non-affine structure, for example the Brusselator chemical reactor systems [17], the flight path angle control systems [21], and the mechanical systems [22], etc.

Remark 2. It is noted that **Assumption 1** guarantees the systems (1) remain controllable in view of $\varepsilon > 0$. Moreover, ε serves as a constrained constant and does not affect the following control design. This assumption is commonly employed in pure-feedback systems, see, for example, [16] and [23]. **Assumption 2** requires a priori knowledge of the $(n+1)$th-order derivatives $y_d^{(n+1)}$, which is essential for achieving asymptotic tracking control for a desired reference, as demonstrated in [20] and [24].

Lemma 1[16]. Given a bounded and continuously differentiable function $h_i(\bar{x}_i, x_{i+1})$ and a positive constant ε, if $\left|\frac{\partial h_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right| \geq \varepsilon$, so x_{i+1} is bounded, for $i = 1, \cdots, n$.

Lemma 2[19]. For any $|a| < 1$, $a \in R$, the following inequality is true

$$\log \frac{1}{1-a^2} \leq \frac{a^2}{1-a^2}. \tag{3}$$

Lemma 3[25]. Let $N = R^l \times Z \in R^{l+1}$ be open sets along with $Z = \{\xi \in R : |\xi| < 1\} \subset R$. Consider the following system

$$\dot{\eta} = h(t, \eta), \tag{4}$$

where $\eta = [\omega, \xi]^{\mathrm{T}} \in N$, together with $h : R_+ \times N \to R^{l+1}$ is piecewise continuous in t as well as locally Lipschitz in η, uniformly with respect to t, on N. Assume there are functions $U : R^l \times R_+ \to R_+$ along with $\upsilon_1 : Z \to R_+$, both continuously differentiable and positive definite within their respective domains, so that

$$|\xi| \to 1, \upsilon_1(\xi) \to \infty \tag{5}$$

$$\gamma_1(\|\omega\|) \leq U(\omega, t) \leq \gamma_2(\|\omega\|), \tag{6}$$

where γ_1, γ_2 are class-K_∞ functions. Define $\upsilon(\eta) = \upsilon_1(\xi) + U(\omega, t)$, as well as $\xi(0) \in Z$. If the following inequality holds

$$\dot{\upsilon} = \frac{\partial \upsilon}{\partial \eta} h(t, \eta) \leq 0, \xi \in Z, \tag{7}$$

in the set $\xi \in Z$, it follows that $\xi(t) \in Z$ for all $t \in [0, \infty)$.

This article focuses on designing the control input u to ensure asymptotic tracking of the system output y to the reference trajectory y_d, with the tracking error $z_1 = y - y_d$ satisfies the prescribed performance F

$$F = \left\{(t, z_1) \in R_{t \geq 0} \times R \,|\, \varphi_0^-(t) < z_1(t) < \varphi_0^+(t)\right\}, \tag{8}$$

where $\varphi_o^+(t)$ and $\varphi_o^-(t)$ are smooth functions that represent the predetermined performance, and meet the following requirements:

(1) It is assumed that $\varphi_o^+(t)$ and $\varphi_o^-(t)$, along with their $(n+1)$th-order derivatives, are bounded;

(2) As $t \to \infty$, we have $\lim\limits_{t\to\infty} \varphi_o^-(t) = \varphi^-$ and $\lim\limits_{t\to\infty} \varphi_o^+(t) = \varphi^+$, where φ^+ as well as φ^- are predefined constants, and moreover, $\varphi^- < \varphi^+$.

III. Adaptive Control Design

As a result of the pure-feedback structure as well as the non-affine form of the control input u, we add an auxiliary integrator as

$$\dot{u} = v, \tag{9}$$

where v denotes an auxiliary control input. The following system, offering convenience for control design, is introduced

$$\left\{\begin{array}{rcl} \dot{x}_i &=& \theta^{\mathrm{T}} f_i(\bar{x}_i) + g_i(\bar{x}_i, x_{i+1}), i = 1, \cdots, n-1 \\ \dot{x}_n &=& \theta^{\mathrm{T}} f_n(\bar{x}_n) + g_n(\bar{x}_n, u) \\ \dot{u} &=& v. \end{array}\right. \tag{10}$$

Remark 3. With the addition of an auxiliary integrator, the systems (1) with non-affine structure is converted to the systems (10). In this case, the u can be computed through (9) and the v in **Step** $n+1$, thus solving the issue of the control input u is involved in $g_n(\bar{x}_n, u)$.

Different from the standard backstepping, we perform the changes of coordinates

$$z_1 = x_1 - y_d \quad (11)$$

$$z_{i+1} = g_i - \alpha_i, i = 1, \cdots, n-1 \quad (12)$$

$$z_{n+1} = g_n - \alpha_n, \quad (13)$$

where α_i represents the virtual control law in **Step** i.

Remark 4. Since the system state x_{i+1} is hidden in the function g_i, the standard backstepping method is difficult to apply in the controller design. To address this design challenge, this paper adopts the approach of treating the non-affine function g_i as a virtual control variable and incorporates it into a new backstepping method.

Next, we will design the controller using a recursive method.

Step 1: We select the following augmented BLF:

$$V_1 = \frac{1}{2} \ln \frac{1}{1-\xi^2} + \frac{1}{2} \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta}, \quad (14)$$

where $\hat{\theta}$ represents the estimate of θ with the error $\tilde{\theta} = \theta - \hat{\theta}$, and Γ denotes a positive definite matrix gain, with

$$\xi = \frac{2z_1 - (\varphi_o^+ + \varphi_o^-)}{\varphi_o^+ - \varphi_o^-}, \quad (15)$$

In terms of (11), (14) and (15), we have

$$\begin{aligned} \dot{V}_1 = & \frac{2\left(\theta^{\mathrm{T}} f_1 + z_2 + \alpha_1 - \dot{y}_d\right)\xi}{(1-\xi^2)\left(\varphi_o^+ - \varphi_o^-\right)} + \frac{2\xi \dot{\varphi}_o^+ \varphi_o^-}{\left(\varphi_o^+ - \varphi_o^-\right)^2} \\ & - \frac{2z_1\left(\dot{\varphi}_o^+ - \dot{\varphi}_o^-\right)\xi}{\left(\varphi_o^+ - \varphi_o^-\right)^2} - \frac{2\xi \dot{\varphi}_o^- \varphi_o^+}{\left(\varphi_o^+ - \varphi_o^-\right)^2} \\ & + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} \dot{\hat{\theta}}. \end{aligned} \quad (16)$$

