Prescribed-time Attitude Tracking Control of Quadrotor UAVs with Unknown Disturbances

Xuejiao Wang, Yingsen Li, Ming Chen, and Yulin Gai

Abstract—**The issue of prescribed-time attitude tracking control is addressed for quadrotor UAVs with unknown external disturbances. Firstly, a stability criterion theorem of practically prescribed-time stabilization is proposed, and a rigorous proof is derived. On this basis, a quadrotor attitude stabilization controller employing the backstepping technology is proposed to guarantee that the attitude errors converge to a small vicinity of the origin within a specified time frame. This scheme stands out from traditional fixed-time and finite-time controls by offering the unique advantage of a preset convergence time that is independent of both initial conditions and system design parameters. The efficacy of the proposed control strategy is demonstrated by simulation results.**

Index Terms—**Practically prescribed-time control, attitude tracking control, backstepping, quadrotor UAV.**

I. Introduction

WITH the rapid development of technology, quadrotor unmanned aerial vehicles (UAVs) have become indispensable tools in the military, civil and commercial fields which are favored for their unique vertical takeoff and landing capabilities, high mobility and adaptability to complex environments [1], [2], [3], [4], [5]. The quadrotor attitude control systems, as an important part of the carriers, are vital to ensure that the quadrotor UAVs can complete various flight tasks successfully. The increasingly diverse flight tasks put forward higher performance requirements for the quadrotor attitude control systems in terms of speediness, stability, accuracy, robustness and disturbance immunity, and also bring more new challenges to the formulation of the quadrotor attitude control systems.

When the UAVs execute various tasks such as accurate delivery, target tracking or visual surveillance, it must realize the precise control of its combat attitude in a short time, especially the adjustment and maintenance of a specific attitude in a predetermined time, which puts forward extremely high requirements for the aviation control system. Furthermore, during the flight of the UAVs, they will inevitably be affected by some disturbances such as gravity and wind, which will affect the attitude control effect of the quadrotors. Therefore, to guarantee the stability of the quadrotor attitude control systems, it is necessary to address the challenge of quadrotor interference. At present, many classical methods in the control field have been utilized for the attitude control of quadrotors, including PID control [6], [7], [8], [9], backstepping control [10], [11] and sliding mode control [12], [13], [14], [15] and so on. In [8], a robust controller based on PID is proposed, which not only has good robustness but also realizes the power reduction of the controller. In [11], the backstepping method is combined with command filter, and a disturbance observer is also added to solve the trouble of input saturation and filtering errors of quadrotor UAVs. [13] adopts the two-loop integral sliding mode control method and introduces an extended state observer to estimate coupling dynamics, unpredictable uncertainties and external disruptions effectively, which made the quadrotor achieve a good trajectory tracking effect. Besides, popular algorithms also include adaptive control [16], [17], [18], [19], model predictive control [20], [21], [22], optimal control [23], [24], [25] and the combination control method of the above control algorithms [26], [27], [28], [29]. For example, [28] adopts a self-adjust control method to self-adjust the PID controller by using the adaptive mechanism based on second-order sliding mode control, and uses the fuzzy compensator to diminish the chattering phenomenon, which achieves a better performance of the autonomous flight system. In [29], a control strategy with finite-time control, adaptive control, integral backstepping control and fast terminal sliding mode control is developed by integrating the recursive control method with the robust control technique, which improves comprehensive performance in the face of uncertainty of the quadrotor UAVs.

Over the past decade, there have been numerous study results on finite-time control algorithms, which can be divided into traditional finite-time control, fixed-time control and prescribed finite-time control [30]. Different from asymptotically stable control algorithm, finite-time control can ensure that the system variables of the systems reach the equilibrium points within a finite time. Therefore, the finite-time control algorithm can achieve good control effect. Fixed-time control can be regarded as a special form of finite-time control. In contrast to the traditional finite-time control, the biggest benefit of fixed-time control is that it provides an upper bound on the convergence time, regardless of the initial conditions, which makes the fixed-time control more feasible and effective in engineering. As a new finite-time control method, prescribed-time control has attracted much attention by scholars over the past few years. And its convergence time upper bound can be related to only one control parameter. In contrast, fixed-time control method involves too many parameters to be adjusted. Soon afterwards, the theory of arbitrary time control was initially introduced by Polyakov in [31] for nonautonomous nonlinear systems, and the con-

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X. J. Wang is a postgraduate student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, P. R. China. (e-mail: wangxuejiao0205@163.com).

