Tension of Some Graphs

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***Abstract*—Centrality measures are scalar values given to each vertex in the graph to quantify its importance based on an assumption. Stress and tension are the two centrality measures which depends on shortest paths. Stress of a vertex v is the number of shortest paths passing through vertex v whereas, tension on an edge e is the number of shortest paths passing through the edge e. In this paper, authors obtain formulae for the evaluation of stress and tension of certain graphs and graph operations with diameter less than or equal to two. Also obtain tension of some wheel related graphs such as gear graph, helm graph, flower graph and sunflower graph.**

***Index Terms*—Geodesic, centrality measure,stress, tension,**

I. Introduction

LET $G = (V, E)$ be a simple, finite, undirected and connected graph. The order and size of G is given by $|V| = n$ and $|E| = m$ respectively. The degree of a vertex v in a graph G, denoted by $\deg(v)$ is the number of edges incident on the vertex v. Let $P = v_o, v_1, \ldots, v_n$ be a $v_o v_n$ path in G. The length of P, denoted by $l(P)$ is the number of edges in the $v_o v_n$ path. Let $d(u, v)$ denote the distance between any two vertices u and v in G. The shortest path between any two vertices in G is called the geodesic. The diameter of G is the length of any longest geodesic, denoted by $diam(G)$. The maximum distance from vertex v to all other vertices in G is the eccentricity $e(v)$ of v. In 1953, Alfonso Shimbel defined the concept of stress of a vertex in a graph. Stress of a vertex v in a graph G is the number of shortest paths in G having v as an internal vertex and is denoted by $st(v)$ [1]. A graph G is $k-$stress regular, if all vertices of G have stress k. The total stress of a graph G is defined by $st(G) = \sum\limits_{i=1}^{n} st(v_i)$. For more studies on stress of a graph, one can refer [2]–[7]. K. Bhargava and others [8], introduced the concept of tension on an edge in a graph. Let e be an edge in the graph G. The tension on e is defined as the number of geodesics in G passing through e. Total tension of G, denoted by $N_\tau(G)$, is defined as $N_\tau(G) = \sum\limits_{e \in E} \tau(e)$.

This paper is organised as follows. In section 3, authors compute stress and tension of complete graph, wheel graph, star, Petersen, triangular book graph, complete bipartite graph, friendship graph, fan graph and cocktail party graph by calculating number of geodesics of different length. Also, obtained stress and tension of some operations on graphs such as join, amalgamation, strong product, lexicographic product and tensor product. In section 4, tension of some wheel related graphs in terms of tension of wheel graph are obtained.

Note that

1) Geodesics of length n contribute $(n - 1)$ to the stress of a graph.
2) Geodesics of length n contribute n to the tension of a graph.

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II. Preliminaries

A. Definitions

Triangular book graph is a planar undirected graph with $n + 2$ vertices and $2n + 1$ edges constructed by n triangles sharing a common edge.

Friendship graph is a planar, undirected graph with $2n + 1$ vertices and $3n$ edges.

Cocktail party graph is a graph consisting of two rows of paired nodes in which all nodes except the paired once are connected with a straight lines.

The **join** $G_1 + G_2$ of two graphs G_1 and G_2 has the vertex set $V(G) = V(G_1) \cup V(G_2)$ and the edge set $E(G) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.

First Zagreb and Second Zagreb indices are degree based topological indices defined as,

$$M_1(G) = \sum_{u \in V(G)} d_u^2.$$

$$M_2(G) = \sum_{uv \in E(G)} d_u.d_v.$$

The **cartesian product** $G \square H$ of graphs G and H is a graph with vertex set $V(G) \times V(H)$ in which any two vertices (u, u') and (v, v') are adjacent if and only if either

1) $u = v$ and $u' \sim v'$ in H or
2) $u' = v'$ and $u \sim v$ in G.

The **tensor product** $G \times H$ of graphs G and H is a graph with vertex set $V(G) \times V(H)$ in which any two vertices (u, u') and (v, v') are adjacent iff

1) $u \sim u'$ in G and
2) $v \sim v'$ in H.

