Development of DMA Control Charts within Range to Enhance Detection of Changes in Process Variation

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***Abstract*—** This research aims to create a double-moving average control chart for monitoring a change in process dispersion with a range of so-called DMA_R chart and propose the explicit formulas of average run length (ARL) of DMA_R chart. It compares the performance of the moving average control chart based on range (MA_R chart). The proposed control chart is an effective alternative to the MA_R chart using the double moving average based on the sample range. The coefficients for the control limits of the DMA_R chart varying the sample sizes and the width for moving average calculation are presented. Comparison and application to real data sets show that the DMA_R chart detects variations at all levels more effectively than the MA_R chart. Furthermore, when the magnitudes of the variation changes are small, the DMA_R chart becomes more effective as the width increases.

***Index Terms*—Time-varying chart, Variation, Efficiency, Monitoring, Average run length**

I. Introduction

CONTROL charts are effective tools used to control the quality of processes to ensure they are always effective. Control charts help track the progress of the production process to monitor data change trends until changes outside the control limit (out-of-control) are detected. Control charts can also assess the manufacturing process's efficiency and determine the cause of variations to reduce variation and improve production processes. Another important aspect is the standard configuration of the product to achieve the goal. It can also be used to improve the production process to meet the standards of manufacturers and consumers. The quality control charts can be divided into two types: the control chart for variables consists of a variable control chart such as an average chart ($\bar{x}$ chart), range chart *(R* chart*)*, and standard deviation chart *(S chart)*. The second type is control charts for attributes, which are control charts used for detecting the number of defects or the number of nonconformities which is counting data and is an integer. An example of this type of control chart is the defect proportion chart *(p* chart*)*, the number of defect chart *(np* chart*)*, and the number of nonconforming products per unit chart *(u* chart*)*, etc [1].

In 1924, Shewhart proposed the Shewhart control chart (Shewhart chart), an effective control chart for detecting significant average changes [2]. However, the small mean change could not be detected. Subsequently, other quality control charts have been developed to detect small changes more efficiently than the Shewhart chart. For example, in 1959, Roberts proposed an exponentially weighted moving average control chart (*EWMA* chart) based on taking data over time from observations in the collection process [3]. It was found that the EWMA chart outperformed the Shewhart chart when the magnitude of the change in the process mean was small. Later, in 2004, Khoo developed a moving average control chart (*MA* chart) using a simple idea to calculate the MA statistics by giving a width of average (w) [4]. This control chart is also easy to calculate and implement, as its efficiency suits small to moderate shifts [5-7]. Next, Khoo and Wong intensely studied and extended the MA chart, a double moving average control chart (*DMA* chart) whose ability to detect a small change in process is better than the former [8]. In 2016, Olatunde and Olaomia developed the *MA* chart for the detection of a change in variation based on a standard, namely the *MA-S* chart. They proposed explicit formulas to determine the average run length (ARL) and compare the results in detecting variation changes with the *S* chart [9-11]. Later, in 2019, Olatunde et al. proposed a *DMA-S* chart that enhanced the ability to detect changes in process variability, presented an explicit formula for finding the ARL [12], and compared the performance with the *S* chart, *MA-S* chart and the *DMA-S* chart outperform other charts, which is suitable for detecting small to moderate changes in the process variation when the process has a normal distribution [13]. Recently, Chananet et al. [14] designed a moving average based on range, namely MA_R chart for detection process variation and suitable for small sample sizes *(n<10)*. Process variance measurements, such as process consistency checks, are more

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critical than process averages in some situations. Therefore, a process variability control chart must be developed to restore the process as smoothly as possible.

The most commonly used control chart performance comparison is average run length (ARL), which is divided into two states: in-control average run length (ARL_0) and out-of-control average run length (ARL_1), see more details by [15-19]. The classical method for determining the ARL is generally used in the Monte Carlo simulation (MC) to estimate the ARL from a simulation under given circumstances. It is a simple and convenient method to validate the results obtained by other methods. However, such methods have limitations in processing results that vary in time consumption. Subsequently, an explicit formula method took less time to calculate. Nevertheless, it may only be found in some cases of the study.

In this research, the *DMA* chart is developed to create a new control chart for detecting a change in variation based on range, namely the DMA_R chart. In addition, the performance of the DMA_R chart is compared with the MA_R chart for detecting process variations and applying them to actual data. The control chart gives the lowest value, ARL_1, indicating that the control chart is most effective in detecting variation changes.

II. MATERIALS AND METHODS

In this research, a new control chart named "Double Moving Average of Range control chart (DMA_R chart)" for detecting process variability is investigated, and the statistics of the control chart and the control limits are presented. The performance of the proposed control chart is compared with the performance of the DMA_R chart. Generally, the *R* chart is more prevalent among quality control practitioners, especially when dealing with small sample sizes, because of the simplicity of calculating the range from each sample. Therefore, the study control charts, and related research are discussed in this section.

A. Range control chart (R chart)

A range chart is a statistical process control (SPC) tool that displays the variation within a data set. It tracks the variation in a process over time and helps identify any changes in the process variance. It plots the range of the data in each subgroup, where the range is calculated from the difference between the highest and lowest values in each subgroup over time. The *R* chart is suitable if the sample sizes (*N*) are small $(N \le 10)$. For developing a quality control chart, it is essential to always consider this *R* chart in conjunction with the *x-bar* chart, which can be calculated to find the average of the range $(\bar{R})$ as $\bar{R} = \sum_{j=1}^{m} R_j / m$, where R_j is the difference between the highest value in sample *j* and the lowest value in sample *j*. The calculation of the upper control limit (UCL) and lower control limit (LCL) is divided into two cases: known and unknown parameters σ. For the latter case, the parameter must be estimated. Montgomery [1] stated that in the process variability, an unbiased estimator σ, is $\hat{\sigma} = \frac{\bar{R}}{d_2}$ for the *R* chart and is $\hat{\sigma} = \frac{\bar{S}}{C_4}$ for the *S* chart, respectively. Consequently, the control limits are as follows:

1. *Known* σ

$$UCL = d_2\sigma + 3d_3\sigma = D_2\sigma \text{ and } LCL = d_2\sigma - 3d_3\sigma = D_1\sigma \quad (1)$$

where the values from Equation (1), $D_1 = (d_2 - 3d_3)$ and $D_2 = (d_2 + 3d_3)$, are coefficients for control limits and depend on the sample size *(N).*

2. *Unknown* σ

The estimate $\hat{\sigma} = \frac{\bar{R}}{d_2}$ is then substituted $\hat{\sigma}$ into Equation (1) as follows:

$$UCL = \bar{R} + 3\frac{d_3}{d_2}\bar{R} = D_4\bar{R} \text{ and } LCL = \bar{R} - 3\frac{d_3}{d_2}\bar{R} = D_3\bar{R}. \quad (2)$$

Then the values from Equation (2), $D_3 = \left(1 - 3\frac{d_3}{d_2}\right)$ and $D_4 = \left(1 + 3\frac{d_3}{d_2}\right)$.