Y. S. Li is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, P. R. China. (e-mail: kddxzdh@sina.com).

M. Chen is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, P. R. China. (e-mail: cm8061@sina.com).

Y. L. Gai is a postgraduate student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, P. R. China. (e-mail: 17641242347@163.com).

vergence time can be specified and without depending on the initial state and control parameters. In fact, it can be regarded as a new type of prescribed-time control theory. In practical industrial processes, practically finite-time and practically fixed-time control are commonly applied, which can realize the convergence of system state variables to a small region close to the equilibrium points in finite time or fixed time.

Motivated by the preceding results, this study introduces a stability criterion theorem for practically prescribed finite-time and develops a control algorithm for attitude path tracking of quadrotor UAVs under the effect of unknown interferences, utilizing the backstepping method. In comparison to current tracking control algorithms, the primary contributions of this research are evident in two distinct aspects:

1) This study introduces, for the first time, the stability criterion theorem for practically prescribed-time stabilization, accompanied by a rigorous proof. Consequently, a new practically prescribed-time attitude control strategy is devised specifically for quadrotor UAVs.

2) In this design approach, backstepping technology is integrated with practically prescribed-time control. The incorporation of the former simplifies the controller's design and facilitates its implementation. The latter ensures that the quadrotor UAVs' attitude could be stabilized within a set time frame, despite external interferences. Moreover, its convergence time remains unaffected by system design parameters or initial conditions.

The remainder of this paper is divided into the following sections. Section II provides the model description and the relevant definitions and theorems. Section III gives the design of the practically prescribed-time control controller with its stability analysis. A sample numerical simulation is provided to further validate the effectiveness of our proposed design in Section IV. Conclusion of this paper is drawn in Section V.

II. Model Description and Preliminaries

A. Notation

The definitions provided below are employed consistently in this paper. $[\cdot]^{\mathrm{T}}$ represents the transpose of a matrix. The notation diag$(\cdot)$ denotes a matrix with non-zero elements only on its diagonal. For $\forall y \in R^n$, $\|y\|$ indicates the Euclidean norm of a vector which is defined as $\|y\| = \sqrt{y^{\mathrm{T}} y}$.

B. Model Description

In this study, a quadrotor UAV is selected as the subject of research. As illustrated in Fig.1, the structural layout of the quadrotor is provided. The quadrotor is characterized by a compact design with a symmetrical crossover frame and four rotors. To describe the attitude orientation of the quadrotor, two reference frames are introduced: the Body-fixed frame and the Earth-fixed frame. The Euler angles in the Earth-fixed frame describe the quadrotor's attitude, denoted in terms of the vector. $\Theta = [\phi, \theta, \varphi]^{\mathrm{T}} \in R^3$, in this case, ϕ denotes the roll angle, θ denotes the pitch angle, and φ denotes the yaw angle. Additionally, the angular velocity vector in the Body-fixed frame is expressed as $\omega = [p, q, r]^{\mathrm{T}} \in R^3$.

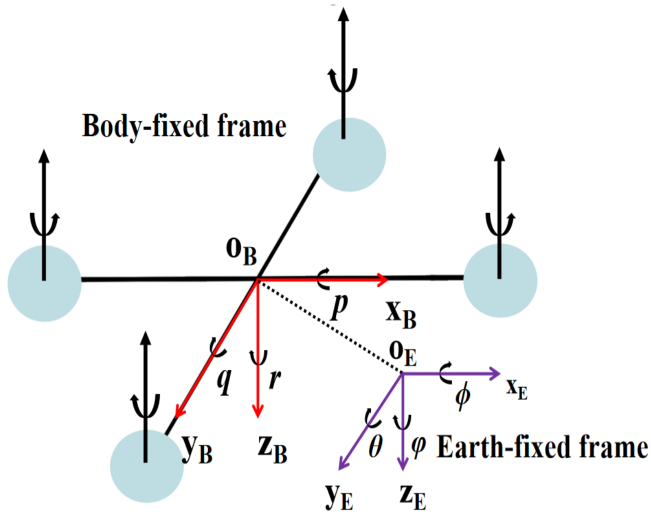

Fig. 1. Quadrotor model framework.