Then, we design the virtual control law α_1 along with the tuning function τ_1 as shown below

$$\alpha_1 = -c_1 z_1 - \hat{\theta}^{\mathrm{T}} f_1 + \frac{1}{2} c_1 \left(\varphi_o^+ + \varphi_o^-\right) + \vartheta \quad (17)$$

$$\tau_1 = \frac{2\xi \Gamma f_1}{(1-\xi^2)\left(\varphi_o^+ - \varphi_o^-\right)}, \quad (18)$$

where $c_1 > 0$ is a design constant and $\vartheta = \dot{y}_d + \frac{z_1\left(\dot{\varphi}_o^+ - \dot{\varphi}_o^-\right) - \dot{\varphi}_o^+ \varphi_o^- + \dot{\varphi}_o^- \varphi_o^+}{\varphi_o^+ - \varphi_o^-}$.

By combining (17) and (18), we know

$$\begin{aligned} \dot{V}_1 = & -c_1 \frac{\xi^2}{1-\xi^2} + \frac{2\xi z_2}{(1-\xi^2)\left(\varphi_o^+ - \varphi_o^-\right)} \\ & + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_1\right). \end{aligned} \quad (19)$$

Step 2: Consider $\zeta = (y_d, \varphi_o^+, \varphi_o^-)^{\mathrm{T}}$, and it is known from **Assumption 2** that its $(n+1)$-order time derivatives $\zeta^{(i)}$ (for $i = 1, 2, \cdots, n+1$) are bounded as well as known.

Take a candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2, \quad (20)$$

also from (12), (19) and (20), we have

$$\begin{aligned} \dot{V}_2 = & -c_1 \frac{\xi^2}{1-\xi^2} + \frac{2\xi z_2}{(1-\xi^2)\left(\varphi_o^+ - \varphi_o^-\right)} \\ & + z_2 \left\{ \frac{\partial g_1}{\partial x_2} \left(\theta^{\mathrm{T}} f_2 + z_3 + \alpha_2\right) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \right. \\ & - \left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1} \right) \left(\theta^{\mathrm{T}} f_1 + g_1\right) \\ & \left. - \sum_{j=0}^{1} \frac{\partial \alpha_1}{\partial \zeta^{(j)}} \zeta^{(j+1)} \right\} + \left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_1\right). \end{aligned} \quad (21)$$

Then α_2 and τ_2 are defined, as described below

$$\begin{aligned} \alpha_2 = & \frac{1}{\frac{\partial g_1}{\partial x_2}} \left\{ - \frac{2\xi}{(1-\xi^2)\left(\varphi_o^+ - \varphi_o^-\right)} - c_2 z_2 \right. \\ & + \sum_{j=0}^{1} \frac{\partial \alpha_1}{\partial \zeta^{(j)}} \zeta^{(j+1)} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \tau_2 \\ & - \hat{\theta}^{\mathrm{T}} \left(\frac{\partial g_1}{\partial x_2} f_2 - \left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1} \right) f_1 \right) \\ & \left. + \left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1} \right) g_1 \right\} \end{aligned} \quad (22)$$

$$\tau_2 = \tau_1 + z_2 \Gamma \left(\frac{\partial g_1}{\partial x_2} f_2 - \left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1} \right) f_1 \right), \quad (23)$$

with $c_2 > 0$ a design constant.

Then, by directly applying (22) and (23) in (21), we can acquire

$$\begin{aligned} \dot{V}_2 = & -c_1 \frac{\xi^2}{1-\xi^2} - c_2 z_2^2 + \frac{\partial g_1}{\partial x_2} z_2 z_3 \\ & + \left(\left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_2\right). \end{aligned} \quad (24)$$

Step i ($3 \leq i \leq n$): Assuming that α_{i-1} and τ_{i-1} are designed, with $z_{i-1} = g_i - \alpha_{i-1}$. From (24), we get

$$\begin{aligned} \dot{V}_{i-1} = & \left(\left(\hat{\theta} - \theta\right)^{\mathrm{T}} \Gamma^{-1} - \sum_{j=1}^{i-2} z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_{i-1}\right) \\ & + \frac{\partial g_{i-2}}{\partial x_{i-1}} z_{i-1} z_i - c_1 \frac{\xi^2}{1-\xi^2} - \sum_{j=2}^{i-1} c_j z_j^2. \end{aligned} \quad (25)$$

Then, we will demonstrate that the property (25) also applies in **Step** i as well. For this purpose, V_i is chosen as follows

$$V_i = V_{i-1} + \frac{1}{2} z_i^2. \quad (26)$$

In view of (12), (25) and (26), $\dot{V}_i$ is possible to obtain as

$$\begin{aligned} \dot{V}_i = & -c_1 \frac{\xi^2}{1-\xi^2} - \sum_{j=2}^{i-1} c_j z_j^2 + z_i \left\{ \frac{\partial g_{i-2}}{\partial x_{i-1}} z_{i-1} \right. \\ & + \frac{\partial g_{i-1}}{\partial x_i} \left(\theta^{\mathrm{T}} f_i + z_{i+1} + \alpha_i\right) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ & + \sum_{j=1}^{i-1} \left(\frac{\partial g_{i-1}}{\partial x_j} - \frac{\partial \alpha_{i-1}}{\partial x_j} \right) \left(\theta^{\mathrm{T}} f_j + g_j\right) \\ & \left. - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \zeta^{(j)}} \zeta^{(j+1)} \right\} + \left(\dot{\hat{\theta}} - \tau_{i-1}\right) \end{aligned} \quad (27)$$

$$\cdot\Big((\hat{\theta}-\theta)^{\mathrm{T}}\Gamma^{-1}-\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Big).$$