The **strong product** $G \boxtimes H$ of graphs G and H is the graph with vertex set $V(G) \times V(H)$ and any two vertices (u, u') and (v, v') are adjacent if and only if either

1) $u = v$ and $u' \sim v'$ or
2) $u' = v'$ and $u \sim v$ or
3) $u \sim v$ and $u' \sim v'$.

The **lexicographic product** $G \cdot H$ of graphs G and H is a graph with vertex set $V(G) \times V(H)$ and any two vertices (u, u') and (v, v') are adjacent in $G \cdot H$ iff either

1) $u \sim u'$ in G or
2) $u = u'$ and $v \sim v'$ in H.

The **symmetric product** $G \oplus H$ of graphs G and H is a graph with vertex set $V(G) \times V(H)$ and any two vertices (u, u') and (v, v') are adjacent iff either

1) $u \sim u'$ and $v \nsim v'$ or
2) $v \sim v'$ and $u \nsim u'$.

Vertex Amalgamation- Let $\{G^i | i \in 1, 2, 3, ..., m\}$ for $m \in N$ and $m \geq 2$ be a collection of finite graphs and each G^i has a fixed vertex, say v_{oi}, which is called terminal. $Amal(G^i, v_{oi})$ is a graph formed by taking all vertices and edges on G^i where $v_{oi} = v_{oj}$, for all $i \neq$ j. If $G^i = G^j = G$ and $|G| = n$, then we write $Amal(G^i, v_{oi})$ with $Amal(G_n)_m$.

B. Definitions of wheel related graphs

A **simple graph** with $n \geq 3$ vertices forming a cycle of length n is called a cycle graph, denoted by C_n.

Wheel Graph W_{n+1} is formed by connecting a single universal vertex to all vertices of cycle.

Gear Graph G_n is obtained from wheel graph by inserting an extra vertex between each pair of adjacent vertices on C_n.

Helm Graph H_n is obtained from W_n by attaching a single edge to the outer circuit of W_n.

Flower Graph Fl_n is obtained from Helm by joining each pendant vertex to the central vertex of H_n.

Sunflower graph SF_n is a graph on $2n + 1$ vertices obtained by taking a wheel with hub x, an $n-$cycle $v_1, v_2, \ldots, v_n$ and additional m vertices $w_1, w_2, \ldots, w_m$, where w_i is joined by edges to v_i, v_{i+1} for $i = 1, 2, \ldots, m$, and $i + 1$ is taken modulo m.

Fan graph $F_{n,1}$ is obtained from removing one peripheral edge from the wheel graph.

Friendship Graph F_n consists of n triangles with a common vertex.

Theorem II.1. *[2], [8] For any graph with n vertices and diameter d, we have*

$$N_{str}(G) = \sum_{i=1}^{d} (i-1) f_i \tag{1}$$

$$N_\tau(G) = \sum_{i=1}^{d} i f_i \tag{2}$$

where, f_i is the number of geodesics of length i in G.

III. Stress and tension of graphs with diameter less than or equal to 2

Theorem III.1. *Let G be a graph with diameter less than or equal to 2.*

Case 1. If G has no C_3 cycles in it, then

(i) $Str(G) = \frac{M_1(G)}{2} - m.$

(ii) $N_\tau(G) = M_1(G) - m.$

Case 2. If x is the number of C_3 cycles in G, then

(i) $Str(G) = \frac{M_1(G)}{2} - m - 3x.$

(ii) $N_\tau(G) = M_1(G) - m - 6x.$

Proof: For any graph with diameter less than or equal to 2,

1) if graph has no C_3 cycles in it, then number of geodesics of length 1 = m.
Number of geodesics of length 2 = $\sum \binom{degv}{2} = \frac{M_1(G)}{2} - m.$
From the theorem II.1,

$$\begin{aligned}(i)\ Str(G) &= 0(m) + 1\left(\frac{M_1(G)}{2} - m\right) \\ &= \frac{M_1(G)}{2} - m.\end{aligned}$$

$$\begin{aligned}(ii)\ N_\tau(G) &= 1(m) + 2\left(\frac{M_1(G)}{2} - m\right) \\ &= m + M_1(G) - 2m \\ &= M_1(G) - m.\end{aligned}$$