B. Moving average - range control chart (MA_R chart)

The moving average range (MA_R) chart can detect a change in the process mean and variability [14]. The MA_R chart is implemented to see a change in process variation based on the range value, which depends on the sample size (*N*). The MA_R statistic of width w at times i is calculated as

$$MA_{R_i} = \begin{cases} \dfrac{R_i + R_{i-1} + R_{i-2} + \ldots}{i} & ; i < w \\ \dfrac{R_i + R_{i-1} + \ldots + R_{i-w+1}}{w} & ; i \ge w \end{cases} \quad (3)$$

where R_j is the range of each sample number *j*.

The expectation of the MA_R chart when $i < w$ is presented in Equation (4),

$$E(MA_R) = E\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) = \frac{1}{i}\sum_{j=1}^{i} E\left(R_j\right) = d_2\sigma. \quad (4)$$

Also, the expectation of the MA_R chart, when $i \ge w$ shown in Equation (5),

$$E(MA_R) = E\left(\frac{1}{w}\sum_{j=i-w+1}^{w} R_j\right) = \frac{1}{w}\sum_{j=i-w+1}^{w} E\left(R_j\right) = d_2\sigma. \quad (5)$$

The variance of the MA_R chart, when $i < w$ is presented in Equation (6),

$$Var(MA_R) = Var\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) = \frac{1}{i^2}\sum_{j=1}^{i} Var\left(R_j\right) = \frac{d_3^2\sigma^2}{i}. \quad (6)$$

Also, the variance of the MA_R chart, when $i \geq w$ as shown on Equation (7),

$$Var(MA_R) = Var\left(\frac{1}{w}\sum_{j=i-w+1}^{i} R_j\right) = \frac{d_3^2\sigma^2}{w}. \quad (7)$$

Therefore, the upper control limit (UCL) and lower control limit (LCL) of the MA_R chart can be calculated in two cases following:

1. *Known* σ

1.1 when $i < w$, *then*

$$\begin{aligned} UCL &= d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma + 3\frac{d_3\sigma}{\sqrt{i}} = D_6^*\sigma, \\ LCL &= d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma - 3\frac{d_3\sigma}{\sqrt{i}} = D_5^*\sigma \end{aligned} \quad (8)$$

where $D_5^* = \left(d_2 - 3\frac{d_3}{\sqrt{i}}\right)$ and $D_6^* = \left(d_2 + 3\frac{d_3}{\sqrt{i}}\right)$, they are the coefficients of control limits which are calculated and proposed in the next section.

1.2 when $i \geq w$, *then*

$$\begin{aligned} UCL &= d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma + 3\frac{d_3\sigma}{\sqrt{w}} = D_8^*\sigma, \\ LCL &= d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma - 3\frac{d_3\sigma}{\sqrt{w}} = D_7^*\sigma \end{aligned} \quad (9)$$

where $D_7^* = \left(d_2 - 3\frac{d_3}{\sqrt{w}}\right)$ and $D_8^* = \left(d_2 + 3\frac{d_3}{\sqrt{w}}\right)$, are the coefficients of control limits from the proposed chart.

2. *Unknown* σ

2.1 when $i < w$, *then*

$$UCL = \bar{R} + 3\bar{R}\frac{d_3}{d_2\sqrt{i}} = D_{10}^*\bar{R},\ LCL = \bar{R} - 3\bar{R}\frac{d_3}{d_2\sqrt{i}} = D_9^*\bar{R} \quad (10)$$

where $D_9^* = \left(1 - 3\frac{d_3}{d_2\sqrt{i}}\right)$ and $D_{10}^* = \left(1 + 3\frac{d_3}{d_2\sqrt{i}}\right)$.

2.2 when $i \geq w$, *then*

$$UCL = \bar{R} + 3\bar{R}\frac{d_3}{d_2\sqrt{w}} = D_{12}^*\bar{R},\ LCL = \bar{R} - 3\bar{R}\frac{d_3}{d_2\sqrt{w}} = D_{11}^*\bar{R} \quad (11)$$

where $D_{11}^* = \left(1 - 3\frac{d_3}{d_2\sqrt{w}}\right)$ and $D_{12}^* = \left(1 + 3\frac{d_3}{d_2\sqrt{w}}\right)$.

C. Double moving average - range control chart (DMA_R chart)

This research aims to construct a new control chart and the table of the coefficients of control limits, which depend on the sample size (N) to monitor changes in process variation, namely the Double Moving Average-Range chart (DMA_R chart). This research implements the DMA_R chart to detect a change in process variation based on the range value. This modified DMA_R chart with the range value. The DMA_R statistic of width w at times i is calculated as

$$DMA_{R_i} = \frac{MA_{R_i} + MA_{R_i} + \ldots + MA_{R_{i-w+1}}}{w} \text{ for } i \geq w. \quad (12)$$

Note that the moving average of the subgroup standard deviation, MA_{R_i} of span w at time i is computed using (12(when $i \geq w.$ For period $i < w,$ the DMA_R statistic is calculated to be the average of all moving average standard deviations up to period i. That is,

$$DMA_{R_i} = \frac{\sum_{j=1}^{i} MA_{R_i}}{i}. \quad (13)$$

The mean of the DMA_R statistic based on an in-control process where the underlying assumption follows a normal distribution, $N(\mu, \sigma^2)$ for the period, $i \geq w$ is given as

$$E(DMA_R) = E\left(\frac{1}{w}\sum_{j=i-w+1}^{i} MA_{R_j}\right) = \frac{1}{w}\sum_{j=i-w+1}^{i} E\left(MA_{R_j}\right) = d_2\sigma. \quad (14)$$

The variation of DMA_R is given as follows for $w > 2$,

$$Var(DMA_R) = \begin{cases} \dfrac{d_3^2\sigma^2}{i^2} & ; i \leq w \\ \dfrac{d_3^2\sigma^2}{w^2}\left[\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (i-w+1)\right]; & w < i < 2w-1. \\ \dfrac{d_3^2\sigma^2}{w^2} & ; i \geq 2w-1 \end{cases} \quad (15)$$

The calculation of the upper control limit (UCL) and lower control limit (LCL) of the DMA_R chart is divided into three cases as follows:

1. *Known* σ

1.1 when $i \leq w,$

$$UCL = d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{i^2}} = d_2\sigma + 3\frac{d_3\sigma}{\sqrt{i^2}} = D_{14}^*\sigma,$$

$$LCL = d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{i^2}} = d_2\sigma - 3\frac{d_3\sigma}{\sqrt{i^2}} = D_{13}^{*}\sigma \tag{16}$$

where $D_{13}^{*} = \left(d_2 - 3\frac{d_3}{\sqrt{i^2}}\right)$ and $D_{14}^{*} = \left(d_2 + 3\frac{d_3}{\sqrt{i^2}}\right)$.