Consequently, the 3-DOF dynamic model for the quadrotor's attitude system can be derived as

$$\begin{cases} \dot{\Theta} = S(\Theta)\varpi \\ J\dot{\varpi} = -\varpi \times J\varpi + \tau + d_\tau \end{cases} \tag{1}$$

where J =diag(J_{xx}, J_{yy}, J_{zz}) represents the inertia matrix, $d_\tau = [d_\phi, d_\theta, d_\varphi]^{\mathrm{T}}$ is the disturbance torque, $\tau = [\tau_1, \tau_2, \tau_3]^{\mathrm{T}}$ denotes the system input, where τ_1, τ_2 and τ_3 represent roll, pitch and yaw torque, respectively. $S(\Theta)$ is interpreted as the transfer matrix that maps the Body-fixed frame to the Earth-fixed frame, denoted by

$$S(\Theta) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \tag{2}$$

The control inputs are calculated as

$$\begin{bmatrix} F \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} c_\upsilon & c_\upsilon & c_\upsilon & c_\upsilon \\ 0 & -c_\upsilon l & 0 & c_\upsilon l \\ c_\upsilon l & 0 & -c_\upsilon l & 0 \\ c_\tau & -c_\tau & c_\tau & -c_\tau \end{bmatrix} \begin{bmatrix} W_1^2 \\ W_2^2 \\ W_3^2 \\ W_4^2 \end{bmatrix} \tag{3}$$

In (3), l stands for the arm length of the quadrotor. and F represents the total thrust produced by four propellers along the z-axis. $c_\upsilon > 0$ represents the lift performance coefficient, and $c_\tau > 0$ represents the torque-related coefficient. The angular velocity of the four motors is represented by $W_i (i = 1, 2, 3, 4)$.

The goal of control in this article is to setup a practically prescribed-time control strategy for (1) to ensure that the attitude tracking errors approach a tiny range close to zero within any given time even in the presence of unknown disturbances.

C. Problem Formulation and Preliminaries

To execute the algorithm outlined in this paper, here, we present several key definitions and lemmas that will be used throughout the paper.

The following describes a dynamical system

$$\dot{\varsigma}(t) = f(t, \varsigma, d_\upsilon), \varsigma(t_0) = \varsigma_0 \tag{4}$$

in the given system, $\varsigma \in R^n$ represents the state vector, $d_v \in R^n$ is a vector of disturbances that are not known but are constrained within certain bounds, and consider $f : R_{\geq 0} \times R^n \to R^n$, a nonlinear function, where $f(t, 0; d_v) = 0$, which indicates that the origin $\varsigma = 0$ represents an equilibrium point of the system described by (4). The time variable t takes values within the interval $[t_0, \infty)$. Let $t_0 \in R_{\geq 0}$ be the starting time.

Definition 1[32]

The origin of (4) is globally finite-time stable provided that

1) it exhibits globally stable and asymptotically converges;

2) each solution $\varsigma(t; t_0, \varsigma_0) = 0$ of (4) reaches the origin within a finite time, i.e., $\forall t \geq t_0 + T(t_0, \varsigma_0)$ s.t. $\varsigma(t; t_0, \varsigma_0) = 0$,where $T : R_{\geq 0} \times R^n \to R_{\geq 0}$ is the settling time function.

Definition 2[33]

The origin of (4) is considered to be fixed-time stable if is considered to be fixed-time stable if it meets the following two conditions

1) it is global finite-time stability;

2) the settling time function has an upper bound, i.e., $\exists T_{max} > 0$, such that for all $\varsigma_0 \in R^n$ and $t_0 \in R_{\geq 0}$, $T(t_0, \varsigma_0) \leq T_{max}$.

Definition 3[31]

The origin of system (4) is called arbitrary-time stable if

1) it exhibits fixed-time stability;

2) $\exists T_s > 0$ could be selected freely beforehand, for all $\varsigma_0 \in R^n$ and $t_0 \in R_{\geq 0}$, the convergence time function $T(t_0, \varsigma_0) \leq T_s$.

Remark 1 Through the above definitions, it is found that, in comparison to finite-time/fixed-time stability, the arbitrary time stabilization is an advanced outcome since its settling time is able to be predefined without taking into account the system's initial states and parameters. In addition, in a certain sense, the arbitrary time stability is termed as the prescribed-time stability since they belong to the category of prescribed-time control.