Then, α_i and τ_i can be given as follows

$$\begin{aligned}\alpha_i &= \frac{1}{\frac{\partial g_{i-1}}{\partial x_i}}\Bigg\{\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}-\frac{\partial g_{i-1}}{\partial x_j}\right)g_j+\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\tau_i\\ &\quad+\Big(\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma-\hat{\theta}^{\mathrm{T}}\Big)\Big(\frac{\partial g_{i-1}}{\partial x_i}f_i\\ &\quad-\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}-\frac{\partial g_{i-1}}{\partial x_j}\right)f_j\Big)-c_iz_i \qquad (28)\\ &\quad-\frac{\partial g_{i-2}}{\partial x_{i-1}}z_{i-1}+\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta^{(j)}}\zeta^{(j+1)}\Bigg\}\\ \tau_i &= \tau_{i-1}+\Gamma z_i\Bigg\{\frac{\partial g_{i-1}}{\partial x_i}f_i\\ &\quad-\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}-\frac{\partial g_{i-1}}{\partial x_j}\right)f_j\Bigg\}, \qquad (29)\end{aligned}$$

where c_i's are positive design constants.

Consequently, we can conclude from (27), (28), and (29) that

$$\begin{aligned}\dot{V}_i &= \Big((\hat{\theta}-\theta)^{\mathrm{T}}\Gamma^{-1}-\sum_{j=1}^{i-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Big)\Big(\dot{\hat{\theta}}-\tau_i\Big)\\ &\quad-c_1\frac{\xi^2}{1-\xi^2}-\sum_{j=2}^{i}c_jz_j^2+\frac{\partial g_{i-1}}{\partial x_i}z_iz_{i+1}. \qquad (30)\end{aligned}$$

Step $n+1$: At this stage, we are ready to design the actual control and the adaptive law. For this purpose, a candidate Lyapunov function is chosen in the following manner

$$V_{n+1}=V_n+\frac{1}{2}z_{n+1}^2. \qquad (31)$$

From (13) and (31), we obtain

$$\begin{aligned}\dot{V}_{n+1} &= -c_1\frac{\xi^2}{1-\xi^2}-\sum_{j=2}^{n}c_jz_j^2+z_{n+1}\Bigg\{\frac{\partial g_{n-1}}{\partial x_n}z_n\\ &\quad+\sum_{j=1}^{n}\left(\frac{\partial g_n}{\partial x_j}-\frac{\partial\alpha_n}{\partial x_j}\right)g_j-\frac{\partial\alpha_n}{\partial\hat{\theta}}\dot{\hat{\theta}}\\ &\quad+\frac{\partial g_n}{\partial u}v+\sum_{j=1}^{n}\left(\frac{\partial g_n}{\partial x_j}-\frac{\partial\alpha_n}{\partial x_j}\right)\theta^{\mathrm{T}}f_j \qquad (32)\\ &\quad-\sum_{j=0}^{n}\frac{\partial\alpha_n}{\partial\zeta^{(j)}}\zeta^{(j+1)}\Bigg\}+\Big(\dot{\hat{\theta}}-\tau_n\Big)\\ &\quad\cdot\Big(\Big(\hat{\theta}-\theta\Big)^{\mathrm{T}}\Gamma^{-1}-\sum_{j=1}^{n-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Big).\end{aligned}$$

Define the auxiliary control law v and the adaptive law $\dot{\hat{\theta}}$ in the following way

$$\begin{aligned}v &= \frac{1}{\frac{\partial g_n}{\partial u}}\Bigg\{-c_{n+1}z_{n+1}-\frac{\partial g_{n-1}}{\partial x_n}z_n\\ &\quad+\frac{\partial\alpha_n}{\partial\hat{\theta}}\tau_{n+1}+\Big(\hat{\theta}^{\mathrm{T}}-\sum_{j=1}^{n-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma\Big) \qquad (33)\\ &\quad\cdot\sum_{j=1}^{n}\left(\frac{\partial\alpha_n}{\partial x_j}-\frac{\partial g_n}{\partial x_j}\right)f_j+\sum_{j=0}^{n}\frac{\partial\alpha_n}{\partial\zeta^{(j)}}\zeta^{(j+1)}\\ &\quad-\sum_{j=1}^{n}\left(\frac{\partial g_n}{\partial x_j}-\frac{\partial\alpha_n}{\partial x_j}\right)g_j\Bigg\}\\ \dot{\hat{\theta}} &= \tau_{n+1} \qquad (34)\\ &= \tau_n+\Gamma z_{n+1}\sum_{j=1}^{n}\left(\frac{\partial\alpha_n}{\partial x_j}-\frac{\partial g_n}{\partial x_j}\right)f_j,\end{aligned}$$

with $c_{n+1}>0$ a design constant.

For any specified initial value of the control $u(0)$, substituting (33) into (9) gives the real control input u.

Combing (33) as well as (34), yields

$$\dot{V}_{n+1}=-c_1\frac{\xi^2}{1-\xi^2}-\sum_{j=2}^{n+1}c_jz_j^2. \qquad (35)$$

Here we provide one theorem on the considered systems (1).

Theorem 1. If the systems (1) satisfy **Assumptions 1-2**, the presented controller, which includes the auxiliary control law (33) with (9) and the adaptive law (34) will guarantee that the following properties are observed in the closed-loop system:

1) All signals in the closed-loop system remain bounded.

2) The tracking error $z_1=y-y_d$ tends towards a prescribed compact set within two predefined performance boundary constraints, furthermore

$$\lim_{t\to\infty}z_1(t)=\frac{\varphi^-+\varphi^+}{2}. \qquad (36)$$

Proof: To start, we will demonstrate that the closed-loop solution is well-defined and remains bounded over the interval $[0,\infty)$. Next, we will show that the tracking error tends towards a given compact set while staying within the established performance bounds.

Firstly, the right-hand sides of the closed-loop system are locally Lipschitz around the initial conditions, which guarantees the existence of a unique solution over a small interval $[0,t_f)$. Suppose that the result is given on a right-maximal interval $[0,T_f)$ where $0<T_f\le\infty$. We will confirm that $T_f=\infty$. As a result, we know from (35) that V_{n+1} is a non-increasing function on $[0,T_f)$ when $\varphi_0^-(0)<z_1(0)<\varphi_0^+(0)$ and $|\xi(0)|<1$ in (8). Then, we can conclude that V_{n+1}, $\xi(t)$, $z_i(t)$ (for $i=1,2,\cdots,n+1$), as well as $\tilde{\theta}(t)$ remain bounded over $[0,T_f)$, and moreover $|\xi(t)|<1$, $t\in[0,T_f)$. Considering θ is a constant vector and $\tilde{\theta}(t)=\theta-\hat{\theta}(t)$, we further get that $\hat{\theta}(t)$ also remains bounded over $[0,T_f)$.

