A Modified Reliability Optimization Design Method Assisted with Mean First-order Reliability Method and Its Application in Pile Foundation

Zhijun Xu, Pengyang Zeng, Zhaoxiang Guo, and Qingnian Yang

Abstract—**Reliability-based design optimization (RBDO) is a valuable tool for optimizing while considering the impact of uncertainties. However, its application in engineering, specifically in pile foundation design, is complicated due to high computational costs and the potential for nonlinear iteration misconvergence. To address these challenges, we propose a modified optimization calculation method utilizing the mean first-order reliability method (MFORM). The revised function of the reliability index is introduced to ensure computational accuracy and linear regression is employed for its calculation. The results of the case study demonstrate that the modified optimization calculation method not only improves computational efficiency but also enhances computational accuracy. While the form of the performance function significantly influences initial and local optimizations, it has minimal impact on global optimization. Through local and global optimizations, the objective function values are reduced by 20.2% and 24.9%, respectively, for the first form of the performance function. For the second form of the performance function, reductions of 15.0% and 24.9%, respectively, are achieved through local and global optimizations.**

Index Terms—**reliability optimization; first-order reliability method; revised function; linear regression; objective function**

Ⅰ. INTRODUCTION

Previous researchers have conducted numerous studies on the reliability of pile capacity, yielding significant research findings. For instance, Yang [1] used three standard methods, namely Classification and Regression Tree (CART), Artificial Neural Network (ANN), and Support Vector Machine (SVM), to examine and evaluate the performance of reliability state methods in a ten-bar truss project. Huang [2] developed a general stochastic method to verify the reliability index of single piles and pile groups based on load tests. Bian [3] introduced setup effects and proposed a methodology for separately calculating the ultimate base and shaft resistance factors in reliability-based design (RBD) for driven piles. Fan [4] considered multiple failure modes and developed a probabilistic reliability analysis framework that accounted for soil spatial variability to assess serviceability performance. Fan 5] presented a sampling-based algorithm for efficient reliability evaluations of axially loaded piles to enhance computational efficiency. Li [6] employed a bootstrap method to characterize uncertainty in probabilistic models and analyze its impact on pile reliability. Zhang [7] demonstrated how to characterize the uncertainty of a pile capacity prediction model and formulate resistance factors for designing large-diameter bored piles, explicitly considering both types of uncertainties. Zheng [8] utilized the entropy principle and Newton iteration method to establish a reliability research approach for the vertical bearing capacity of single piles. Zhang et al. [9] employed the bearing capacity reduction factor of single piles to investigate the influence of the debris on the reliability index. Dithide [10] defined the model factor of pile bearing capacity and examined the influence of model factors on the reliability index by analyzing data collected from 87 driven and 87 bored piles. Kwak [11] compiled field measurement data from 52 steel pipe piles, optimized the data using Bayesian theory and subsequently studied the reliability index and target reliability index of driven steel piles. ZHANG [12] collected extensive settlement data for piles in bridge engineering and proposed a settlement reliability research method utilizing probability theory. The research above primarily focuses on the ultimate bearing or serviceability limit state without considering economic benefits. However, in engineering practice, construction costs profoundly impact the project. Therefore, optimizing engineering design becomes increasingly crucial.

Implementing the reliability-based design optimization process faces challenges such as high computation costs and potential misconvergence of nonlinear iterations. Currently, there is limited research on the reliability-based optimization design of single piles under vertical load. In the geotechnical engineering field, Zheng [13] proposes a Bayesian optimization method for geotechnical data based on the findings in reference [5]. Furthermore, Wang [14, 15] introduces an optimization design method for foundation engineering and extended foundations, considering bearing capacity limit state, serviceability limit state, and economic benefits. Zhang [16] presents a reliability optimization design method for geotechnical engineering systems. Babu [17], on the other hand, describes an inverse reliability

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method for determining the depth and section modulus of a cantilever sheet pile wall. Basha [18] investigates stability in sandy soils using reliability analysis and formulates an optimization problem for targeted stability. Ching [19, 20] establishes a sufficient condition for equivalence between reliability and the factor of safety in reliability-based design optimization.