2) If a graph G has C_3 cycles in it, then number of geodesics of length 1 = m.
Number of geodesics of length 2 = $\sum \binom{degv}{2}-$ 3(number of C_3 cycles in G).

$$= \frac{M_1(G)}{2} - m - 3x$$

where, x is the number of C_3 cycles in G
From the theorem II.1,

$$\begin{aligned}(i)\ Str(G) &= 0(m) + 1\left(\frac{M_1(G)}{2} - m - 3x\right) \\ &= \frac{M_1(G)}{2} - m - 3x.\end{aligned}$$

$$\begin{aligned}(ii)\ N_\tau(G) &= 1(m) + 2\left(\frac{M_1(G)}{2} - m - 3x\right) \\ &= m + M_1(G) - 2m - 6x \\ &= M_1(G) - m - 6x.\end{aligned}$$

■

Using III.1 we find stress and tensions of some standard graphs whose diameter is less than or equal to two I II.

IV. Tension of some graphs

Let $w_0,\ w_i,\ 1 \leq i \leq n$ denote central and peripheral vertices of the wheel graph W_{n+1}. Then $w_0 w_i,\ 1 \leq i \leq n$ is the radial edge and $w_i w_{i+1}, 1 \leq i \leq n-1,\ w_1 w_n$ are the peripheral edges of wheel graph. In the graphs mentioned in section 2.2, all the radial and peripheral edges have equal tension.

Theorem IV.1. *The total tension of gear graph is,*

$$N_\tau(G_n) = 16n^2 - 14n.$$

Proof: Let w_i' be the vertex inserted to the edge $w_i w_{i+1}$, $1 \leq i \leq n-1$ and w_n' be the vertex inserted to the edge $w_1 w_n$. **Case 1:** Consider the edge $w_0 w_1$.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_i) : 2 \leq i \leq n,$ $(w_1', w_1, w_0), (w_n', w_1, w_0)$
3	$(w_1, w_0, w_i, w_i') : 2 \leq i \leq n-1,$ $(w_1, w_0, w_{i+1}, w_i') : 2 \leq i \leq n-1,$ $(w_1', w_1, w_0, w_i) : 3 \leq i \leq n,$ $(w_i, w_0, w_1, w_n') : 2 \leq i \leq n-1$
4	$(w_1', w_1, w_0, w_i, w_i') : 3 \leq i \leq n-1,$ $(w_1', w_1, w_0, w_{i+1}, w_i') : 3 \leq i \leq n-1,$ $(w_i', w_i, w_0, w_1, w_n') : 2 \leq i \leq n-2,$ $(w_n', w_1, w_0, w_{i+1}, w_i') : 2 \leq i \leq n-2$

Among them, $n-2$ are from wheel graph and the remaining $8n-16$ are the new paths added in gear graph that are passing through w_0w_1.

Case 2: Consider the edge w_1w_1'.

Length of path	Path
1	(w_1, w_1')
2	$(w_1, w_1', w_2), (w_1', w_1, w_0), (w_1', w_1, w_n')$
3	$(w_1', w_1, w_0, w_i) : 3 \leq i \leq n,$ $(w_1', w_1, w_n', w_n), (w_2, w_1', w_1, w_n'), (w_1, w_1', w_2, w_2'),$
4	$(w_1', w_1, w_0, w_i, w_i') : 3 \leq i \leq n-1,$ $(w_1', w_1, w_0, w_{i+1}, w_i') : 3 \leq i \leq n-1,$ $(w_n', w_1, w_1', w_2, w_2'), (w_1', w_1, w_n', w_n, w_{n-1}')$