1.2 when $w < i < 2w-1$,

$$\begin{aligned} UCL &= d_2\sigma + 3\frac{d_3\sigma}{\sqrt{w^2}}\sqrt{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}} = D_{16}^{*}\sigma, \\ LCL &= d_2\sigma - 3\frac{d_3\sigma}{\sqrt{w^2}}\sqrt{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}} = D_{15}^{*}\sigma \end{aligned} \tag{17}$$

where $D_{15}^{*} = \left(d_2 - 3d_3\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}}\right)$

and $D_{16}^{*} = \left(d_2 + 3d_3\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}}\right)$.

1.3 when $i \ge 2w-1$,

$$\begin{aligned} UCL &= d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{w^2}} = d_2\sigma + 3d_3\sigma\sqrt{\frac{1}{w^2}} = D_{18}^{*}\sigma, \\ LCL &= d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{w^2}} = d_2\sigma - 3d_3\sigma\sqrt{\frac{1}{w^2}} = D_{17}^{*}\sigma \end{aligned} \tag{18}$$

where $D_{17}^{*} = \left(d_2 - 3d_3\sqrt{\frac{1}{w^2}}\right)$, and $D_{18}^{*} = \left(d_2 + 3d_3\sqrt{\frac{1}{w^2}}\right)$.

2. Unknown σ

2.1 when $i \le w$,

$$UCL = \bar{R} + 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{i^2}} = D_{20}^{*}\sigma,\ LCL = \bar{R} - 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{i^2}} = D_{19}^{*}\sigma \tag{19}$$

where $D_{19}^{*} = \left(1 - 3\frac{d_3}{d_2}\sqrt{\frac{1}{i^2}}\right)$, and $D_{20}^{*} = \left(1 + 3\frac{d_3}{d_2}\sqrt{\frac{1}{i^2}}\right)$.

2.2 when $w < i < 2w-1$,

$$UCL = \bar{R} + 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}} = D_{22}^{*}\sigma,$$

$$LCL = \bar{R} - 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}} = D_{21}^{*}\sigma \tag{20}$$

where $D_{21}^{*} = \left(1 - 3\frac{d_3}{d_2}\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}}\right)$

and $D_{22}^{*} = \left(1 + 3\frac{d_3}{d_2}\sqrt{\frac{\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(i-w+1)}{i}}{w^2}}\right)$.

2.3 when $i \ge 2w-1$,

$$UCL = \bar{R} + 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{w^2}} = D_{24}^{*}\sigma,\ LCL = \bar{R} - 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{w^2}} = D_{23}^{*}\sigma \tag{21}$$

where $D_{23}^{*} = \left(1 - 3\frac{d_3}{d_2}\sqrt{\frac{1}{w^2}}\right)$ and $D_{24}^{*} = \left(1 + 3\frac{d_3}{d_2}\sqrt{\frac{1}{w^2}}\right)$.

D. Average run length (ARL)

The control chart's effectiveness is gauged through the Average Run Length (ARL), categorized into two states: in-control and out-of-control processes. Refer to Montgomery's details [1] under normal circumstances, in-control processes should yield high ARL values, whereas out-of-control processes should result in minimal values. Historically, various analytical methods have been employed to compute ARL, with Monte Carlo (MC) simulation emerging as the most widely used and accurate technique. Nonetheless, this method faces limitations in handling large datasets and time constraints. The calculation for ARL using Monte Carlo is outlined as follows.

$$ARL = \sum_{i=1}^{T} RL_i \Big/ T \tag{22}$$

In this context, RL_i signifies the sample being analyzed before the process surpasses the control limits for the initial occurrence. In the i^{th} simulation round, N denotes the count of experiment repetitions.

Moreover, various approaches are available; the Markov chain approach (MCA) is a widely adopted and effective technique that applies matrix inversion to the principles of Markov chains. While there is no theoretical impact on accuracy, the results are compared with Monte Carlo (MC) simulations [15]. The integral equation (IE) also represents a contemporary method utilizing fundamental mathematical formulas and the central limit theorem. This approach is another method capable of accurately assessing the performance of a control chart [16]. Although the mentioned techniques are suitable for evaluating the efficiency of control charts, they may only be optimal for optimizing some control charts as the process changes. Furthermore, an explicit formula is suggested for assessing ARL see [5 - 7] and [9 - 14] for additional details.

III. Results and Discussion

In this section, the results of this new control chart design can be described as follows: The First Part determines the control chart, and the Second Part finds the explicit formula of the DMA_R chart. Next, the Third Part compares the control chart, and the Final Part applies to actual data.

A. Coefficient of the DMA_R Chart

This section showed the factor of control limits for the known and unknown parameters (σ). Adjust the width (w) of the DMA_R chart to 5. When the parameters (σ) are identified, their explanation can be provided as follows. Table I indicates the coefficient of control limits of the DMA_R chart for w = 1, 2, 3, 4, and 5, which is calculated from (16) $i \le w$. Next, Table II illustrates the coefficient of control limits of the DMA_R chart for w = 6, 7, and 8 when $w < i < 2w-1$ computed from (17). Later, Table III specifies that the coefficient of control limits of the DMA_R chart for w = 9, 10, 15, and 20 when $i \ge 2w-1$ found from (18). In addition, unknown parameters (σ) can be explained as such: Table IV shows the coefficient of control limits of the DMA_R chart for w = 1, 2, 3, 4, and 5, which is calculated from (19), while $i \le w$. Table V illustrates that the coefficient of control limits of the DMA_R chart for w = 6, 7, and 8 when $w < i < 2w-1$ compute from (20). Finally, Table VI specifies that the coefficient of control limits of the DMA_R chart for w = 9, 10, 15, and 20 when $i \ge 2w-1$ found from (21). Therefore, the coefficient of control limits tables is very useful and prompt for practitioners.