Reliability-based design optimization has also been applied in various other fields. For instance, Caitlyn [21] synthesizes information on reliability-based optimization in systems similar to offshore renewable energy systems. Kamjoo [22] utilizes RBDO to develop a design load model for bridge girders subjected to location-specific traffic loads. Ho-Huu [23] proposes a novel approach combining multi-objective evolutionary optimization and reliability analysis. Meng [24], on the other hand, suggests a target performance approach (TPA) to reduce computational costs in nonprobabilistic reliability analyses.

This paper uses reliability theory to present a research model for optimizing the design of single piles under vertical loads. We propose an improved optimization calculation method to tackle challenges such as high computation costs and the misconvergence of nonlinear iterations. Additionally, we validate the effectiveness of our approach through a case study.

II. Establishment of reliability-based optimization model

The reliability optimization design method is considered superior to traditional approaches as it takes uncertainties into account. Fig. 1 illustrates two design methods [25].

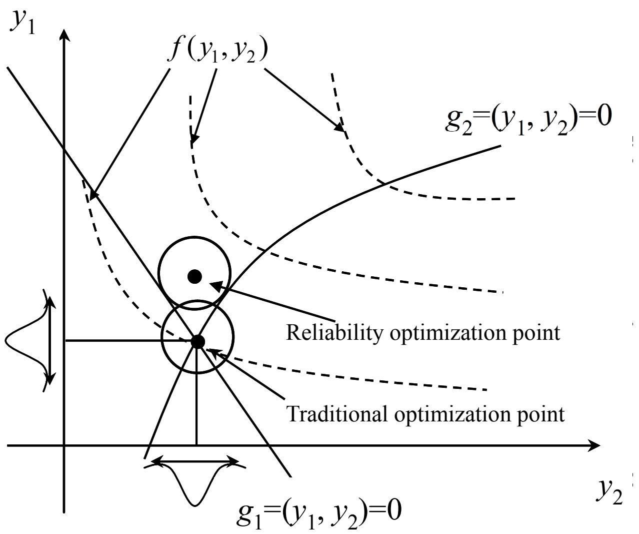

Fig. 1. Reliability optimization diagram

Assuming that $X = (X_1, X_2, \cdots, X_M)$ are the design parameters, which should be treated as random variables. In civil engineering, the optimization design should consider construction costs, such as material costs, labor costs, and quality inspection fees. However, it is challenging to fully account for all aspects of construction costs due to various uncertainties. This paper primarily focuses on construction material costs, making the objective function the quantity of construction materials. According to optimization principles, the reliability-based optimization design model is as follows [16, 22-24]:

Objective function
$$\min m(X) \tag{1a}$$

Boundary conditions
$$\begin{aligned} &\beta_i(X) > \beta_i^{\mathrm{T}} \quad i = 1, 2, \cdots, n_{\mathrm{f}} \\ &X_j^{\mathrm{L}} < X_j < X_j^{\mathrm{U}} \quad j = 1, 2, \cdots, M \end{aligned} \tag{1b}$$

Where $m(X)$ is the objective function; β_i and β_j^{T} are the reliability index and target reliability index, respectively, based on the i-th failure mode; X_j^{L} and X_j^{U} are the minimum and maximum values of Xj, respectively. Based on Eq. (1), the reliability optimization design model uses the target reliability index as a boundary condition to overcome uncertainties in civil engineering.

In Eq. (1), the reliability index is often calculated using the advanced first-order reliability method (AFORM), and the calculation formula is [26]

$$\beta = \min_{X \in F} \sqrt{(X - \boldsymbol{\mu})^{\mathrm{T}} C^{-1} (X - \boldsymbol{\mu})} \tag{2}$$

or

$$\beta = \min_{X \in F} \sqrt{\left(\frac{x_i - \mu_i}{\sigma_i}\right)^{\mathrm{T}} R^{-1} \left(\frac{x_i - \mu_i}{\sigma_i}\right)} \tag{3}$$

Where μ is the mean of X; C is the covariance matrix; R is the matrix of correlation coefficients; F is the failure zone; μ_i and σ_i are the mean and standard variance of x_i, respectively. Fig.2 shows the schematic diagram of first-order reliability method (FORM). Based on Fig.2, scholars have improved this method and obtained meaningful results.