The boundedness of z_1 together with **Assumption 2** and the transformation $z_1=x_1-y_d$ implies that x_1 remains bounded over $[0,T_f)$. Given the boundedness of $(z_1,\xi,\hat{\theta})$ and (17), we can conclude that α_1 is bounded on $[0,T_f)$ and the boundedness of z_2 further means that g_1 remains bounded due to $z_2 = g_1 - \alpha_1$. Then, it is known from **Assumption 1** and **Lemma 1** that x_2 remains bounded. Since z_2, g_1 and ξ remain bounded over $[0, T_f)$, we get that α_2 is bounded over $[0, T_f)$. Further, the boundedness of $z_3 = g_2 - \alpha_2$ means that g_2 remains bounded, which yields that x_3 also remains bounded over $[0, T_f)$. Similarly, it can be shown that $x_4, \cdots, x_n, \alpha_3, \cdots, \alpha_n, u, v$ are all bounded over $[0, T_f)$ in terms of $u = x_{n+1}$. Therefore, there is no finite escape on $[0, T_f)$ and hence, $T_f = \infty$. Up to this point, all signals of the closed-loop system remain bounded over $[0, \infty)$ thus completing property 1).

Secondly, according to **Lemma 2**, we further know that $\dot{V}_{n+1}$ in (35) further turns into

$$\dot{V}_{n+1} \leq -c_1 \log \frac{1}{1-\xi^2} - \sum_{j=2}^{n+1} c_j z_j^2. \quad (37)$$

According to the LaSalle-Yoshizawa theorem in [4], we can derive that

$$\lim_{t\to\infty} \log \frac{1}{1-\xi^2(t)} = 0, \quad (38)$$

then

$$\lim_{t\to\infty} \xi(t) = 0. \quad (39)$$

Given that $|\xi(t)| < 1$ for $t \geq 0$, we know

$$\varphi_0^-(t) < z_1(t) < \varphi_0^+(t),\ t \geq 0. \quad (40)$$

This shows that the tracking error $z_1 = y - y_d$ tends towards a prescribed compact set within the predefined performance constraints for all $t \geq 0$. Moreover, in terms of the transformation in (15) and $\lim_{t\to\infty} \xi(t) = 0$ in (39), we can also infer that

$$\lim_{t\to\infty} z_1(t) = \frac{\varphi^- + \varphi^+}{2}. \quad (41)$$

This demonstrates that the property 2) holds, and the proof is hence completed.

Remark 5. The predetermined performance constraint $\varphi_0^- < z_1 < \varphi_0^+$ can be written as $\varphi_0^- + y_\mathrm{d} < y < \varphi_0^+ + y_\mathrm{d}$, that is, the predetermined performance constraint problem of systems (1) can be transformed into a time-varying output constraint problem of systems (1). Compared to literature [11], this paper chooses a simple BLF V_1, which directly guarantees the predetermined performance constraint and simplifies the designed controller. Meanwhile, in the **Step 1** of the virtual control design, this paper avoids using inequality scaling, thereby reducing the conservativeness of the resulting control algorithm.

Remark 6. In this study, the tracking error could tend towards zero under the additional condition that the predefined bounding constants satisfy $\varphi^- + \varphi^+ = 0$. In this case, the asymptotic output tracking control for time-varying desired references can also be realized with predefined performance such as [25]. This finding is verified in the subsequent simulation examples.

Remark 7. The tracking control is examined for a category of nonlinear uncertain pure-feedback systems with prescribed performance. It is shown that the static uncertainty such as the unknown parameters can be well handled in the control synthesis. However, some kinds of more complicated dynamic uncertainties, such as the (input) unmodeled dynamics are not considered in this article. This result has not been reported in the existing literature, and it remains an open issue yet to be resolved.

IV. Simulation Examples

This section presents three examples to demonstrate the success of the proposed control scheme in this paper.

Example 1: We analyze the subsequent nonlinear system, which describes the controlled oscillator circuit system [26] (as shown in Fig. 1), in the form presented below:

$$\ddot{v}_1 + \epsilon h'(v_1)\dot{v}_1 + v_1 = u, \quad (42)$$

where u represents the controller, v_1 denotes the voltage across the resistive element, $h'(v_1)$ characterizes the behavior of the resistive element, and ϵ is a constant parameter. In this case, we consider $x_1 = v_1$, $x_2 = \dot{v}_1$, and $h(v_1) = -v_1 + \frac{1}{3}v_1^3$ like in [26]. Then, the system (42) turns into

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \epsilon\left(1 + x_1^2\right)x_2 + u - x_1 \\ y &= x_1, \end{cases} \quad (43)$$

which falls into the considered systems (1) with $f_1 = 0$, $f_2 = \left(1 + x_1^2\right)x_2$, $g_1 = x_2$, $g_2 = u - x_1$ and $\epsilon = 1$. Then, we formulate the control input

$$\begin{aligned} v &= -c_3 z_3 - z_2 + \Big(\frac{\partial \alpha_2}{\partial x_1} + 1\Big)g_1 + \frac{\partial \alpha_2}{\partial x_2}g_2 \\ &\quad + \frac{\partial \alpha_2}{\partial \hat{\theta}_2}\dot{\hat{\theta}}_2 + \sum_{j=0}^{2}\frac{\partial \alpha_2}{\partial \zeta^{(j)}}\zeta^{(j+1)} + \hat{\theta}_2\frac{\partial \alpha_2}{\partial x_2}f_2 \quad (44) \\ \dot{u} &= v, \quad (45) \end{aligned}$$

and the adaptive updating law

$$\dot{\hat{\theta}}_2 = z_2 f_2 - z_3 \frac{\partial \alpha_2}{\partial x_2} f_2, \quad (46)$$