B. Explicit formulas ARL of the DMA_R Chart

The efficiency of the control chart is measured from the average run length, which can be obtained from the explicit formula of DMA_R as Equation (23). The detail of the ARL is shown in the appendix.

$$
\begin{aligned}
ARL \cong \Bigg\{ 1 - \sum_{i=1}^{w} \Bigg(& P\left(Z > \frac{\bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}} \right) + P\left(Z < \frac{\bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}} \right) \Bigg) \\
- \sum_{j=i-w+1}^{2w-2} \Bigg(& P\left(Z > \frac{\bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}} \right) \Bigg) \\
+ & P\left(Z < \frac{\bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}} \right) \Bigg\} \\
\times \Bigg\{ & P\left(Z > \frac{\bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}} \right) + P\left(Z < \frac{\bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}} \right) \Bigg\}^{-1} \\
+ & (2w-2). \qquad (23)
\end{aligned}
$$

TABLE I

Coefficient of DMA_R Control Limit for the Known Parameter σ When $i \le w$

Sample (N)	d_2	d_3	w =1 D_{13}^*	w =1 D_{14}^*	w =2 D_{13}^*	w =2 D_{14}^*	w =3 D_{13}^*	w =3 D_{14}^*	w =4 D_{13}^*	w =4 D_{14}^*	w =5 D_{13}^*	w =5 D_{14}^*
2	1.128	0.853	0.000	3.687	0.000	2.695	0.000	2.283	0.205	2.051	0.355	3.343
3	1.693	0.888	0.000	4.357	0.062	3.324	0.491	2.895	0.732	2.654	0.888	3.950
4	2.059	0.88	0.000	4.699	0.442	3.676	0.868	3.251	1.106	3.012	1.261	4.260
5	2.326	0.864	0.000	4.918	0.739	3.913	1.156	3.496	1.391	3.261	1.543	4.459
6	2.534	0.848	0.000	5.078	0.976	4.092	1.386	3.682	1.616	3.452	1.765	4.604
7	2.704	0.833	0.205	5.203	1.174	4.234	1.576	3.832	1.802	3.606	1.949	4.922
8	2.847	0.82	0.387	5.307	1.341	4.353	1.737	3.957	1.959	3.735	2.104	5.199
9	2.97	0.808	0.546	5.394	1.486	4.454	1.876	4.064	2.095	3.845	2.237	5.436
10	3.078	0.797	0.687	5.469	1.614	4.542	1.999	4.157	2.215	3.941	2.355	5.645
11	3.173	0.787	0.812	5.534	1.727	4.619	2.107	4.239	2.321	4.025	2.459	5.829
12	3.258	0.778	0.924	5.592	1.829	4.687	2.205	4.311	2.416	4.100	2.553	5.994
13	3.336	0.77	1.026	5.646	1.921	4.751	2.293	4.379	2.503	4.170	2.638	6.145
14	3.407	0.762	1.121	5.693	2.007	4.807	2.375	4.439	2.582	4.232	2.716	6.283
15	3.472	0.755	1.207	5.737	2.085	4.859	2.450	4.494	2.655	4.289	2.787	6.408
16	3.532	0.749	1.285	5.779	2.156	4.908	2.518	4.546	2.721	4.343	2.853	6.524
17	3.588	0.743	1.359	5.817	2.223	4.953	2.582	4.594	2.784	4.392	2.914	6.633
18	3.64	0.738	1.426	5.854	2.284	4.996	2.641	4.639	2.841	4.439	2.971	6.734
19	3.689	0.733	1.490	5.888	2.342	5.036	2.697	4.682	2.896	4.483	3.024	6.828
20	3.735	0.729	1.548	5.922	2.396	5.074	2.748	4.722	2.949	4.524	3.074	6.917
25	3.931	0.709	1.804	6.058	2.629	5.234	2.971	4.891	3.164	4.699	3.288	7.296

TABLE II
COEFFICIENT OF DMA_R CONTROL LIMIT FOR THE KNOWN PARAMETER σ WHEN $w < i < 2w-1$

Sample (N)	d_2	d_3	w =6 D^*_{15}	w =6 D^*_{16}	w =7 D^*_{15}	w =7 D^*_{16}	w =8 D^*_{15}	w =8 D^*_{16}
2	1.128	0.853	0.408	3.112	0.475	2.823	0.528	2.593
3	1.693	0.888	0.943	3.678	1.013	3.337	1.068	3.064
4	2.059	0.880	1.316	3.966	1.385	3.598	1.440	3.305
5	2.326	0.864	1.597	4.151	1.664	3.766	1.718	3.460
6	2.534	0.848	1.818	4.286	1.885	3.889	1.938	3.572
7	2.704	0.833	2.001	4.597	2.066	4.189	2.118	3.865
8	2.847	0.820	2.155	4.867	2.219	4.451	2.270	4.120
9	2.970	0.808	2.288	5.099	2.351	4.677	2.402	4.340
10	3.078	0.797	2.405	5.303	2.468	4.875	2.517	4.533
11	3.173	0.787	2.509	5.483	2.570	5.050	2.620	4.704
12	3.258	0.778	2.601	5.644	2.662	5.206	2.711	4.857
13	3.336	0.770	2.686	5.792	2.746	5.350	2.794	4.997
14	3.407	0.762	2.764	5.926	2.823	5.481	2.871	5.125
15	3.472	0.755	2.835	6.050	2.894	5.600	2.941	5.242
16	3.532	0.749	2.900	6.163	2.958	5.711	3.005	5.350
17	3.588	0.743	2.961	6.269	3.019	5.813	3.065	5.450
18	3.640	0.738	3.017	6.367	3.075	5.909	3.121	5.543
19	3.689	0.733	3.070	6.460	3.128	5.999	3.173	5.631
20	3.735	0.729	3.120	6.547	3.177	6.083	3.222	5.713
25	3.931	0.709	3.332	6.918	3.388	6.443	3.432	6.065