Lemma 1[31] Considering (4), we let $E \in R^n$ be a domain that includes the point of stability $\varsigma = 0$. Then, $\gamma_1(\varsigma)$ and $\gamma_2(\varsigma)$ are two continuous functions with strictly positive definiteness on D. It is assumed that there exists a function with continuous derivatives and real outputs $\vartheta : [t_0, t_s] \times E \to R_{\geq 0}$. Suppose that there exists a real constant $\eta > 1$ such that

(i) $\gamma_1(\varsigma) \leq \vartheta(t, \varsigma) \leq \gamma_2(\varsigma)$, $\forall \varsigma \in E$

(ii)$\vartheta(t, 0) = 0$, $\forall t > t_s$

(iii)$\dot{\vartheta} \leq \frac{-\eta(e^{\vartheta}-1)}{e^{\vartheta}(t_s-t)}$, $\forall \vartheta \neq 0$, then the convergence point is prescribed-time stability with arbitrary convergence moment and the stability time of (4) is $T \leq t_s - t_0$.

Remark 2 Based on Lemma 1, it can be seen that for a system to be prescribed-time stable, its solutions must asymptotically approach equilibrium points within a finite time. Nevertheless, in many existing studies on adaptive finite-time control for uncertain nonlinear systems, such as quadrotor UAV systems, guaranteed practical prescribed-time stability is often not achieved due to various technical challenges. As a result, the goal of our work is to address and resolve this issue.

Theorem 1 For each solution $\varsigma(t, \varsigma_0)$ and (4), assume that there exists a Lyapunov function $\vartheta(\varsigma)$ satisfying the following differential inequality

$$\dot{\vartheta}(\varsigma) \leq \frac{-\eta(1-e^{-\vartheta})}{t_s - t} + \epsilon \tag{5}$$

given that $\eta > 1$ is a system parameter, and $0 < \epsilon < \infty$, then the system is practically prescribed-time stable.

Proof The equation (5) can be expressed as

$$\dot{\vartheta}(\varsigma) \leq \frac{-\eta(1-e^{-\vartheta})}{t_s - t} + \frac{\eta\varrho(1-e^{-\vartheta})}{t_s - t} - \frac{\eta\varrho(1-e^{-\vartheta})}{t_s - t} + \epsilon \tag{6}$$

When $t \to t_s$, since $-\frac{\eta\varrho(1-e^{-\vartheta})}{t_s-t} + \epsilon < 0$, we can get

$$\dot{V}(\varsigma) \leq \frac{-\eta(1-e^{-\vartheta})}{t_s - t} + \frac{-\eta\varrho(1-e^{-\vartheta})}{t_s - t} \tag{7}$$

where $0 < \varrho < 1$ is a auxiliary parameter.

By mathematical means, (7) is rerepresented as

$$\frac{d\vartheta}{dt} \leq \frac{-\tilde{\eta}(1-e^{-\vartheta})}{t_s - t} \tag{8}$$

where $\tilde{\eta} = \eta(1-\varrho) \geq 1$.

The solution to (8) is

$$\vartheta(\varsigma) \leq \ln\left(C(t_s - t)^{\tilde{\eta}} + 1\right) \tag{9}$$

where C is integral constant. From (9), we have

$$\dot{\vartheta}(\varsigma) \leq \frac{-\tilde{\eta}C(t_s - t)^{\tilde{\eta}-1}}{C(t_s - t)^{\tilde{\eta}} + 1} \tag{10}$$

From (10), we could know that when $t \to t_s$, $\dot{\vartheta}(\varsigma) \to 0$. Obviously, from (9), at $t \to t_s$, we have $\vartheta(\varsigma) = 0$. As a result, it implies that for $\forall t \geq t_s$, $\vartheta(\varsigma) = 0$ is maintained. This completes the proof.

Assumption 1 Suppose $d_\tau = [d_\phi, d_\theta, d_\varphi]^{\mathrm{T}}$ in (4) is bounded and $\|d_\tau\| \leq \delta$, where δ represents a known positive constant.

Assumption 2 The reference signal Θ_d along with its first derivative $\dot{\Theta}_d$ exhibit continuity and boundedness.

Assumption 3 The quadrotor features a symmetrical structure and rigid propeller assemblies.

Assumption 4 All states of the quadrotor model are available, and $\phi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ are always true.

III. Prescribed-time Controller Design

In this paper, the backstepping method and the practically prescribed-time stable theory that we present in Theorem 1 are integrated into the quadrotor UAVs system, which not only ensures the attitude tracking errors tend to a tiny neighborhood around zero by a predefined time, but also solves the system external interference problem. First, the system (4) is rewritten as

$$\begin{cases} \dot{\Theta} = S(\Theta)\varpi \\ \dot{\varpi} = J^{-1}(-\varpi \times J\varpi + \tau + d_\tau) \end{cases} \tag{11}$$

Firstly, according to the normal design steps of the backstepping method, the following tracking errors are defined

$$z_1 = \Theta - \Theta_d \tag{12}$$

$$z_2 = \varpi - \alpha \tag{13}$$

where $\Theta_d = [\phi_d, \theta_d, \varphi_d]^{\mathrm{T}}$ is a reference trajectory vector. $\alpha \in R^{3\times 1}$ represents the virtual control law.