It can be known from Eq. (2) that the reliability index is an optimization problem based on first order reliability method (FORM). Therefore, the reliability optimization design model is actually a dual optimization model, and solving Eq. (1) is difficult. To solve this problem, this paper proposes an improved reliability optimization calculation method.

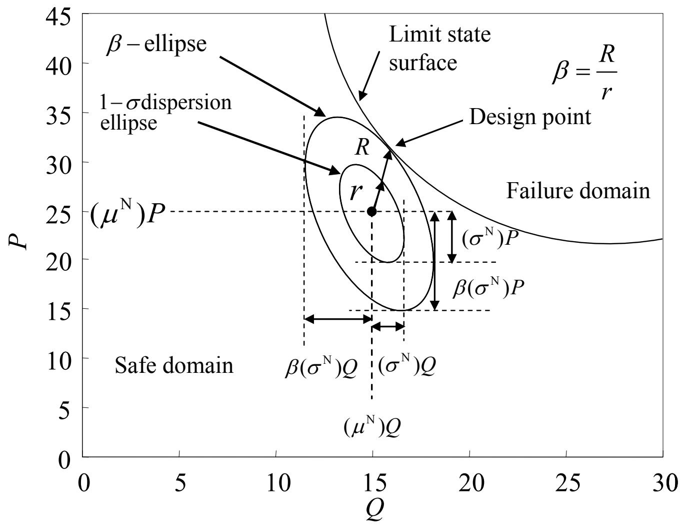

Fig. 2. Schematic diagram of a reliability method

III. Improved reliability optimization algorithm

From Eq. 1, reliability boundary conditions complicate the optimization calculations. According to reliability theory, regardless of whether the limit state equation is linear or nonlinear, the reliability is calculated using the mean first order reliability method (MFORM), and the formula is [26]

$$\beta^{\mathrm{M}} = \frac{g(\boldsymbol{\mu})}{\sqrt{\mathrm{G\,C\,G^T}}} \tag{4}$$

$$G = \left\{ \frac{\partial g(x)}{\partial x_1}\Big|_{x=\boldsymbol{\mu}}, \frac{\partial g(x)}{\partial x_2}\Big|_{x=\boldsymbol{\mu}}, \cdots, \frac{\partial g(x)}{\partial x_{\mathrm{n}}}\Big|_{x=\boldsymbol{\mu}} \right\} \tag{5}$$

Where β^{M} is the reliability index based on the MFORM. If Eq. (4) is used instead of Eq. (2), the calculation of Eq. 3 is greatly simplified. However, the calculation accuracy of the mean first-order reliability method (MFORM) described by Eq. (4) is low, yielding questionable optimization design results. To solve the above problem, a revised function is established as follows:

$$\beta^{\mathrm{M}} = t(\beta) \tag{6}$$

Where $t(\bullet)$ is the revised function, and β is the reliability index calculated from Eq.4.

Eq. (6) is a monotonically increasing function. To introduce Eq. (6) into Eq.1, the following lemma is established:

Lemma: $\beta_i(X) > \beta_i^{\mathrm{T}}$ and $\beta_i^{\mathrm{M}}(X) > \beta_i^{\mathrm{MT}}$ are equivalent. Where $\beta_i^{\mathrm{M}}(X)$ and β_i^{MT} are the reliability index and the target reliability index, respectively, based on Eq.6, and $i = 1, 2, \cdots, n_{\mathrm{f}}$.

Proof: (1) $\beta_i(X) > \beta_i^{\mathrm{T}}$ can infer $\beta_i^{\mathrm{M}}(X) > \beta_i^{\mathrm{MT}}$.

$\because$ $t(\bullet)$ is a monotonically increasing function, and $\beta_i(X) > \beta_i^{\mathrm{T}}$.

$\therefore$ $t(\beta_i(X)) > t(\beta_i^{\mathrm{T}})$.

$\because$ $\beta^{\mathrm{M}} = t(\beta)$.