with $y_d(t) = 0.3\cos t + 0.5\sin(0.6t)$, $\varphi_0^+(t) = e^{-0.4t} + 0.05$, $\varphi_0^-(t) = -\left(e^{-0.4t} + 0.05\right)$, $\alpha_1 = -c_1 z_1 + \dot{y}_d + \frac{z_1\dot{\varphi}_o^+}{\varphi_o^+}$, $z_2 = g_1 - \alpha_1$, $z_3 = g_2 - \alpha_2$, $\zeta = \left(y_d, \varphi_o^+, \varphi_o^-\right)^{\mathrm{T}}$, $\xi = \frac{z_1}{\varphi_o^+}$, and $\alpha_2 = -c_2 z_2 - \frac{\xi}{(1-\xi^2)\varphi_o^+} + \frac{\partial \alpha_1}{\partial x_1}g_1 + \frac{\partial \alpha_1}{\partial \zeta}\dot{\zeta} + \frac{\partial \alpha_1}{\partial \zeta^{(1)}}\zeta^{(2)} - \hat{\theta}_2 f_2$.

The design parameters of system (43) are selected as $[\theta_2, c_1, c_2, c_3]^{\mathrm{T}} = [1, 1, 1, 1]^{\mathrm{T}}$. The initials of system (43) are set as $[x_1(0), x_2(0), \hat{\theta}_2(0), u(0)]^{\mathrm{T}} = [0.2, 0.2, 0.3, 0.2]^{\mathrm{T}}$. Specifically, Fig. 2 illustrates that the system output accurately tracks the desired reference signal. As shown in Figs. 3-4, the control input and the adaptive estimate $\hat{\theta}_2$ remain bounded. Fig. 5 illustrates the tracking error of the system output, and it is evident that the tracking error stays within the PPB at all times.

Example 2: Next, we examine a single-link manipulator [27], illustrated in Fig. 6, whose dynamics are described as

$$I\ddot{q}(t) + B\dot{q}(t) + Mgl\sin(q(t)) = u(t), \quad (47)$$

where $q(t)$ stands for the link angles, $g = 9.81N/s^2$ is the gravitational constant, $u(t)$ denotes the controller, and the meanings of the remaining symbols can be found in [27]. For simplicity, we set $B = 2kg\cdot m/s$, $l = 1m$, $I = 1kg\cdot m^2$ and $M = 1kg$.

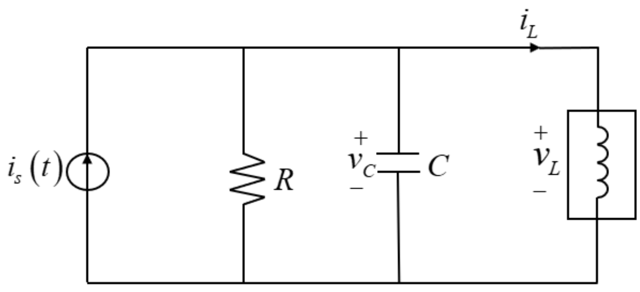

Fig. 1. Controlled oscillator circuit

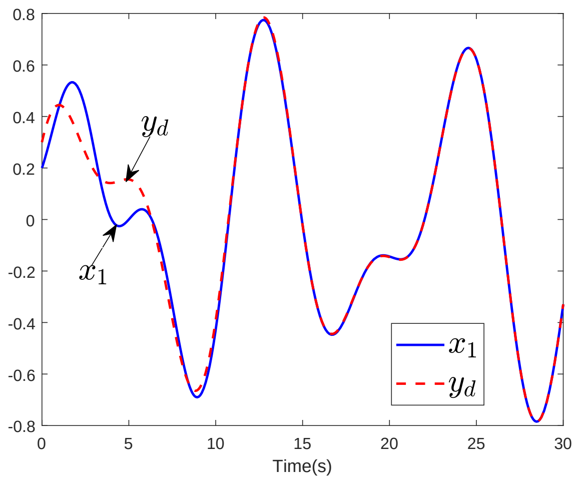

Fig. 2. The system output and tracking signal of closed-loop system (43)–(46)

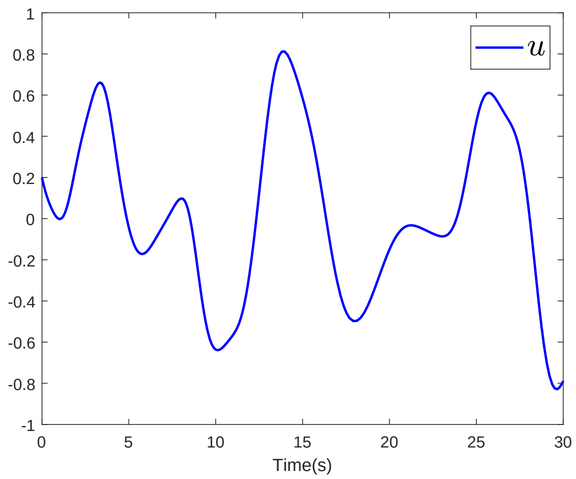

Fig. 3. The control input u of closed-loop system (43)–(46)

Let $x_1 = q(t)$ and $x_2 = \dot{q}(t)$. Then, according to (47), the dynamics are

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -\frac{1}{I}\left(Bx_2 + Mgl\sin(x_1)\right) + \frac{1}{I}u \\ y & = & x_1, \end{array} \right. \quad (48)$$

which falls into the considered systems (1) with $f_1 = 0$, $f_2 = -\left(2x_2 + 9.81\sin(x_1)\right)$, $g_1 = x_2$ and $g_2 = u$. Then we

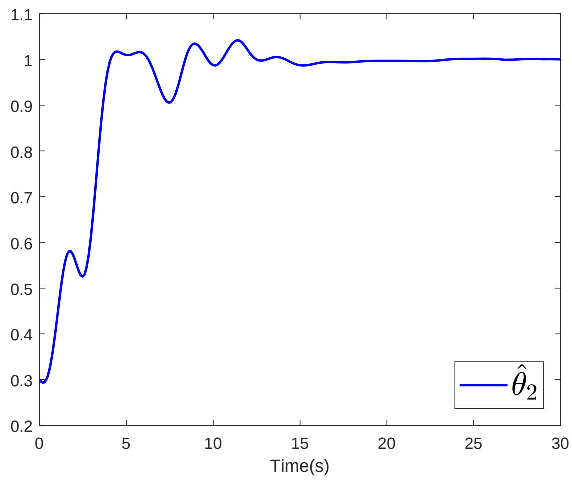

Fig. 4. The adaptive estimate $\hat{\theta}_2$ of closed-loop system (43)–(46)