TABLE III
COEFFICIENT OF DMA_R CONTROL LIMIT FOR THE KNOWN PARAMETER σ WHEN $i \geq 2w-1$

Sample (N)	d_2	d_3	w =9 D^*_{17}	w =9 D^*_{18}	w =10 D^*_{17}	w =10 D^*_{18}	w =15 D^*_{17}	w =15 D^*_{18}	w=20 D^*_{17}	w=20 D^*_{18}
2	1.128	0.853	0.844	0.803	0.872	1.384	0.957	1.299	1.000	1.256
3	1.693	0.888	1.397	1.370	1.427	1.959	1.515	1.871	1.560	1.826
4	2.059	0.88	1.766	1.865	1.795	2.323	1.883	2.235	1.927	2.191
5	2.326	0.864	2.038	2.237	2.067	2.585	2.153	2.499	2.196	2.456
6	2.534	0.848	2.251	2.531	2.280	2.788	2.364	2.704	2.407	2.661
7	2.704	0.833	2.426	2.772	2.454	2.954	2.537	2.871	2.580	2.829
8	2.847	0.82	2.574	2.976	2.601	3.093	2.683	3.011	2.724	2.970
9	2.97	0.808	2.701	3.152	2.728	3.212	2.808	3.132	2.849	3.091
10	3.078	0.797	2.812	3.307	2.839	3.317	2.919	3.237	2.958	3.198
11	3.173	0.787	2.911	3.444	2.937	3.409	3.016	3.330	3.055	3.291
12	3.258	0.778	2.999	3.566	3.025	3.491	3.102	3.414	3.141	3.375
13	3.336	0.77	3.079	3.678	3.105	3.567	3.182	3.490	3.221	3.452
14	3.407	0.762	3.153	3.781	3.178	3.636	3.255	3.559	3.293	3.521
15	3.472	0.755	3.220	3.874	3.246	3.699	3.321	3.623	3.359	3.585
16	3.532	0.749	3.282	3.960	3.307	3.757	3.382	3.682	3.420	3.644
17	3.588	0.743	3.340	4.041	3.365	3.811	3.439	3.737	3.477	3.699
18	3.64	0.738	3.394	4.115	3.419	3.861	3.492	3.788	3.529	3.751
19	3.689	0.733	3.445	4.186	3.470	3.909	3.542	3.836	3.579	3.799
20	3.735	0.729	3.492	4.251	3.516	3.954	3.589	3.881	3.626	3.844
25	3.931	0.709	3.695	4.532	3.718	4.144	3.789	4.073	3.825	4.037

C. The comparison of performance of the DMA_R chart

This section shows the performance of the DMA_R chart compared with the MA_R chart for monitoring process variability. The control chart with the smallest ARL_1 is the most efficient. The width (w) parameter for MA_R and DMA_R charts are set to 2, 3, 5, 10, and 15 and given $ARL_0 = 370$. The shift sizes of the process variation $(\delta = \sigma_1 / \sigma_0)$ where $\sigma_0 = 1$, the process are from Normal (0,1) were 1.02, 1.04, 1.06, 1.08, 1.10, 1.25, 1.75, 2.00, 2.50 and 3.00. The ARL calculations for MA_R and DMA_R charts yield results that can be categorized into three cases. Table VII shows the ARL of DMA_R and MA_R charts for $n = 5$. The result shows that when the magnitudes of changes δ are small to moderate ($\delta < 2$), the proposed chart's detection efficiency is better than the MA_R chart. Otherwise, there are no significant differences for all case studies. Table VIII presents the ARL for MA_R and DMA_R charts for a subgroup size of 10. The findings reveal that when δ are less than 1.75, DMA_R charts demonstrate greater effectiveness in detecting changes than MA_R chart. Conversely, when δ exceed 2.00, MA_R chart outperform DMA_R chart. Finally, Table IX presents the ARL for MA_R and DMA_R charts with a subgroup size 15. The results suggest that when δ are below 2.00, DMA_R charts exhibit greater effectiveness in detecting changes than MA_R charts in all cases. On the other hand, when δ =2.5, DMA_R charts prove to be as adept at capturing process changes as MA_R chart. By calculating ARL values, the MA_R and DMA_R charts reveal an interesting trend: as the magnitude of shifts (δ) increases, the width (w) decreases.

TABLE IV
COEFFICIENT OF DMA$_R$ CONTROL LIMIT FOR THE UNKNOWN PARAMETER σ WHEN $i \le w$

Sample (N)	d_2	d_3	w =1 D^*_{19}	w =1 D^*_{20}	w =2 D^*_{19}	w =2 D^*_{20}	w =3 D^*_{19}	w =3 D^*_{20}	w =4 D^*_{19}	w =4 D^*_{20}	w =5 D^*_{19}	w =5 D^*_{20}
2	1.128	0.853	0.000	3.269	0.000	2.389	0.000	2.024	0.181	1.819	0.143	1.857
3	1.693	0.888	0.000	2.574	0.036	1.964	0.290	1.710	0.432	1.568	0.406	1.594
4	2.059	0.88	0.000	2.282	0.215	1.785	0.421	1.579	0.537	1.463	0.516	1.484
5	2.326	0.864	0.000	2.114	0.318	1.682	0.497	1.503	0.598	1.402	0.579	1.421
6	2.534	0.848	0.000	2.004	0.385	1.615	0.547	1.453	0.638	1.362	0.621	1.379
7	2.704	0.833	0.076	1.924	0.434	1.566	0.583	1.417	0.667	1.334	0.651	1.349
8	2.847	0.82	0.136	1.864	0.471	1.529	0.610	1.390	0.688	1.312	0.674	1.326
9	2.97	0.808	0.184	1.816	0.500	1.500	0.632	1.368	0.706	1.295	0.692	1.308
10	3.078	0.797	0.223	1.777	0.524	1.476	0.649	1.351	0.720	1.280	0.706	1.293
11	3.173	0.787	0.256	1.744	0.544	1.456	0.664	1.336	0.732	1.269	0.719	1.281
12	3.258	0.778	0.284	1.716	0.561	1.439	0.677	1.323	0.742	1.259	0.729	1.271
13	3.336	0.77	0.308	1.692	0.576	1.424	0.688	1.313	0.750	1.250	0.738	1.262
14	3.407	0.762	0.329	1.671	0.589	1.411	0.697	1.303	0.758	1.242	0.746	1.253
15	3.472	0.755	0.348	1.652	0.601	1.400	0.706	1.294	0.765	1.235	0.754	1.246
16	3.532	0.749	0.364	1.636	0.610	1.390	0.713	1.287	0.770	1.230	0.760	1.240
17	3.588	0.743	0.379	1.621	0.620	1.380	0.720	1.280	0.776	1.224	0.765	1.235
18	3.64	0.738	0.392	1.608	0.628	1.373	0.726	1.275	0.781	1.219	0.770	1.230
19	3.689	0.733	0.404	1.596	0.635	1.365	0.731	1.269	0.785	1.215	0.775	1.225
20	3.735	0.729	0.415	1.586	0.641	1.359	0.736	1.264	0.789	1.211	0.779	1.221
25	3.931	0.709	0.459	1.541	0.669	1.331	0.756	1.244	0.805	1.1952	0.796	1.204