$\therefore$ $t(\beta_i(X)) = \beta_i^{\mathrm{M}}(X)$ and $t(\beta_i^{\mathrm{T}}) = \beta_i^{\mathrm{MT}}$ can be obtained.

$\therefore$ $\beta_i^{\mathrm{M}}(X) > \beta_i^{\mathrm{MT}}$.

(2) $\beta_i^{\mathrm{M}}(X) > \beta_i^{\mathrm{MT}}$ can infer $\beta_i(X) > \beta_i^{\mathrm{T}}$.

$\because$ $t(\bullet)$ is a monotonically increasing function, and $\beta_i^{\mathrm{M}}(X) > \beta_i^{\mathrm{MT}}$.

$\therefore$ $t(\beta_i^{\mathrm{M}}(X)) > t(\beta_i^{\mathrm{MT}})$.

$\because$ $\beta^{\mathrm{M}} = t(\beta)$.

$\therefore$ $t(\beta_i^{\mathrm{M}}(X)) = \beta_i(X)$ and $t(\beta_i^{\mathrm{MT}}) = \beta_i^{\mathrm{T}}$ can be obtained.

$\therefore$ $\beta_i(X) > \beta_i^{\mathrm{T}}$.

Thus, the lemma is established, and Eq. (1) can be converted into the following:

Objective function	$\min m(X)$	(7a)
Boundary conditions	$\beta_i(X) > \beta_i^{\mathrm{T}} \quad i = 1, 2, \cdots, n_{\mathrm{f}}$ $X_j^{\mathrm{L}} < X_j < X_j^{\mathrm{U}} \quad j = 1, 2, \cdots, M$	(7b)

Where β_i^{M} and β_i^{MT} are the reliability index and the target reliability index, respectively, based on the MFORM, and $i = 1, 2, \cdots, n_{\mathrm{f}}$. Eq. (7) solves computational efficiency and accuracy problems.

Linear regression method is used to determine the expression of the revised function. The specific steps are as follows:

Step 1: In the range of design variables (i.e., for pile foundation engineering, pile length, and pile diameter), different design values are selected and used to calculate β and β^{M} based on Eq. (1) and Eq. (4).

Step 2: On the basis of step 1, linear regression method is used to calculate the revised function.

Ⅳ. Optimization design calibration

The linear regression method is a numerical approach that requires the calibration of the revised function. Fig.3 illustrates the calibration of the optimal design point on a two-dimensional plane. In Fig.3(a), the design point coincides with the optimization point, allowing for optimization design without needing for calibration. Fig.3(b) reveals that the design point falls within the unsafe zone, rendering the design parameters infeasible. Conversely, Fig.3(c) demonstrates that the design point lies within the feasibility domain, meeting the optimal design requirements. However, this design point may entail unnecessary economic losses, necessitating calibration. Therefore, calibration is necessary for the two scenarios depicted in Fig.3(b) and Fig.3(c). Appropriate calibration of design parameter values, such as pile diameter and length, is required to align the design points with the optimal design point.

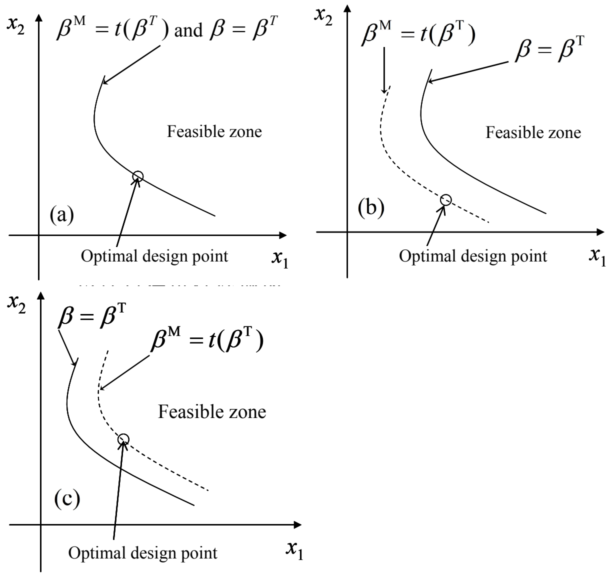

Fig. 3. Calibration of optimal design points: (a) perfect mapping function; (b) optimal design in the infeasible zone; (c) optimal design has potential to be further optimized.