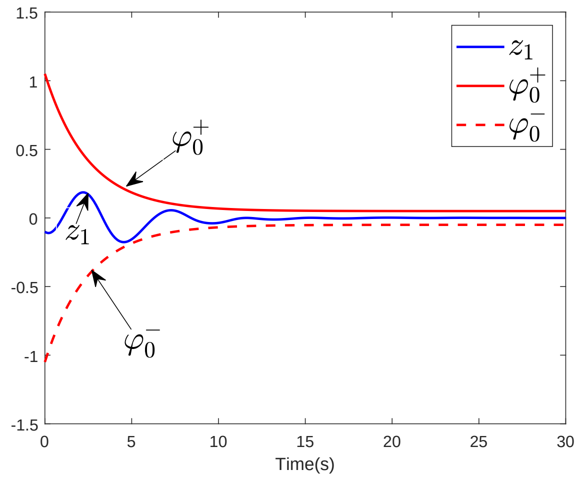

Fig. 5. The output tracking error with predetermined performance of closed-loop system (43)–(46)

formulate the control input

$$\begin{aligned} v &= -c_3 z_3 - z_2 + \frac{\partial \alpha_2}{\partial x_1} g_1 + \frac{\partial \alpha_2}{\partial x_2} g_2 \\ &\quad + \frac{\partial \alpha_2}{\partial \hat{\theta}_2}\dot{\hat{\theta}}_2 + \sum_{j=0}^{2} \frac{\partial \alpha_2}{\partial \zeta^{(j)}} \zeta^{(j+1)} + \hat{\theta}_2 \frac{\partial \alpha_2}{\partial x_2} f_2 \quad (49) \\ \dot{u} &= v, \quad (50) \end{aligned}$$

and the adaptive updating law

$$\dot{\hat{\theta}}_2 = z_2 f_2 - z_3 \frac{\partial \alpha_2}{\partial x_2} f_2, \quad (51)$$

with $y_d(t) = 0.5\cos(0.6t) + 0.5\sin t$, $\varphi_0^+(t) = e^{-t} + 0.1$, $\varphi_0^-(t) = -(e^{-t} + 0.1)$, $\alpha_1 = -c_1 z_1 + \dot{y}_d + \frac{z_1 \dot{\varphi}_o^+}{\varphi_o^+}$, $z_2 = g_1 - \alpha_1$, $z_3 = g_2 - \alpha_2$, $\zeta = (y_d, \varphi_o^+, \varphi_o^-)^{\mathrm{T}}$, $\xi = \frac{z_1}{\varphi_o^+}$, and $\alpha_2 = -c_2 z_2 - \frac{\xi}{(1-\xi^2)\varphi_o^+} + \frac{\partial \alpha_1}{\partial x_1} g_1 + \frac{\partial \alpha_1}{\partial \zeta}\dot{\zeta} + \frac{\partial \alpha_1}{\partial \zeta^{(1)}}\zeta^{(2)} - \hat{\theta}_2 f_2$.

The controller design parameters of system (48) are chosen as $[\theta_2, c_1, c_2, c_3]^{\mathrm{T}} = [-1, 1, 1, 1]^{\mathrm{T}}$. The initials of system (48) are set as $[x_1(0), x_2(0), \hat{\theta}_2(0), u(0)]^{\mathrm{T}} = [0.1, 0.5, 0.3, 0.5]^{\mathrm{T}}$. It is evident from Figs 7-10 that when using the BLF-based controller, the system output tracking error remains within the PPB.

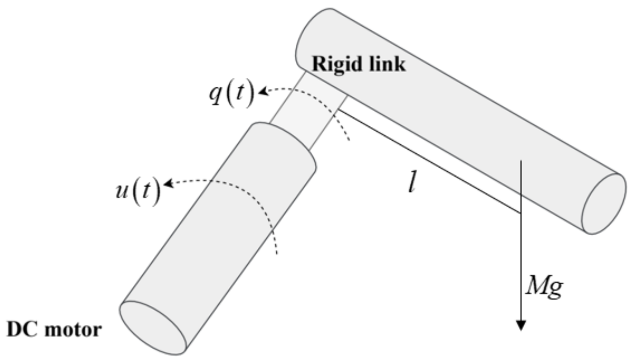

Fig. 6. Single-link manipulator

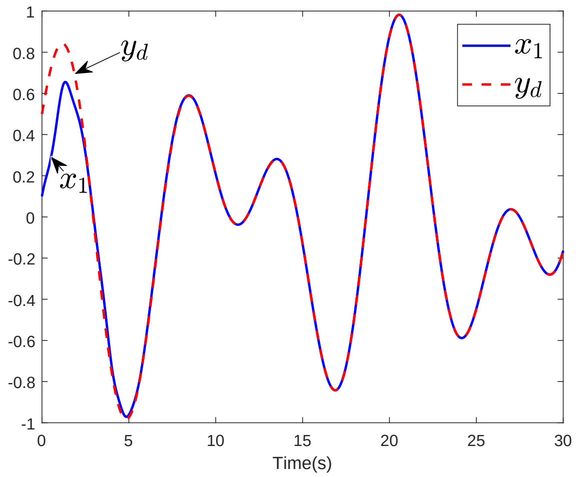

Fig. 7. The system output and tracking signal of closed-loop system (48)–(51)

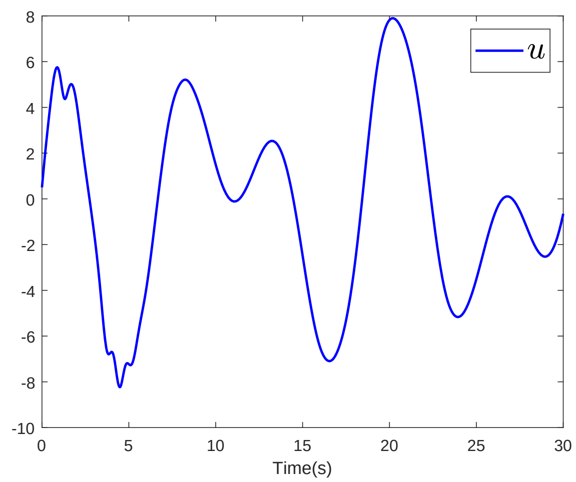

Fig. 8. The control input u of closed-loop system (48)–(51)