Step 1. First Lyapunov candidate function is chosen as

$$V_1 = \frac{1}{2} z_1^{\mathrm{T}} z_1 \tag{14}$$

Taking the derivative of V_1 yields

$$\dot{V}_1 = z_1^{\mathrm{T}} \dot{z}_1 = z_1^{\mathrm{T}} \left(S(\Theta)\varpi - \dot{\Theta}_d \right) \tag{15}$$
$$= z_1^{\mathrm{T}} \left(S(\Theta)(z_2 + \alpha) - \dot{\Theta}_d \right) \tag{16}$$

Now, we design the following virtual control law

$$\alpha = \begin{cases} S^{-1}(\Theta) \left(-\frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t} + \dot{\Theta}_d \right), & \text{if } t_0 \leq t < t_s \\ S^{-1}(\Theta)\dot{\Theta}_d - c_1 z_1, & \text{otherwise} \end{cases} \tag{17}$$

with the help of (17), one obtains

$$\dot{V}_1 = \begin{cases} z_1^{\mathrm{T}} S(\Theta) z_2 - z_1^{\mathrm{T}} \frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t}, & \text{if } t_0 \leq t < t_s \\ -c_1 z_1^{\mathrm{T}} z_1 - z_1^{\mathrm{T}} S(\Theta) z_2, & \text{otherwise} \end{cases} \tag{18}$$

where $\varepsilon = [1,1,1]^{\mathrm{T}}$, t_s indicates the moment when the settling time occurs, irrespective of system parameters and initial states.

Step 2. Choosing the following Lyapunov function

$$V_2 = \frac{1}{2} z_1^{\mathrm{T}} z_1 + \frac{1}{2} z_2^{\mathrm{T}} z_2 \tag{19}$$

Taking the derivative of V_2 results in

$$\begin{aligned} \dot{V}_2 &= z_1^{\mathrm{T}} \dot{z}_1 + z_2^{\mathrm{T}} \dot{z}_2 \\ &= \dot{V}_1 + z_2^{\mathrm{T}} \left(J^{-1}(-\varpi \times J\varpi + \tau + d_\tau) - \dot{\alpha} \right) \end{aligned} \tag{20}$$

Now the attitude control law τ is built as

$$\tau = \begin{cases} \varpi \times J\varpi + \\ J \left(\dot{\alpha} - S^{\mathrm{T}}(\Theta) z_1 - \frac{\eta_2(\varepsilon - e^{-z_2})}{t_s - t} \right), & \text{if } t_0 \leq t < t_s \\ \varpi \times J\varpi + J \left(\dot{\alpha} - S^{\mathrm{T}}(\Theta) z_1 - c_2 z_2 - z_2 \right), & \text{otherwise} \end{cases} \tag{21}$$

Then, based on (21), (20) is rewritten as

$$\dot{V}_2 = \begin{cases} -z_1^{\mathrm{T}} \frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t} - z_2^{\mathrm{T}} \frac{\eta_2(\varepsilon - e^{-z_2})}{t_s - t} \\ +z_2^{\mathrm{T}} J^{-1} d_\tau, & \text{if } t_0 \leq t < t_s \\ -c_1 z_1^{\mathrm{T}} z_1 - c_2 z_2^{\mathrm{T}} z_2 + z_2^{\mathrm{T}} J^{-1} d_\tau - z_2^{\mathrm{T}} z_2, & \text{otherwise} \end{cases} \tag{22}$$