Based on the above analysis, the reliability optimization design process is summarized as follows:

Step 1: Given the design parameters X, objective function $m(X)$ and reliability boundary conditions, Eq. (1) is used to establish the reliability optimization design model.

Step 2: The revised function is solved by linear regression ($\beta^{\mathrm{M}} = t(\beta)$), and Eq. (7) is used to establish the updated optimization design model.

Step 3: Based on step 2, Eq. (7) is solved using an optimization solution method, such as the linear programming technique.

Step 4: According to Fig.(3), the optimization design points are locally and globally calibrated.

Ⅴ. CASE STUDY

To verify the proposed method, this paper will take the single piles under vertical load, for example, shown in Fig.4. The basic parameters are shown in Tab.1. For the reliability optimization design, the influence of groundwater on the pile is not considered.

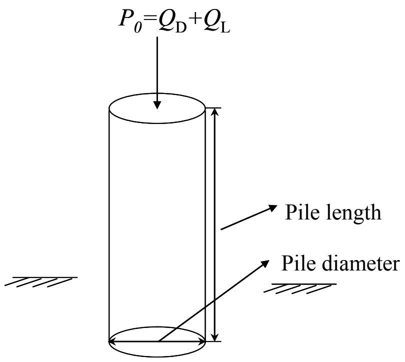

Fig. 4. Schematic diagram of single piles under vertical loads

TABLE Ⅰ
BASIC PARAMETERS OF SOIL

Soil thickness [mm]	q_{sik} [kPa]	q_{rk} [kPa]	Sediment thickness [mm]	E_0 [MPa]
1000	85	6000	0	30

According to pile foundation engineering, two failure modes are considered: (1) the vertical load is greater than the ultimate bearing capacity of the pile, and (2) the vertical settlement is greater than the allowable settlement specified in the specification. The limit state equations of the two failure modes are [26]

$$g_1 = R - Q_D - Q_L \tag{8}$$

$$g_2 = s - s_0 \tag{9}$$

Where g_1 and g_2 are the limit state equations of vertical bearing capacity and settlement, respectively; R is the ultimate vertical bearing capacity (kN); Q_D and Q_L are the dead load and live load (kN), respectively; s is the vertical settlement amount(mm) of single piles under vertical load, and s_0 is the vertical allowable settlement(mm), the specific value of which is based on relevant specifications. Many calculation formulas exist for the vertical bearing capacity and settlement of single piles. This paper adopts the formulas in the technical code for building pile foundations in China to calculate the vertical bearing capacity and settlement. The design parameters that should be random variables are Poisson's ratio (v), the Elastic Modulus (E_s), the dead load (QD), and the live load (QD) in Eq. (8) and Eq. (9), respectively. That is $X = \{Q_D, Q_L, v, E_s\}$. The statistical characteristics of the parameters are shown in Table 2. Therefore, in the objective function, the design variables are the pile length (l) and the pile diameter (d).

TABLE Ⅱ
STATISTICAL CHARACTERISTICS OF RANDOM VARIABLES

	Q_D[kN]	Q_L[kN]	v	E_s[MPa]
Distribution type	Log-normal distribution	Log-normal distribution	Normal distribution	Log-normal distribution
Mean	1500	350	0.35	38
Standard variance	180	55	0.05	7

3.2 and 2.0 are the target reliability indices of the vertical bearing capacity and settlement of single piles [26]. According to the technical code for building pile foundations in China [27], pile length and pile diameter (m) are

$$\begin{aligned} 9 < l \le 16 \\ 0.3 < d \le 1 \end{aligned} \tag{10}$$

The reliability optimization model obtained by Eq. (1) is

Objective function
$$\min m(D, L) = \frac{\pi d^2}{4} l \tag{11a}$$

Boundary conditions
$$\beta_1(X) > 3.2 \tag{11b}$$

$$\beta_2(X) > 2.0 \tag{11c}$$

$$9 < l \le 16 \tag{11d}$$

$$0.3 < d \le 1 \tag{11e}$$

The least squares method calculates the revised function described in Eq. (6). Firstly, different pile lengths and pile diameters are given. Based on Eq. (8) and Eq. (9), the reliability indexes corresponding to different pile lengths and pile diameters are calculated, as shown in Tab.3 and Tab.4. Then, the least squares method is used to fit the revised function, which is shown in Fig.5 and Fig.6. As seen from Fig.5 and Fig.6, the revised function is monotonically increasing, which is consistent with the former results. Additionally, the fitting accuracy of the quartic polynomial for the revised function is satisfactory, which meets the engineering requirement.