TABLE V
COEFFICIENT OF DMA$_R$ CONTROL LIMIT FOR THE UNKNOWN PARAMETER σ WHEN $w < i < 2w-1$

Sample (N)	d_2	d_3	w =6 D^*_{21}	w =6 D^*_{22}	w =7 D^*_{21}	w =7 D^*_{22}	w =8 D^*_{21}	w =8 D^*_{22}
2	1.128	0.853	0.362	1.638	0.421	1.579	0.468	1.532
3	1.693	0.888	0.557	1.443	0.598	1.402	0.631	1.369
4	2.059	0.880	0.639	1.361	0.673	1.327	0.699	1.301
5	2.326	0.864	0.686	1.313	0.716	1.284	0.739	1.261
6	2.534	0.848	0.718	1.282	0.744	1.256	0.765	1.235
7	2.704	0.833	0.740	1.260	0.764	1.236	0.783	1.217
8	2.847	0.820	0.757	1.243	0.779	1.221	0.797	1.203
9	2.970	0.808	0.770	1.230	0.792	1.208	0.809	1.191
10	3.078	0.797	0.781	1.218	0.802	1.198	0.817	1.182
11	3.173	0.787	0.791	1.209	0.810	1.190	0.825	1.174
12	3.258	0.778	0.798	1.201	0.817	1.183	0.832	1.168
13	3.336	0.770	0.805	1.195	0.823	1.177	0.838	1.162
14	3.407	0.762	0.811	1.189	0.829	1.171	0.843	1.157
15	3.472	0.755	0.816	1.184	0.833	1.165	0.847	1.153
16	3.532	0.749	0.821	1.179	0.838	1.162	0.851	1.149
17	3.588	0.743	0.825	1.175	0.841	1.159	0.854	1.146
18	3.640	0.738	0.829	1.171	0.845	1.155	0.857	1.143
19	3.689	0.733	0.832	1.168	0.848	1.152	0.860	1.140
20	3.735	0.729	0.835	1.165	0.851	1.150	0.863	1.137
25	3.931	0.709	0.848	1.152	0.862	1.138	0.873	1.127

D. Applied to real application

1. Application I

This section presents the utilization of the application for the control chart. The observation of real data is five samples and 45 subgroups [1]. Determine the ARL of MA_R, and DMA_R charts in detecting data changes. The statistics of the MA$_R$, versus the DMA_R chart are shown in Fig. 1(a) and 1(b), respectively. The results show that the first sample outside of the control limit of the DMA_R chart is sample no.18, respectively while the MA_R chart cannot detect any changes. Therefore, the DMA_R chart is the best control chart for detecting the process variation change.

2. Application II

The second dataset is derived from Adeoti and Olaomi. The data set represents the five groups and 20 subgroups [9]. Fig. 2(a) and 2(b) show the statistics of the MA_R, and DMA_R charts, respectively. The results show that the DMA_R chart can quickly detect a change in the early process (observation no.3rd) while the MA_R chart cannot detect process variation. Consequently, the DMA_R chart is superior to the MA_R chart, confirming both the results obtained from the explicit formulas and two real data sets.

TABLE VI
COEFFICIENT OF DMA$_R$ CONTROL LIMIT FOR THE UNKNOWN PARAMETER σ WHEN $i \geq 2w-1$

Sample (N)	d_2	d_3	w=9 D^*_{23}	w=9 D^*_{24}	w=10 D^*_{23}	w=10 D^*_{24}	w=15 D^*_{23}	w=15 D^*_{24}	w=20 D^*_{23}	w=20 D^*_{24}
2	1.128	0.853	0.748	1.252	0.773	1.227	0.849	1.151	0.887	1.113
3	1.693	0.888	0.825	1.175	0.843	1.157	0.895	1.105	0.921	1.079
4	2.059	0.880	0.858	1.142	0.872	1.128	0.915	1.086	0.936	1.064
5	2.326	0.864	0.876	1.124	0.889	1.111	0.926	1.074	0.944	1.056
6	2.534	0.848	0.888	1.111	0.300	1.100	0.933	1.067	0.950	1.050
7	2.704	0.833	0.897	1.103	0.908	1.092	0.938	1.062	0.954	1.046
8	2.847	0.820	0.904	1.096	0.914	1.086	0.942	1.058	0.957	1.043
9	2.970	0.808	0.909	1.091	0.918	1.082	0.946	1.054	0.959	1.041
10	3.078	0.797	0.914	1.086	0.922	1.078	0.948	1.052	0.961	1.039
11	3.173	0.787	0.917	1.083	0.926	1.074	0.950	1.050	0.962	1.037
12	3.258	0.778	0.920	1.080	0.928	1.072	0.952	1.048	0.964	1.036
13	3.336	0.770	0.923	1.077	0.931	1.069	0.954	1.046	0.965	1.034
14	3.407	0.762	0.925	1.075	0.933	1.067	0.955	1.045	0.966	1.034
15	3.472	0.755	0.927	1.072	0.935	1.065	0.957	1.044	0.967	1.033
16	3.532	0.749	0.929	1.071	0.936	1.064	0.958	1.042	0.968	1.032
17	3.588	0.743	0.931	1.069	0.938	1.062	0.959	1.041	0.969	1.031
18	3.640	0.738	0.932	1.068	0.939	1.061	0.959	1.041	0.970	1.030
19	3.689	0.733	0.934	1.066	0.940	1.060	0.960	1.040	0.970	1.030
20	3.735	0.729	0.935	1.065	0.941	1.059	0.961	1.039	0.971	1.029
25	3.931	0.709	0.940	1.060	0.946	1.054	0.964	1.036	0.973	1.027