Example 3: In this case, we present the following numerical example

$$\left\{\begin{array}{rcl}\dot{x}_1 & = & \theta_1 x_1 + x_2 + x_2^3 \\ \dot{x}_2 & = & \theta_2 x_1 x_2 + u + \frac{u^3}{7} \\ y & = & x_1,\end{array}\right. \tag{52}$$

where $\theta = [\theta_1, \theta_2]^{\mathrm{T}}$ is the unknown constant parameter

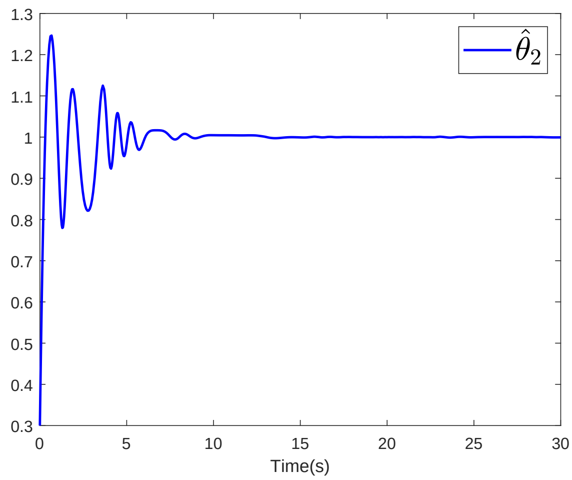

Fig. 9. The adaptive estimate $\hat{\theta}_2$ of closed-loop system (48)–(51)

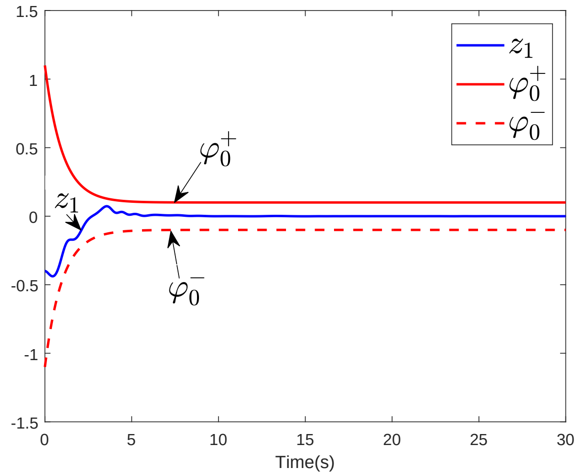

Fig. 10. The output tracking error with predetermined performance of closed-loop system (48)–(51)

vector. This system (52) falls into the considered pure-feedback structure with $f_1 = x_1$, $f_2 = x_1 x_2$, $g_1 = x_2 + x_2^3$, and $g_2 = u + \frac{u^3}{7}$. Then we formulate the control input

$$\begin{aligned} v \;=\; & \frac{1}{\frac{\partial g_2}{\partial u}}\Bigg\{ -c_3 z_3 + \sum_{j=1}^{2}\Big(\frac{\partial \alpha_2}{\partial x_j} - \frac{\partial g_2}{\partial x_j}\Big) g_j \\ & - \frac{\partial g_1}{\partial x_2} z_2 + \frac{\partial \alpha_2}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\Big(\frac{\partial \alpha_2}{\partial x_1} - \frac{\partial g_2}{\partial x_1}\Big) \\ & + \hat{\theta}_1\Big(\frac{\partial \alpha_2}{\partial x_1} - \frac{\partial g_2}{\partial x_1}\Big) f_1 + \frac{\partial \alpha_2}{\partial \hat{\theta}_2}\dot{\hat{\theta}}_2 \\ & + \hat{\theta}_2\Big(\frac{\partial \alpha_2}{\partial x_2} - \frac{\partial g_2}{\partial x_2}\Big) f_2 + \sum_{j=0}^{2}\frac{\partial \alpha_2}{\partial \zeta^{(j)}}\zeta^{(j+1)}\Bigg\} \end{aligned} \tag{53}$$

$$\dot{u} = v, \tag{54}$$

and the adaptive updating laws

$$\begin{aligned} \dot{\hat{\theta}}_1 \;=\; & \frac{2\xi x_1}{(1-\xi^2)\left(\varphi_o^{+} - \varphi_o^{-}\right)} - z_2\Big(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1}\Big) x_1 \\ & - z_3\Big(\frac{\partial \alpha_2}{\partial x_1} - \frac{\partial g_2}{\partial x_1}\Big) x_1 \end{aligned} \tag{55}$$

$$\dot{\hat{\theta}}_2 = z_2 \frac{\partial g_1}{\partial x_2} f_2 - z_3\Big(\frac{\partial \alpha_2}{\partial x_2} - \frac{\partial g_2}{\partial x_2}\Big) f_2, \tag{56}$$

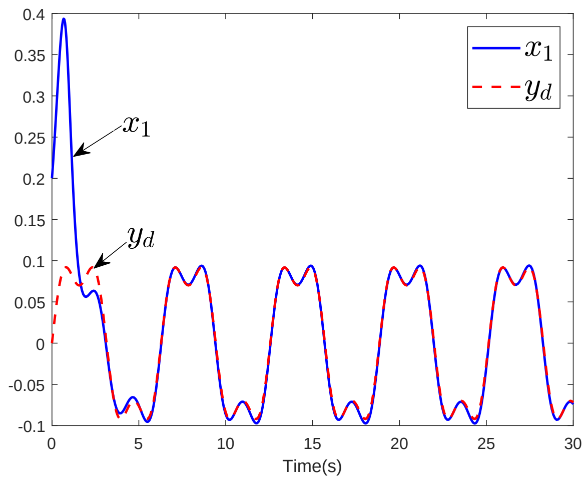

Fig. 11. The system output and tracking signal of closed-loop system (52)–(56)