TABLE Ⅲ
CALCULATION RESULTS OF B AND BM USING DIFFERENT DESIGN VALUES (BASED ON EQ.(8))

d	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
l	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5
β	0.47	0.83	1.10	1.52	1.87	2.18	2.44	2.72	3.03	3.64	3.92	4.19	4.63
β^M	0.48	0.60	0.85	0.99	1.03	1.12	1.19	1.21	1.30	1.34	1.35	1.35	1.37

TABLE Ⅳ
CALCULATION RESULTS OF B AND BM USING DIFFERENT DESIGN VALUES (BASED ON EQ.(9))

d	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
l	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5
β	0.94	1.08	1.40	1.61	1.99	2.40	2.82	3.01	3.34	3.84	3.90	6.88	8.02
β^M	1.06	1.17	1.63	1.98	2.56	3.17	3.89	4.45	5.10	5.91	6.12	6.88	8.03

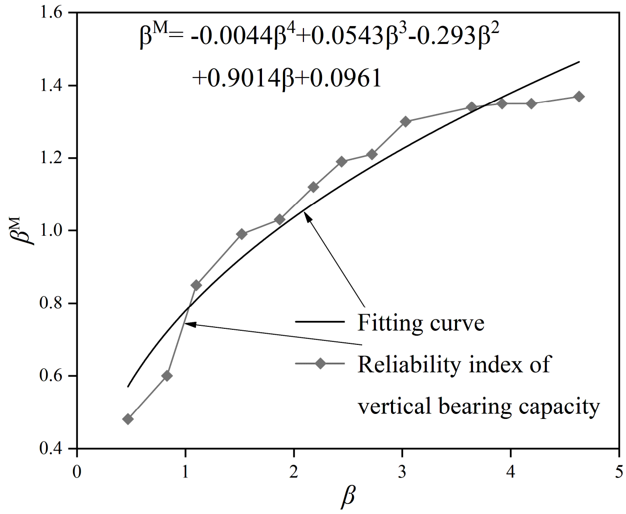

Fig. 5. Fitting results of reliability index for vertical bearing capacity (based on Table 3)

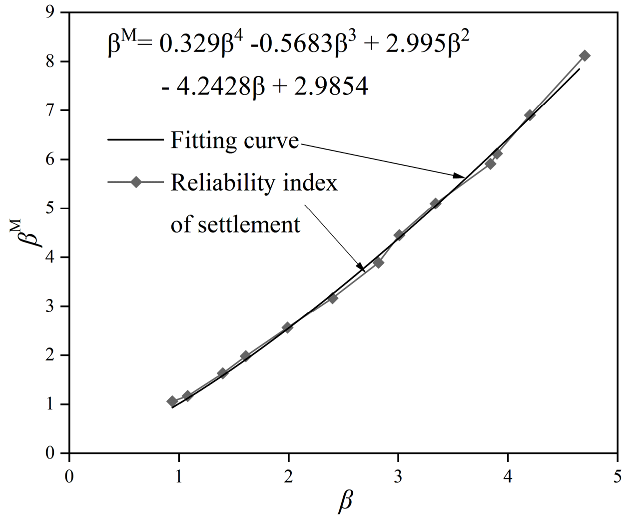

Fig. 6. Fitting results of reliability index for vertical bearing capacity (based on Table 4)

Based on Fig.5 and Fig.6, the updated target reliability index of vertical bearing capacity and the settlement are 1.33 and 2.46, respectively. According to Eq. (7), the improved optimization model is