Hence,

$$\tau_{G_n}(w_0w_i) = \tau_{W_{n+1}}(w_0w_i) + 4 + 2(n-2) + 2(n-2)$$
$$+4(n-3)$$
$$= \tau_{W_{n+1}}(w_0w_i) + 8n^2 - 16n$$
$$= 9n^2 - 18n.$$

$$\tau_{G_n}(w_iw_j') = 1 + 3 + (n+1) + (2n-4)$$
$$= 6n^2 + 2n.$$

Therefore, $N_\tau(G_n) = \tau_{G_n}(w_0w_i) + \tau_{G_n}(w_iw_j') = 15n^2 - 16n.$ ■

Theorem IV.2. *The total tension of helm graph is,*

$$N_\tau(H_n) = 6n^2 + 6.$$

Proof:

Let $w_i',\ 1 \leq i \leq n$ be the pendant vertex with respect to the vertex w_i, $1 \leq i \leq n$.

Case 1: Consider the edge w_1w_2.

Length of path	Path
1	(w_1, w_2)
2	$(w_1', w_1, w_2), (w_1, w_2, w_2'), (w_1, w_2, w_3),$ (w_2, w_1, w_n)
3	$(w_1', w_1, w_2, w_2'), (w_1', w_1, w_2, w_3), (w_1, w_2, w_3, w_3'),$ $(w_2, w_1, w_n, w_n'), (w_2', w_2, w_1, w_n)$
4	$(w_1', w_1, w_2, w_3, w_3'), (w_2', w_2, w_1, w_n, w_n')$

Among these paths, 3 are from the wheel graph and the remaining 9 paths are the newly added in the helm graph which are passing through the edge w_1w_2.

Case 2: Consider the edge w_1w_1'.

Length of path	Path
1	(w_1, w_1')
2	$(w_1', w_1, w_i) : i = 0, 2, n$
3	$(w_1', w_1, w_i, w_i') : i = 2, n$ $(w_1', w_1, w_0, w_i),\ 3 \leq i \leq n-1$ $(w_1', w_1, w_2, w_3), (w_1', w_1, w_n, w_{n-1})$
4	$(w_1', w_1, w_0, w_i, w_i'),\ 3 \leq i \leq n-1.$ $(w_1', w_1, w_n, w_{n-1}, w_{n-1}'), (w_1', w_1, w_2, w_3, w_3')$

Case 3: Consider the edge w_0w_1.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_i) : 3 \leq i \leq n-1, (w_1', w_1, w_0)$
3	$(w_1', w_1, w_0, w_i) : 3 \leq i \leq n-1,$ $(w_1, w_0, w_3, w_i') : 3 \leq i \leq n-1$
4	$(w_1', w_1, w_0, w_i, w_i') : 3 \leq i \leq n-1$

Out of these paths, $n-2$ are from wheel graph and the remaining $3n-8$ are the new paths added in the helm graph that are passing through the edge w_0w_1.

Therefore,

$$\tau_{H_n}(w_0w_i) = \tau_{W_{n+1}}(w_0w_i) + (n-3) + (n-3) + (n-3) + 1$$
$$= \tau_{W_{n+1}}(w_0w_i) + 3n^2 - 8n.$$

$$\tau_{H_n}(w_iw_j) = \tau_{W_{n+1}}(w_iw_j) + 9n.$$

$$\tau_{H_n}(w_iw_i') = 1 + 3 + (n+1) + (n-1) = 2n^2 + 4n.$$

Therefore, $N_\tau(H_n) = N_\tau(W_{n+1}) + 5n^2 + 5n = 6n^2 + 6n.$ ■

Theorem IV.3. *The total tension of flower graph is,*

$$N_\tau(Fl_n) = 4n^2 + 4n.$$

Proof: Let w_i' be the pendant vertex with respect to the vertex w_i, $1 \leq i \leq n$. Note that all peripheral edges have equal tension, all radial edges have equal tension and all the newly added edges have equal tension.