When $t_0 \leq t < t_s$, using the following inequality

$$\begin{aligned} z_2^{\mathrm{T}} J^{-1} d_\tau &\leq \left\| J^{-1} \right\| \| d_\tau \| \sqrt{z_1^{\mathrm{T}} z_1 + z_2^{\mathrm{T}} z_2} \\ &\leq \delta \left\| J^{-1} \right\| \sqrt{2V_2} \end{aligned} \tag{23}$$

otherwise, using the Young's inequality

$$z_2^{\mathrm{T}} J^{-1} d_\tau \leq \left\| z_2^{\mathrm{T}} \right\|^2 + \frac{\delta \left\| J^{-1} \right\|}{4} \tag{24}$$

we can obtain

$$\dot{V}_2 \leq \begin{cases} -z_1^{\mathrm{T}} \frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t} - z_2^{\mathrm{T}} \frac{\eta_2(\varepsilon - e^{-z_2})}{t_s - t} \\ +\delta \left\| J^{-1} \right\| \sqrt{2V_2}, & \text{if } \ t_0 \leq t < t_s \\ -c_1 z_1^{\mathrm{T}} z_1 - c_2 z_2^{\mathrm{T}} z_2 + \frac{\delta \| J^{-1} \|}{4}, & \text{otherwise} \end{cases} \tag{25}$$

So far, the main results of this paper can be given as follows.

Theorem 2 For (1) with Assumption 1, 2 and 3, if the virtual control law (17) and the actual control law (21) are selected, the system (1) not only satisfies that the tracking errors in the closed-loop system tend to the tiny neighborhood near zero but also has a great interference suppression performance against the external interferences.

Proof

First, let us have a discussion about the situation of $t_0 \leq t < t_s$.

From (25), it can be obtained

$$\dot{V}_2 \leq -z_1^{\mathrm{T}} \frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t} - z_2^{\mathrm{T}} \frac{\eta_2(\varepsilon - e^{-z_2})}{t_s - t} + \delta \left\| J^{-1} \right\| \sqrt{2V_2} \tag{26}$$

apparently, $\dot{V}_2 \leq -z_1 \frac{\eta_1(\varepsilon - e^{-z_1})}{t_s - t} + \delta \left\| J^{-1} \right\| \sqrt{2V_2}$ or $\dot{V}_2 \leq -z_2 \frac{\eta_2(\varepsilon - e^{-z_2})}{t_s - t} + \delta \left\| J^{-1} \right\| \sqrt{2V_2}$.

According to (19), we could easily know

$$\sqrt{V_2} \leq \max\{\|z_1\|, \|z_2\|\} \tag{27}$$

It can be devided into two cases that $\sqrt{V_2} \leq \|z_1\|$ or $\sqrt{V_2} \leq \|z_2\|$. Let us take $\sqrt{V_2} \leq \|z_1\|$ as an example, i.e., assuming $\max\{\|z_1\|, \|z_2\|\} = \|z_1\|$, then we can get

$$\dot{V}_2 \leq -\frac{2\eta_1 \sqrt{V_2}(\varepsilon - e^{\sqrt{V_2}})}{t_s - t} + \delta \left\| J^{-1} \right\| \sqrt{2V_2} \tag{28}$$

Let $\mu = \sqrt{V_2}$, we can further obtain $\dot{\mu} = \frac{\dot{V}_2}{2\sqrt{V_2}}$, then

$$\dot{\mu} \leq -\frac{\eta(e^{\mu} - \varepsilon)}{e^{\mu}(t_s - t)} + \frac{\gamma}{2} \tag{29}$$

where $\gamma = \sqrt{2}\delta \left\| J^{-1} \right\|$, $\eta = \frac{\eta_1}{2}$ and $\eta \geq 1$. Obviously, $\frac{\gamma}{2}$ is a known positive constant. According to Theorem 1, (29) satisfies practically prescribed-time stable which leads to the stabilization of μ within the given time t_s. From the above content, it can be inferred that V_2 also converges to zero within prescribed-time t_s. Note that if $\max\{\|z_1\|, \|z_2\|\} = \|z_2\|$, then the same result can be achieved. Thus, when $t_0 \leq t < t_s$, the criterion for practically prescribed-time described in Theorem 1 is satisfied.

Secondly, we are attention to the situation of $t \geq t_s$.

When $t \geq t_s$, (25) can be rewritten as

$$\dot{V}_2 \leq -AV_2 + D \tag{30}$$

where $A = c_1 + c_2$, $D = \frac{\delta \| J^{-1} \|}{4}$. Thus, similar to [34], the attitude tracking errors tends to a tiny neighborhood near zero and remains near zero.

In summary, the control strategy we designed achieves the practically prescribed-time when external interferences are present and can continue to maintain stability after a given time even if the interference is always present. At this point, the proof of Theorem 2 is completed.