Objective function
$$\min m(D,L)=\frac{\pi d^2}{4}l \tag{12a}$$

Boundary conditions
$$\beta_1(X)>1.30 \tag{12b}$$
$$\beta_2(X)>2.0 \tag{12c}$$
$$9<l\le 16 \tag{12d}$$
$$0.3<d\le 1 \tag{12e}$$

To illustrate the method of this paper, the initial pile length and pile diameter values are 13 m and 0.8 m, respectively [27]. The corresponding objective function is 6.53 m^3, and the reliability indexes based on Eq. (2) and Eq. (4) are 3.41 and 2.62, respectively. The initial reliability index is in the security domain, according to Fig.3. (13, 0.8) are taken as the initial design values to optimize the pile length and the pile diameter using Eq. (7), and the results are shown in Tab.5. Tab.5 shows that the optimization result satisfies the reliability boundary condition. β_1 nearly coincides with the boundary conditions and does not need to be corrected. The objective function m(X) value is 5.215, which is 20.2% smaller than the initial objective function value.

TABLE Ⅴ
RELIABILITY OPTIMIZATION RESULTS BASED ON EQ.(8) AND EQ.(9)

	X		MFORM		m(X) [m^3]
	d [m]	l [m]	β_1	β_2	
Initial value	0.800	13.000	3.410	2.620	6.531
Initial optimization	0.736	12.332	3.201	2.493	5.215
Local updating	0.736	12.332	3.201	2.493	5.215
Global updating	0.758	10.874	3.206	2.010	4.905

From the optimization theory, the initial optimization results in Tab.5 are only local optimizations. For example, in Tab.5, β2 is much larger than the target reliability, and the initial result needs to be globally optimal. In addition, the dead load (QD), live load (QL), modulus of elasticity (E_s), and Poisson's ratio (v) have significant impacts on the optimization results. Therefore, according to the global optimization theory, the initial optimization results are globally calibrated considering random variables $X=\{Q_D,Q_L,v,E_s\}$. Table 5 shows that β1 and β2 substantially coincide with the target reliability boundary conditions. Therefore, the global calibrations are the optimization results. The objective function value (m(X)) is 4.905, reduced by 24.9% and 4.7% from the initial function value and the optimization result, respectively. Therefore, the design parameters after local and global optimization can significantly reduce construction costs.

According to the reliability theory, if the form performance function changes, the reliability will change based on the MFORM for the same failure mode. Accordingly, the boundary conditions in Eq. (1) or Eq. (7) differ. Eq. (8) and Eq. (9) are equivalently converted into the following forms:

$$g_3=\ln\frac{R}{Q_D+Q_L} \tag{13}$$

$$g_4=\ln\frac{s}{s_0} \tag{14}$$

TABLE VI
CALCULATION RESULTS OF B AND BM USING DIFFERENT DESIGN VALUES (BASED ON EQ.(13))

d	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
l	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5
β	0.34	0.78	1.09	1.42	1.81	2.18	2.32	2.75	3.03	3.53	4.03	4.29	4.62
β^{M}	0.48	0.82	1.07	1.35	1.57	1.97	2.01	2.16	2.46	2.82	3.00	3.10	3.34

TABLE VII
CALCULATION RESULTS OF B AND BM USING DIFFERENT DESIGN VALUES (BASED ON EQ.(14))

d	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
l	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5
β	0.96	1.01	1.24	1.58	1.97	2.43	2.85	3.11	3.30	3.89	4.27	4.61	4.91
β^{M}	0.92	1.01	1.15	2.00	2.00	2.54	2.93	3.17	3.45	3.99	4.39	4.74	4.98

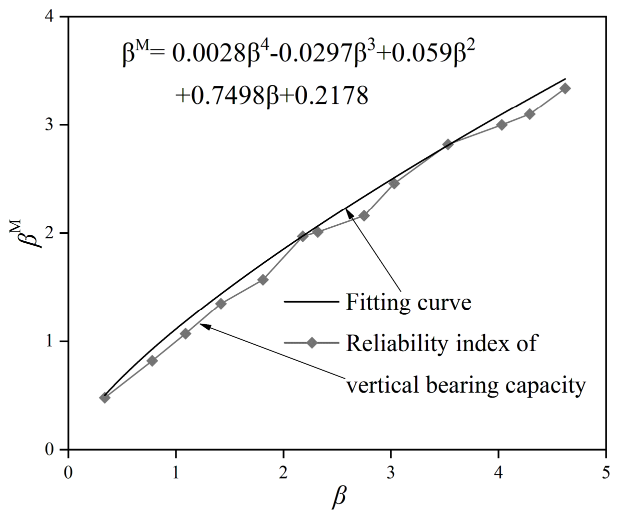

Fig. 7. Fitting results of bearing capacity reliability indicators (based on Table 6)