Case 1: Consider the radial edge w_0w_1.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_2'), (w_1, w_0, w_i) : 3 \leq i \leq n-1,$ $(w_1, w_0, w_i') : 3 \leq i \leq n$

Among them, $n-2$ are from wheel graph and the remaining $n-1$ are the new paths added in flower graph that are passing through w_0w_1.

Case 2: Consider a peripheral edge w_1w_2.

Length of path	Path
1	(w_1, w_2)
2	$(w_1', w_1, w_2), (w_1, w_2, w_2'), (w_1, w_2, w_3),$ (w_2, w_1, w_n)

Among them, 3 are from wheel graph and the remaining 2 are the new paths added in flower Graph that are passing through the edge w_1w_2.

Case 3: Consider the edge w_iw_1'.

Length of path	Path
1	(w_1, w_1')
2	$(w_1', w_1, w_2), (w_1', w_1, w_n)$

Case 4: Consider the edge w_0w_1'.

Length of path	Path
1	(w_0, w_1')
2	$(w_1', w_0, w_i) : 2 \leq i \leq n$ $(w_1', w_o, w_i') : 2 \leq i \leq n$

We have,

$$\begin{aligned} N_\tau(w_0w_i : Fl_n) &= \tau_{H_n}(w_0w_i) + (n-1) - 1 - 3(n-3) \\ &= \tau_{H_n}(w_0w_i) - 2n^2 + 7n \\ &= \tau_{W_{n+1}}(w_0w_i) + n^2 - n. \end{aligned}$$

$$\begin{aligned} N_\tau(w_iw_j : Fl_n) &= \tau_{H_n}(w_iw_j) - 7n \\ &= \tau_{W_{n+1}}(w_iw_j) + 2n. \end{aligned}$$

$$\begin{aligned} T(w_iw_i' : Fl_n) &= \tau_{H_n}(w_iw_i') - 2n^2 + 3n \\ &= 2n^2 - 2n^2 + 3n = 3n. \end{aligned}$$

$$T(w_0w_i' : Fl_n) = 1 + 2(n-1) = 2n^2 - n.$$

Therefore,

$$\begin{aligned} N_\tau(Fl_n) &= N_\tau(H_n) - 2n^2 + 2n \\ &= N_\tau(W_{n+1}) + 3n^2 + 3n \\ &= 4n^2 + 4n. \end{aligned}$$

■

Theorem IV.4. *The total tension of sunflower graph is,*

$$T(SF_n) = 15n^2 - 36n.$$

Proof: Let w_i' be the pendant vertex with respect to the vertex w_i, $1 \leq i \leq n$.

Case 1: Consider the edge w_0w_1.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_i) : 3 \leq i \leq n-1,$ $(w_1', w_1, w_0), (w_n', w_1, w_0)$
3	$(w_1, w_0, w_i, w_i'),$ $(w_1, w_0, w_{i+1}, w_i') : 3 \leq i \leq n-1,$ $(w_1', w_1, w_0, w_i) : 4 \leq i \leq n-1,$ $(w_n', w_1, w_0, w_i) : 3 \leq i \leq -2$
4	$(w_1', w_1, w_0, w_i, w_i'),$ $(w_1', w_1, w_0, w_{i+1}, w_i') : 4 \leq i \leq n-2,$ $(w_n', w_1, w_0, w_i, w_i'),$ $(w_n', w_1, w_0, w_{i+1}, w_i') : 3 \leq i \leq n-3$

Among them, $n-2$ are from wheel graph and the remaining $8n-34$ are new paths added in sunflower graph that are passing through w_0w_1.

Case 2: Consider the edge w_1w_2.

Length of path	Path
1	(w_1, w_2)
2	$(w_1, w_2, w_2'), (w_1, w_2, w_3), (w_2, w_1, w_n), (w_2, w_1, w_n')$
3	$(w_2', w_2, w_1, w_n'), (w_2', w_2, w_1, w_n), (w_1, w_2, w_3, w_3'), (w_2, w_1, w_n, w_{n-1}'), (w_3, w_2, w_1, w_n')$
4	$(w_2', w_2, w_1, w_n, w_{n-1}'), (w_3', w_3, w_2, w_1, w_n')$

Among them, 3 are from wheel graph and the remaining 9 are the new paths in sunflower graph that are passing through w_1w_2.