IV. Simulation Studies

This section demonstrates a MATLAB/Simulink-based numerical simulation to verify the effectiveness of the designed controller in attitude tracking control for the quadrotor UAV. Setting the simulation time to be 30s, the nominal inertia moment is J =diag$(0.039, 0.039, 0.071)$kg·m^2, the disturbance torgue $d_\tau = [-0.5\cos t, 0, -0.5\sin t]^{\mathrm{T}}$ and the parameters assumed for simulation are $c_1 = 5, c_2 = 5$, $\eta_1 = 1.76$, $\eta_2 = 2.32$ for $t_s = 6$s and $t_s = 12$s, respectively. The initial values of the Euler angle and angular rate are gave as $\Theta(0) = [0.3, 0.2, -0.4]^{\mathrm{T}}$rad and $\varpi(0) = [0, 0, 0]^{\mathrm{T}}$rad/s, respectively. In addition, $\Theta_d = [0.3\sin(t), 0.3\cos(t), 0]^{\mathrm{T}}$rad is selected as the desired value of the Euler angles.

Emphasize here, the simulation results reflect the reality. As depicted in Fig.2, it is evident that for a final time $t_s = 6$s, the Euler angle closely follows the anticipated trajectory within the specified duration. The curves of tracking errors are shown in Fig.3. Obviously, the proposed methodology evidently facilitates swift and precise attitude tracking behavior within $t_s = 6$s. This further substantiates the interference rejection performance of the theory practically prescribed-time stability. Additionally, for $t_s = 6$s, the response plots of the practical control laws denoted as τ_1, τ_2 and τ_3 are illustrated in Fig.4.

To distinctly verify and emphasize the effectiveness of the control approach outlined in this document, we additionally illustrate the simulation outcome for $t_s = 12$s. The outcomes of the simulation are depicted in Fig.5. and the curves of tracking errors are shown in Fig.6. It is evident that the system states converge within $t_s = 12$s. Concurrently, for $t_s = 12$s, the plots representing the actual control laws labeled as τ_1, τ_2 and τ_3 are exhibited in Fig.7. The above results verify that the outstanding tracking performance has been attained utilizing our suggested approach.

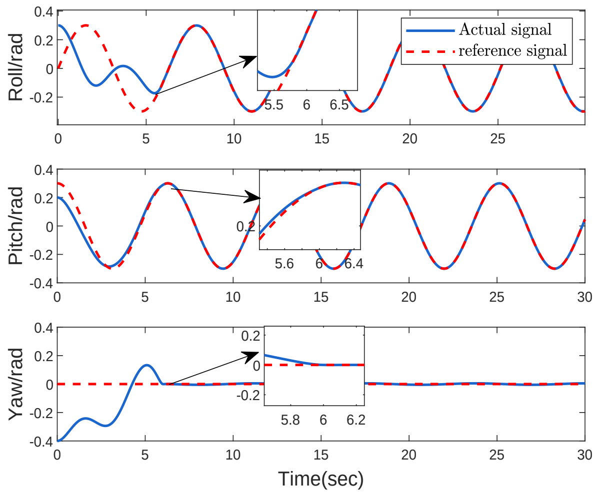

Fig. 2. Trajectories of Θ and $\Theta_d, t_s = 6$s.

V. Conclusion

A practically prescribed-time attitude stability control method is proposed for the attitude system of quadrotor UAVs with external interference. The convergence time is independent of any initial conditions and parameters, and the desired trajectory of the attitude system of the quadrotor UAV is achieved within a given time. At the same time, the external interference problem of the system has been effectively solved by adopting the practically prescribed-time theory. As a future work, we are intention to integrate more methods, such as adaptive control, sliding mode control and active disturbance rejection technology, into the actual preset time stability theory proposed in this paper, so as to solve more external and internal uncertainties and achieve better trajectory tracking effect.

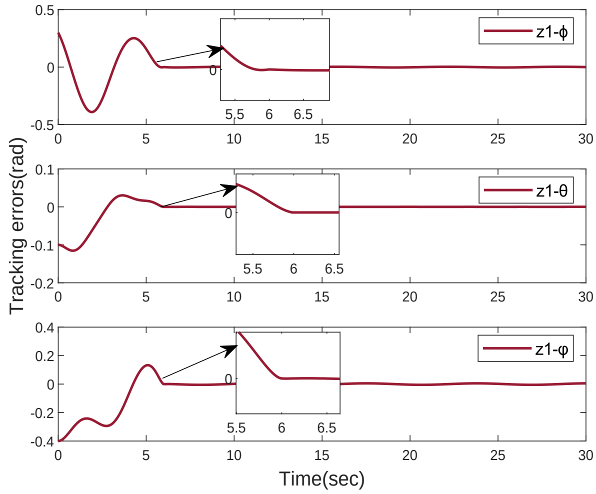

Fig. 3. Tracking errors of $\Theta, t_s = 6s$.