TABLE VII
COMPARISON OF ARL$_1$ FOR THE CONTROL CHART WHEN ARL$_0$ = 370 AND N = 5

δ	w = 2		w = 3		w = 5		w = 10		w = 15	
	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R
0.00	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
1.02	290.447	319.555	273.193	314.007	229.241	304.019	130.491	282.870	**80.933**	265.274
1.04	212.792	255.354	175.363	240.908	110.442	217.391	43.235	176.262	**33.628**	149.109
1.06	150.990	195.440	108.341	175.868	55.322	147.285	**24.066**	105.638	27.029	83.235
1.08	106.552	147.121	68.049	126.663	31.160	99.515	**18.703**	65.437	25.079	49.986
1.10	75.963	110.748	44.414	91.811	19.932	68.604	**16.689**	42.826	23.523	32.867
1.25	11.687	20.059	6.799	14.864	**6.585**	10.481	9.0740	8.685	8.4523	9.861
1.75	**2.197**	2.521	2.344	2.397	2.451	2.646	2.4333	3.273	2.4333	3.461
2.00	**1.775**	1.851	1.871	1.877	1.887	2.097	1.8851	2.339	1.8851	2.363
2.50	1.399	**1.391**	1.421	1.445	1.421	1.532	1.42105	1.564	1.4210	1.564
3.00	1.233	**1.225**	1.239	1.260	1.239	1.291	1.239	1.297	1.239	1.296

Note: the bold number gives the minimum of ARL$_1$

TABLE VII
COMPARISON OF ARL$_1$ FOR THE CONTROL CHART WHEN ARL$_0$ = 370 AND N = 10

δ	w = 2		w = 3		w = 5		w = 10		w = 15	
	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R
0.00	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
1.02	275.620	308.364	244.922	298.694	179.247	281.683	*79.116*	247.763	**49.266**	221.846
1.04	180.010	227.035	129.358	205.682	65.521	173.624	**26.023**	125.330	27.659	98.570
1.06	112.907	158.374	67.883	134.105	29.030	102.586	**18.158**	64.611	24.818	48.236
1.08	71.568	109.494	38.113	87.681	16.395	62.394	**16.031**	36.715	22.312	27.925
1.10	46.825	76.679	23.266	58.812	**11.363**	39.876	14.733	23.263	18.808	18.981
1.25	6.178	10.919	4.374	7.911	**5.252**	6.041	5.386	6.510	5.235	8.031
1.75	**1.666**	1.712	1.711	1.759	1.712	1.920	1.712	2.006	1.712	2.008
2.00	1.376	**1.374**	1.385	1.423	1.385	1.478	1.385	1.487	1.385	1.487
2.50	1.136	**1.134**	1.137	1.148	1.137	1.153	1.137	1.153	1.137	1.153
3.00	1.058	**1.058**	1.058	1.061	1.058	1.062	1.058	1.062	1.058	1.062

Note: the bold number gives the minimum of ARL$_1$

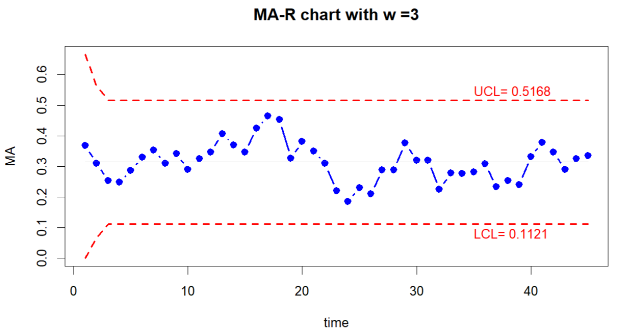
IV. DISCUSSION

The proposed new chart is the DMA_R chart for detecting process variability changes. The prompt coefficient tables for the DMA_R charts are supplied for cases of known and unknown parameters σ, with different sample sizes (N) and width (w) values. The explicit formulas are derived and proved by the central limit theorem. The numerical results from the explicit formulas found that the performance of DMA_R chart is superior to the MA_R chart. Additionally, the explicit formulas are accurate, easy to calculate and have less time for calculation. Two real applications are shown: the performance comparison of the DMA_R chart versus the MA_R chart, in which the proposed chart is superior to the MA_R chart for small and moderate shifts in process dispersion. Otherwise, the performance of MA_R and DMA_R charts are in the same manner.

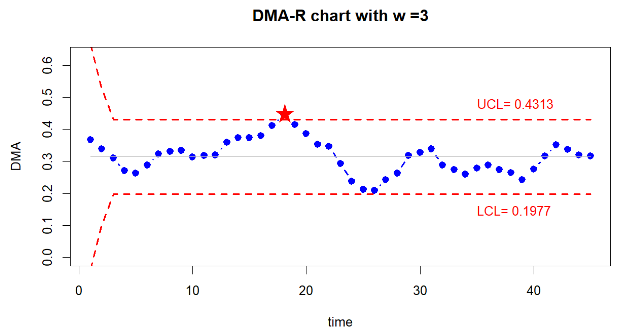
TABLE VII
COMPARISON OF ARL_1 FOR THE CONTROL CHART WHEN $ARL_0 = 370$ AND N = 15

δ	w = 2		w = 3		w = 5		w = 10		w =15	
	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R	DMA_R	MA_R
0.00	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398	370.398
1.02	264.538	300.234	225.565	287.724	151.237	266.179	59.807	225.170	**40.034**	195.669
1.04	159.397	208.829	105.454	184.149	48.207	149.055	**21.664**	100.760	26.315	76.588
1.06	92.737	137.426	50.887	112.185	20.893	81.586	**16.799**	48.406	23.578	35.862
1.08	55.477	90.454	27.285	69.566	**12.425**	47.097	15.102	26.831	19.748	21.154
1.10	34.827	60.913	16.404	44.858	**9.244**	29.144	13.475	17.182	15.209	15.122
1.25	4.592	7.848	**3.727**	5.785	4.465	4.812	4.320	5.769	4.300	7.007
1.75	**1.458**	1.464	1.469	1.519	1.469	1.589	1.469	1.603	1.469	1.603
2.00	**1.223**	1.221	1.224	1.247	1.224	1.259	1.224	1.259	1.224	1.259
2.50	**1.060**	**1.060**	**1.060**	1.063	**1.060**	1.063	1.060	1.063	1.060	1.063
3.00	**1.019**	**1.019**	**1.019**	1.020	1.020	1.020	**1.019**	**1.019**	1.020	**1.019**