Case 3: Consider the edge w_1w_1'.

Length of path	Path
1	(w_1, w_1')
2	$(w_1', w_1, w_0), (w_1', w_1, w_n), (w_1', w_1, w_n')$
3	$(w_1', w_1, w_0, w_i) : 4 \leq i \leq n-1,$ $(w_1', w_1, w_n, w_{n-1}), (w_1', w_1, w_n, w_{n-1}')$
4	$(w_1', w_1, w_0, w_i, w_i'),$ $(w_1', w_1, w_0, w_{i+1}, w_i') : 4 \leq i \leq n-2,$ $(w_1', w_1, w_n, w_{n-1}, w_{n-2}')$

Now,

$$\begin{aligned} N_\tau(w_0w_i : SF_n) &= \tau_{W_{n+1}}(w_0w_i) + 2 + 4(n-4) + 4(n-5) \\ &= \tau_{W_{n+1}}(w_0w_i) + 8n^2 - 34n. \end{aligned}$$

$$\begin{aligned} N_\tau(w_iw_j : SF_n) &= \tau_{W_{n+1}}(w_iw_j) + 2 + 5 + 2 \\ &= \tau_{W_{n+1}}(w_iw_j) + 9n. \end{aligned}$$

$$\begin{aligned} N_\tau(w_iw_j' : SF_n) &= 2(7 + (n-3) + 2(n-5)) \\ &= 6n^2 - 12n. \end{aligned}$$

Therefore,
$N_\tau(SF_n) = N_\tau(W_{n+1}) + 14n^2 - 37n = 15n^2 - 36n.$ ■

Theorem IV.5. *The total tension of fan graph is,*

$$N_\tau(F_{n,1}) = n^2 + n - 3.$$

Proof: Let w_1w_n be the peripheral edge that is removed from W_{n+1}.

Case 1: Consider the edge w_0w_1.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_i) : 3 \leq i \leq n-1, (w_1, w_0, w_n)$

But, (w_1, w_0, w_n) is a new path in fan graph since w_1w_n is deleted. Therefore, there is increase of 2 in total tension.

Case 2: Consider the edge w_2w_3.

Length of path	Path
1	(w_2, w_3)
2	$(w_1, w_2, w_3), (w_2, w_3, w_4)$

Since, the edge w_1w_n is removed from the wheel graph to obtain fan graph, there is a decrease of 5 from the total tension.

$$N_\tau(w_0w_i : F_{n,1}) = \tau_{W_{n+1}}(w_0w_i) - 5.$$

$$N_\tau(w_iw_j : F_{n,1}) = \tau_{W_{n+1}}(w_iw_j) + 2.$$

Therefore,
$N_\tau(F_{n,1}) = N_\tau(W_{n+1}) - 3 = n^2 + n - 3.$ ■

Theorem IV.6. *The total tension of friendship graph is,*

$$T(F_n) = n^2 - \frac{n}{2}.$$

Proof: Let w_0 be the central vertex and $w_i,\ 1 \leq i \leq n$ be the peripheral vertices of the friendship graph.

Case 1: Consider the edge w_0w_1.

Length of path	Path
1	(w_0, w_1)
2	$(w_1, w_0, w_i) : 3 \leq i \leq n-1, (w_1, w_0, w_n)$

But, there are 2 paths of length 2 passing through each peripheral edge in wheel graph.

Case 2: Consider the edge w_1w_2.