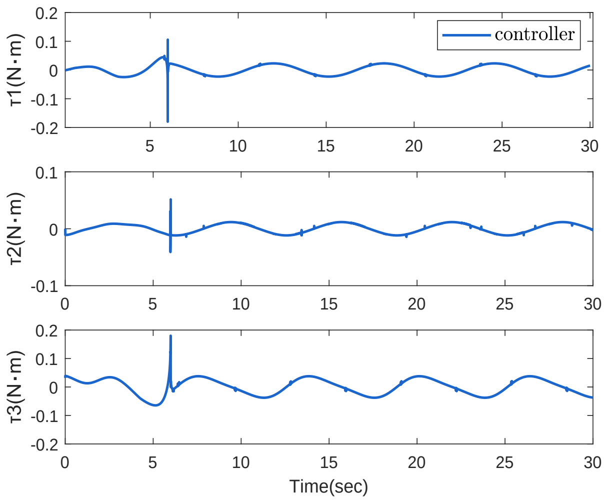

Fig. 4. Control law $\tau, t_s = 6$s.

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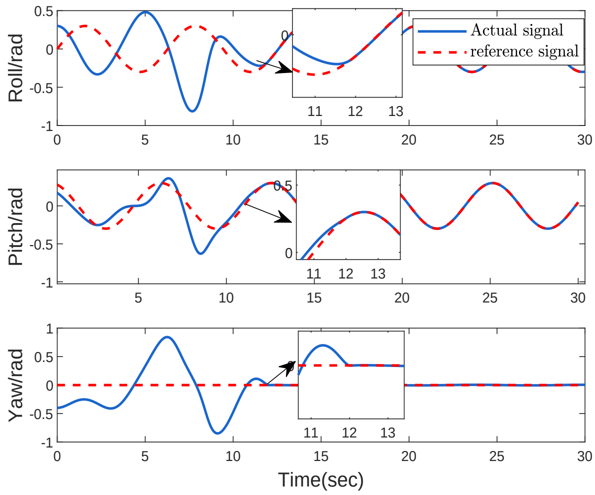

Fig. 5. Trajectories of Θ and $\Theta_d, t_s = 12\text{s}$.

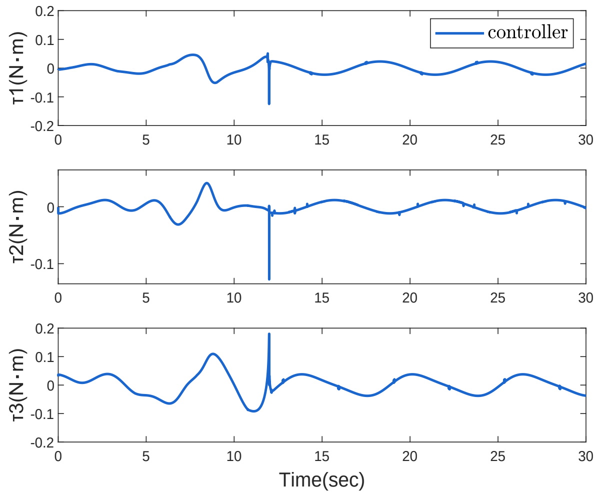

Fig. 7. Control law $\tau, t_s = 12\text{s}$.

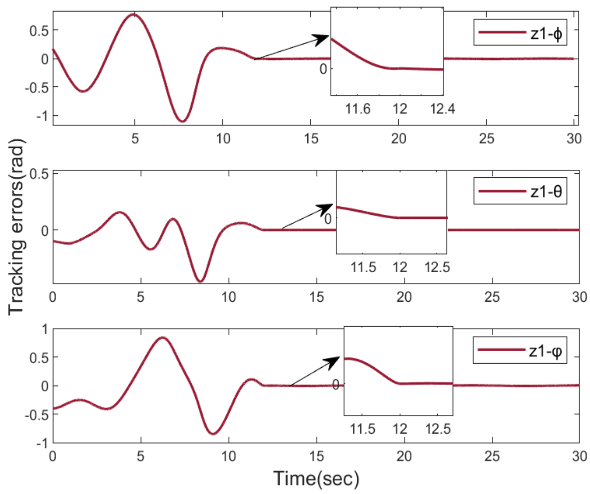

Fig. 6. Tracking errors of $\Theta, t_s = 12s$.

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