Note: the bold number gives the minimum of ARL_1

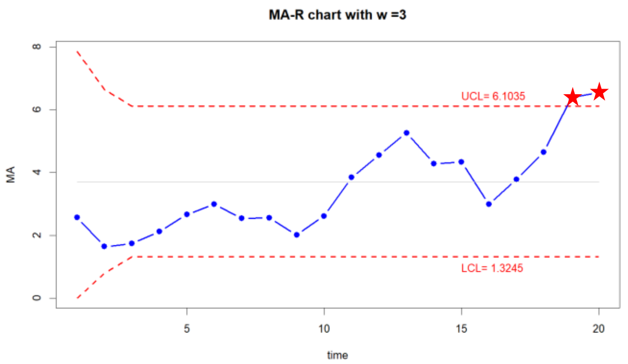


(a)

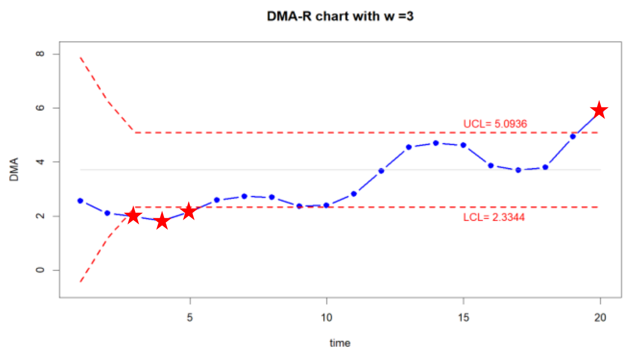


(b)

Fig. 1. (a) The statistics of the MA_R chart and (b) The statistics of the DMA_R chart for real application I



(a)



(b)

Fig. 2. (a) The statistics of the MA_R chart and (b) The statistics of the DMA_R chart for real application II

Also, the DMA_R chart performs better for small and large sample sizes for small shifts in process variability. The mixed control charts are an alternative effective to detecting a very small change in process dispersion, which is currently an extensive study.

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APPENDIX

Let $ARL = n$, then the ARL with $w \geq 2$ is computed as

$$
\begin{aligned}
\frac{1}{ARL} &\cong \frac{1}{n} P\left(\text{out of control signal at time } i \leq w\right) \\
&\quad + \frac{1}{n} P\left(\text{out of control signal at time } w < i < 2w-1\right) \\
&\quad + \frac{n-(2w-2)}{n} P\left(\text{out of control signal at time } i \geq 2w-1\right) \\
&\cong \frac{1}{n}\sum_{i=1}^{w}\left(P\left(MA_{R_i} > UCL_{i<w}\right) + P\left(MA_{R_i} < LCL_{i<w}\right)\right) \\
&\quad + \frac{1}{n}\sum_{j=i-w+1}^{2w-2}\left(P\left(M_j > UCL_{w<i<2w-1}\right) + P\left(M_j < LCL_{w<i<2w-1}\right)\right) \\
&\cong \frac{1}{n}\sum_{i=1}^{w}\left(P\left(\frac{\sum_{j=1}^{i} MA_{R_j}}{i} > \bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}\right)\right. \\
&\quad \left. + P\left(\frac{\sum_{j=1}^{i} MA_{R_j}}{i} < \bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}\right)\right) \\
&\quad + \frac{1}{n}\sum_{j=i-w+1}^{2w-2}\left(P\left(\frac{\sum_{j=i-w+1}^{i} MA_{R_j}}{w} > \bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}\right)\right. \\
&\quad \left. + P\left(\frac{\sum_{j=i-w+1}^{i} MA_{R_j}}{w} < \bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}\right)\right) \\
&\quad + \frac{n-(2w-1)}{n}\left(P\left(\frac{\sum_{j=i-w+1}^{i} MA_{R_j}}{w} > \bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}\right)\right. \\
&\quad \left. + P\left(\frac{\sum_{j=i-w+1}^{i} MA_{R_j}}{w} > \bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}\right)\right) \\
&\cong \frac{1}{n}\sum_{i=1}^{w} P\left(\left(Z > \frac{\bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right. \\
&\quad \left. + P\left(Z < \frac{\bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right) \\
&\quad + \frac{1}{n}\sum_{j=w+1}^{2w-2} P\left(\left(Z > \frac{\bar{R} + H\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}} - \bar{R}_1}{\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}}\right)\right. \\
&\quad \left. + P\left(Z < \frac{\bar{R} - H\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}} - \bar{R}_1}{\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=w+1}^{w-1}\frac{1}{j} + \frac{(j-w+1)}{w}}}\right)\right) \\
&\quad + \frac{n-(2w-2)}{n}\left(P\left(Z > \frac{\bar{R} + H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right)\right. \\
&\quad \left. + P\left(Z < \frac{\bar{R} - H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}} - \bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right)\right). \qquad \text{(a1)}
\end{aligned}
$$

Let A_1, A_2 and A_3 be as follows:

$$A_1=\sum_{i=1}^{w}\left(P\left(Z>\frac{\bar{R}+H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right)$$

$$A_2=\sum_{j=i-w+1}^{2w-2}\left(P\left(Z>\frac{\bar{R}+H\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}-\bar{R}_1}{\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}-\bar{R}_1}{\sqrt{\frac{(1-D_4^2)}{w^2}\sum_{j=w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}}\right)\right)$$

$$A_3=P\left(Z>\frac{\bar{R}+H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right)$$

$$+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right).$$

When substituting A_1, A_2 and A_3 in Equation (a1), then

$$n\cong\left(1-A_1-A_2\right)A_3^{-1}+(2w-2).$$

The explicit formulas of ARL can be calculated as in (a2)

$$ARL\cong\left(1-A_1-A_2\right)A_3^{-1}+(2w-2);\ \ w\neq\frac{n}{2}-1. \qquad \text{(a2)}$$

The explicit formulas of the $\mathrm{DMA_R}$ chart can be written as

$$ARL\cong\left\{1-\sum_{i=1}^{w}\left(P\left(Z>\frac{\bar{R}+H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.$$

$$\left.+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{i^2}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right)$$

$$-\sum_{j=i-w+1}^{2w-2}\left(P\left(Z>\frac{\bar{R}+H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}}\right)\right.$$

$$\left.\left.+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+\frac{(j-w+1)}{w}}}\right)\right)\right\}$$

$$\times\left\{P\left(Z>\frac{\bar{R}+H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right)\right.$$

$$\left.+P\left(Z<\frac{\bar{R}-H\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}-\bar{R}_1}{\sqrt{\frac{\left(1-D_4^2\right)}{w^2}}}\right)\right\}^{-1}+(2w-2). \qquad \text{(a3)}$$