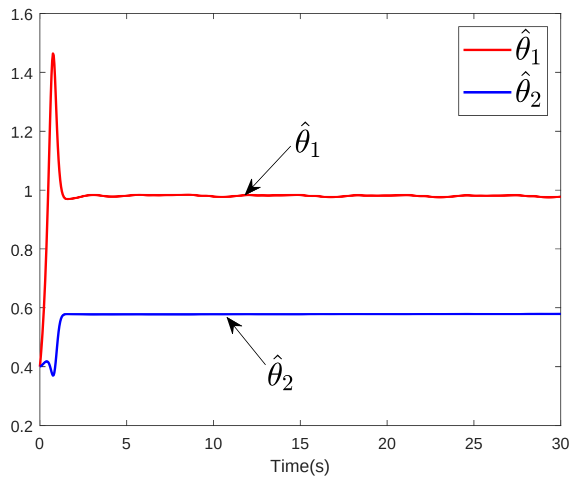

Fig. 13. The adaptive estimates $\hat{\theta}_1$, $\hat{\theta}_2$ of closed-loop system (52)–(56)

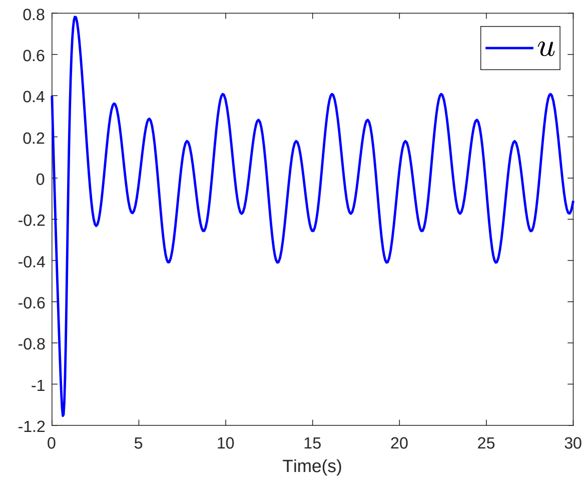

Fig. 12. The control input u of closed-loop system (52)–(56)

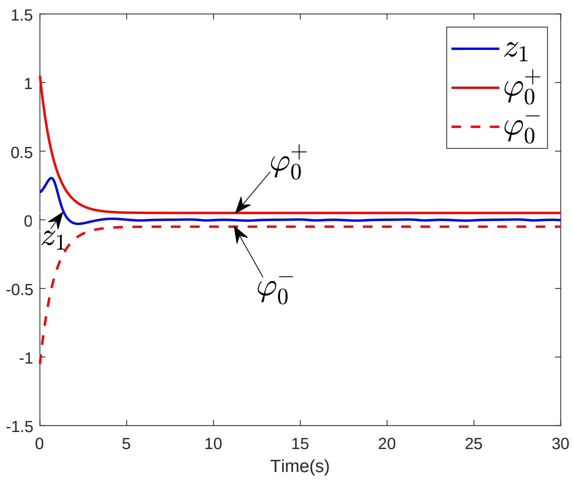

Fig. 14. The output tracking error with predetermined performance of closed-loop system (52)–(56)

with $y_d(t) = 0.1\left(\sin t + 0.3\sin(3t)\right)$, $\varphi_0^+(t) = e^{-1.2t} + 0.05$, $\varphi_0^-(t) = -\left(e^{-1.2t} + 0.05\right)$, $\alpha_1 = -\hat{\theta}_1 x_1 - c_1 z_1 + \frac{1}{2}c_1\left(\varphi_o^+ + \varphi_o^-\right) + \dot{y}_d + \frac{z_1\left(\dot{\varphi}_o^+ - \dot{\varphi}_o^-\right)}{\varphi_o^+ - \varphi_o^-}$, $\xi = \frac{z_1}{\varphi_o^+}$, $\alpha_2 = \frac{1}{\frac{\partial g_1}{\partial x_2}}\Big\{ - c_2 z_2 - \frac{2\xi}{(1-\xi^2)(\varphi_o^+ - \varphi_o^-)} + \left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1}\right)g_1 + \frac{\partial \alpha_1}{\partial \zeta}\dot{\zeta} + \frac{\partial \alpha_1}{\partial \zeta^{(1)}}\zeta^{(2)} - \hat{\theta}_2 \frac{\partial g_1}{\partial x_2} f_2 + \hat{\theta}_1\left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1}\right)x_1 + \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\tau_{21}\Big\}$, and $\tau_{21} = \frac{2\xi x_1}{(1-\xi^2)(\varphi_o^+ - \varphi_o^-)} - z_2\left(\frac{\partial \alpha_1}{\partial x_1} - \frac{\partial g_1}{\partial x_1}\right)x_1$.

For simulation purposes, the controller design parameters of system (52) are taken as $[\theta_1, \theta_2, c_1, c_2, c_3]^{\mathrm{T}} = [-1, 1, 1, 1, 1]^{\mathrm{T}}$. The initials of system (52) are set as $[x_1(0), x_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0), u(0)]^{\mathrm{T}} = [0.2, 0.07, 0.4, 0.4, 0.4]^{\mathrm{T}}$. Fig. 11 demonstrates that the output can asymptotically follow the desired trajectory. The control input and the adaptive estimates $(\hat{\theta}_1, \hat{\theta}_2)$ are presented in Fig. 12 and Fig. 13, in that order. As seen in Fig. 14, the tracking error tends towards zero, which is attributed to the condition $\varphi^- + \varphi^+ = 0$, ensuring that the PPB are never violated throughout.

V. Conclusion

A method for output tracking control is presented for a category of nonlinear systems in pure-feedback structure, which involves a non-affine control input and constant parameter uncertainty in this paper. Furthermore, using BLF technique and the integrator, we design an adaptive controller with prescribed performance via backstepping. Simultaneously, this controller ensures that the system output asymptotically tracks a given reference signal, while also guaranteeing that all signals in the closed-loop system remain bounded and the tracking error satisfies the predefined performance constraints. A potential direction for future research is to expand the findings of this paper to pure-feedback systems with time-varying parameters.

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