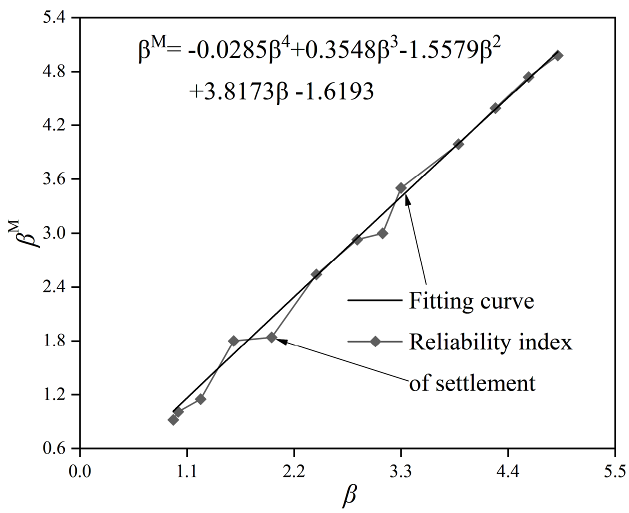

Fig. 8. Fitting reliability index fitting results (based on Table 7)

TABLE VIII
RELIABILITY OPTIMIZATION RESULTS BASED ON EQ.(15) AND EQ.(16)

	X		MFORM		m(X) [m^3]
	d [m]	l [m]	β_a	β_b	
Initial value	0.800	13.000	3.410	2.620	6.531
Initial optimization	0.772	11.864	3.204	2.647	5.551
Local updating	0.772	11.864	3.204	2.647	5.551
Global updating	0.749	11.131	3.197	2.001	4.902

According to foregoing calculation method, the reliability index is calculated using different pile lengths and pile diameters. β and β M are shown in Tab.6 and Tab.7 based on Eq. (13) and Eq. (14).

According to Table 6 and Table 7, the quartic polynomial fits the reliability index using MATLAB software. The fitting results are shown in Fig.7 and Fig.8.

From Fig.7 and Fig.8, the revised functions based on Eq. (13) and Eq. (14) are

$$\beta^{M} = 0.0028\beta^4 - 0.0297\beta^3 + 0.059\beta^2 + 0.7498\beta + 0.2178 \quad (13)$$

$$\beta^{M} = -0.0285\beta^4 + 0.3548\beta^3 - 1.5579\beta^2 + 3.8173\beta - 1.6193 \quad (14)$$

It can be seen from Eq. (15) and Eq. (16) that as the form of the performance function changes, the revised function also changes. Given (13, 0.8) as the initial design values, the design variables are optimized and updated according to the methods above. The results in Table 8 show that the initial optimization and local calibration results are 5.551 m^3, which is different from the previous analysis. Therefore, the form of the performance function has a specific influence on local optimization. Table 8 also shows that the global optimization result is near the previous analysis results. Therefore, the effect of the form of the performance function on the global optimization result is negligible.

VI. SUMMARY AND CONCLUSIONS

The reliability optimization design method of single piles under vertical loads is proposed. The conclusions are summarized as follows:

This article proposes a reliability-based design optimization (RBDO) method, which simplifies reliability analysis by using the mean first-order reliability method (MFORM), with high computational efficiency and strong generalizability.

In order to ensure calculation accuracy, a modification function was established between the reliability index and the target reliability index, and the calculation results were corrected, greatly improving the calculation accuracy.

For the same pile failure mode, the reliability correction function varies with different functional functions, leading to diverse local optimization results. Nevertheless, the disparity in global optimization results is negligible, allowing us to disregard the impact of the functional function's form on the overall optimization outcomes.

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