Length of path	Path
1	(w_1, w_2)
2	0

Therefore,

$$\tau_{F_n}(w_0w_i) = \tau_{W_{n+1}}(w_0w_i) + 2 \times \frac{n}{2}.$$

$$\tau_{F_n}(w_iw_j) = \frac{n}{2} = 3n - \frac{5n}{2} = \tau_{W_{n+1}}(w_iw_j) - \frac{5n}{2}.$$

Therefore,

$$\begin{aligned} N_\tau(F_n) &= N_\tau(W_{n+1}) + n - \frac{5n}{2} \\ &= N_\tau(W_{n+1}) - \frac{3n}{2} \\ &= n^2 - \frac{n}{2}. \end{aligned}$$

■

Table I: Stress and tension of some standard graphs with diameter less than or equal to 2

Graph	**Geodesics of length 1**	**Geodesics of length 2**	**Number of C_3 cycles**	**Stress**	**Tension**
Complete graph	$\binom{n}{2}$	0	$\binom{n}{3}$	0	$\binom{n}{2}$
Wheel graph	$2(n-1)$	$\frac{M_1(G)}{2} - 5(n-1)$	$(n-1)$	$\frac{M_1(G)}{2} - 5(n-1)$	$M_1(G) - 8(n-1)$
Star	n	$\frac{M_1(G)}{2} - n$	0	$\frac{M_1(G)}{2} - n$	$M_1(G) - n$
Petersen graph	15	30	0	30	75
Triangular book graph	$2n+1$	$\frac{M_1(G)}{2} - 5n - 1$	n	$\frac{M_1(G)}{2} - 5n - 1$	$M_1(G) - 8n - 1$
Complete bipartite graph	mn	$\frac{M_1(G)}{2} - mn$	0	$\frac{M_1(G)}{2} - mn$	$M_1(G) - mn$
Friendship graph	$3n$	$\frac{M_1(G)}{2} - 6n$	n	$\frac{M_1(G)}{2} - 6n$	$M_1(G) - 9n$
Fan graph	$2n-3$	$\frac{M_1(G)}{2} - 5n + 9$	$n-2$	$\frac{M_1(G)}{2} - 5n + 9$	$M_1(G) - 8n + 15$
Cocktail party graph	$2n(n-1)$	$\frac{M_1(G)}{2} - 2n(n-1) - 3(2\binom{n}{3} + 2n\binom{n-1}{2})$	$2\binom{n}{3} + 2n\binom{n-1}{2}$	$\frac{M_1(G)}{2} - 2n(n-1) - 6\binom{n}{3} - 6n\binom{n-1}{2}$	$M_1(G) - 2n(n-1) - 12\binom{n}{3} - 12n\binom{n-1}{2}$

Table II: Stress and tension of some graph operations

Graph	**Geodesics of length 1**	**Geodesics of length 2**	**Number of C_3 cycles**	**Stress**	**Tension**
$G_1 + G_2$ G_1/G_2 has n number of C_3 cycles	$q_1 + q_2 + p_1p_2$	$\frac{M_1(G)}{2} - (q_1 + q_2 + p_1p_2) - 3(p_1q_2 + p_2q_1 + n)$	$(p_1q_2 + p_2q_1) + n$	$\frac{M_1(G)}{2} - (q_1 + q_2 + p_1p_2) - 3(p_1q_2 + p_2q_1 + n)$	$M_1(G) - (q_1 + q_2 + p_1p_2) - 6(p_1q_2 + p_2q_1 + n)$
$G_1 + G_2$ G_1/G_2 has no C_3 cycles	$q_1 + q_2 + p_1p_2$	$\frac{M_1(G)}{2} - (q_1 + q_2 + p_1p_2) - 3(p_1q_2 + p_2q_1)$	$p_1q_2 + p_2q_1$	$\frac{M_1(G)}{2} - (q_1 + q_2 + p_1p_2) - 3(p_1q_2 + p_2q_1)$	$M_1(G) - (q_1 + q_2 + p_1p_2) - 6(p_1q_2 + p_2q_1)$
$Amal$ (k, K_n)	$n_1(\frac{n_1-1}{2}) + n_2(\frac{n_2-1}{2}) + ... + n_k(\frac{n_k-1}{2})$	$\frac{M_1(G)}{2}$ - $\{n_1\left(\frac{n_1-1}{2}\right) + n_2\left(\frac{n_2-1}{2}\right) + \cdots + n_k\left(\frac{n_k-1}{2}\right)\}$- $3\{\binom{n_1}{3} + \binom{n_2}{3}$ +...+$\binom{n_k}{3}\}$	$\binom{n_1}{3} + \binom{n_2}{3} + \cdots + \binom{n_k}{3}$	$\frac{M_1(G)}{2}$ -$\{n_1\left(\frac{n_1-1}{2}\right)$ + $n_2\left(\frac{n_2-1}{2}\right)$ +...+$n_k\left(\frac{n_k-1}{2}\right)\}$ -$3\{\binom{n_1}{3} + \binom{n_2}{3}$ +...+ $\binom{n_k}{3}\}$	$M_1(G)$-$\{n_1\left(\frac{n_1-1}{2}\right)$ +$n_2\left(\frac{n_2-1}{2}\right)$ +...+ $n_k\left(\frac{n_k-1}{2}\right)\}$- $6\{\binom{n_1}{3} + \binom{n_2}{3}$ +...+ $\binom{n_k}{3}\}$
$K_n \boxtimes K_m$	$v_1e_2 + e_1(v_2)^2$	$\frac{M_1(G)}{2} - (v_1e_2 + e_1(v_2)^2) - 3\binom{mn}{3}$	$\binom{mn}{3}$	$\frac{M_1(G)}{2} - (v_1e_2 + e_1(v_2)^2) - 3\binom{mn}{3}$	$M_1(G) - (v_1e_2 + e_1(v_2)^2) - 6\binom{mn}{3}$
$K_n \times K_m$	$2e_1e_2$	$\frac{M_1(G)}{2} - 2e_1e_2$	0	$\frac{M_1(G)}{2} - 2e_1e_2$	$M_1(G) - 2e_1e_2$
$K_n \cdot K_m$	$v_1e_2 + e_1(v_2)^2$	$\frac{M_1(G)}{2} - (v_1e_2 + e_1(v_2)^2) - 3\binom{mn}{3}$	$\binom{mn}{3}$	$\frac{M_1(G)}{2} - (v_1e_2 + e_1(v_2)^2) - 3\binom{mn}{3}$	$M_1(G) - (v_1e_2 + e_1(v_2)^2) - 6\binom{mn}{3}$

V. Conclusion

In this paper, the authors obtained stress and tension of some graphs with diameter less than or equal to two by calculating number of geodesics of different length. Also, calculated tension of some wheel related graphs in terms of the number of vertices of wheel graph. The tensions of gear graph, helm graph, flower graph, sunflower graph fan graph and friendship graph are obtained by noting the tension of each edge of all the graphs.

References

[1] A. Shimbel. Structural parameters of communication networks. *The Bulletin of Mathematical Biophysics*, 15:501–507, 1953.

[2] N. N. Dattatreya K. Bhargava and R. Rajendra. On stress of a vertex in a graph. *Palestine Journal of Mathematics*, 9(2):1–11, 2022.

[3] S. Arumugam P. Raksha, K. A. Bhat and K. Manjunatha Prasad. The stress of a graph. *Communications in Combinatorics and Optimization*, 8(1):53–65, 2023.

[4] J. Shiny. Induced stress of some graph operations. *Malaya Journal of Matematik*, 9(1):259–261, 2021.

[5] S. Arumugam P. Raksha, K. Arathi Bhat and K. Manjunatha Prasad. Stress of wheel related graphs. *National Academy Science Letters*, 45(5):427–431, 2023.

[6] Sabitha D'Souza Madhumitha K. V., Swati Nayak. Stress and tension of generalized complements of graphs. *Engineering Letters*, 32(3):512–519, 2024.

[7] S. Arumugam P. Raksha, K. Arathi Bhat and K. Manjunatha Prasad. Stress of a graph and its computation. *AKCE International Journal of Graphs and Combinatorics*, 20(2):200–208, 2023.

[8] N. N. Dattatreya K. Bhargava and R. Rajendra. Tension on an edge in a graph. *Boletim da Sociedade Paranaense de Matemática*, 41:1–9, 2023.