Comparison of Robustness Non-Linearity Test in Computational Statistics when Outlier Detected

Dwi Rantini*, Arip Ramadan, Alhassan Sesay

Abstract— **In regression modeling, we often encounter data with different ranges. Such data usually have outliers. If an outlier has a value far from the mean, it can cause an error in modeling. For example, data has a quadratic pattern, but because there are outliers, it can be indicated that the data is linear. This research will prove which non-linearity test is more robust if outlier data is shown. To prove this, data is generated, and outliers are found in the variable response of the non-linear model. Using the RESET, the Terasvirta and White tests will prove to be more robust. The results show that the Terasvirta test is more robust than the RESET and White tests. This statement applies to models that are non-linear in parameters and variables. Therefore, if we want to test the goodness of a non-linear model and outliers are detected from our research, we recommend using the Terasvirta test. We prove that 53.42% of Terasvirta performs better than the RESET and White tests. Because the Terasvirta test is proven to be more robust if outliers are found in the non-linear model, this is very important to increase knowledge in education, especially in computing statistics.**

Index Terms— **Education, LM Test, Outlier, Ramsey's RESET Test, Terasvirta Test, White Test**

I. Introduction

There is a lot of modeling in statistics. One of them is a causal relationship that is modeled through regression analysis, and the regression analysis requires response variables and predictor variables. In regression analysis, there are generally two types, namely linear regression models and non-linear regression models. Non-linear regression can be divided into three types: non-linear in parameters, non-linear in variables, and non-linear in parameters and variables [1]. Examples of non-linear regression models are quadratic regression models, cubic regression models, and others. There are generally two types of non-linearity tests: Ramsey's or regression specification error (RESET) test and Lagrange Multiplier (LM) test. Both of these tests detect equation specification errors. Then, there are several types of LM tests, such as the Terasvirta and the White test.

There are various kinds of errors when doing statistical modeling [2], [3], [4]. One of them is mis-specified model. Suppose a data follows an exponential pattern, it turns out that when linearity is tested, a linear model is found, one of the causes is the existence of an outlier. In a regression analysis, it is very important to detect an outlier, because it can be very sensitive to the results of the regression analysis [5] [6].

Outliers come from various causes. For example, outliers from data errors, outliers from intentional or motivated misreporting, outliers from sampling error, outliers from standardization failure, and others [7]. Osborne's research can provide a more detailed explanation of the types of outliers [7]. Then, research that provides a methodologist to detect the presence of outliers was conducted by Hodge and Austin [8], where outliers are considered anomalies. Several similar studies have also been carried out on detecting the presence of outliers, some of which were carried out by Rousseeuw and Hubert [9].

After discussing linear and non-linear regression and types of non-linearity tests, the causes of outliers, and ways to detect outliers, the present study contributes to which non-linearity tests are more robust to outliers detection in non-linear regression analysis. Nonlinear data must be modeled appropriately [4]. Similar research has been done, but the results given are that there are no tests that dominate other tests [10]. As explained earlier, if data contains one or two outliers, a weak test can be used to indicate that the data has a linear pattern. This situation can make the analysis results quite significantly different. It is important to know which test is more robust against outliers. Research conducted by Li and Hao has examined data containing outliers using maximum likelihood estimation (MLE) and Bayesian approaches [11]. Furthermore, research conducted by Hsiao et al. used classification in the view of outliers [12].

II. Properties

A. *Ramsey's RESET Test*

A simple example will be used to illustrate Ramsey's test or regression specification error (RESET) test. Suppose a response variable Y is influenced by a predictor variable X. Therefore, it can be written as Equation (1)

$$Y_i = \lambda_1 + \lambda_2 X_i + u_{3i} \quad (1)$$

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The analysis steps for the RESET test method can be followed as follows [13]

1. Based on the Equation (1), Y_i can be estimated by ordinary least square (OLS), so that $\hat{Y}_i$ is obtained.
2. Recalculate Equation (1) by calculating $\hat{Y}_i$ as an additional predictor variable. Suppose the additional predictor variables are $\hat{Y}_i^2$ and $\hat{Y}_i^3$. Therefore it can be written as Equation (2)
$$Y_i = \theta_1 + \theta_2 X_i + \theta_3 \hat{Y}_i^2 + \theta_4 \hat{Y}_i^3 + u_i \quad (2)$$
3. Calculate R^2 in Equation (1) as R_{old}^2 and in Equation (2) as R_{new}^2. Then use the F test in Equation (3) to find out if the increase R^2 from using Equation (2) is statistically significant.
$$F = \frac{\left(R_{new}^2 - R_{old}^2\right) / \text{NR}}{\left(1 - R_{new}^2\right) / \left(\text{NObs-NPNM}\right)} \quad (3)$$
where NR is the number of new regressors, NObs is the number of observations, and NPNM is the number of parameters in the new model.
4. If the computed F value is significant, it can be accepted that the model in Equation (1) is mis-specified.

B. Lagrange Multiplier (LM) Test

To illustrate the LM test, we will continue with the preceding illustrative. The LM test proceeds as follows [13]

1. Do the estimation on the Equation (1), so we get residuals $\hat{u}_i$.
2. If in fact the unrestricted regression on the Equation (4) is the true regression, the residuals obtained on the Equation (1) should be related to the squared and cubed terms, that is X_i^2 and X_i^3
$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + \beta_4 X_i^3 + u_i \quad (4)$$
3. Regress the $\hat{u}_i$ obtained in Step 1 on all the predictor variables (including those in the restricted regression), so that it can be written into Equation (5)
$$\hat{u}_i = \alpha_1 + \alpha_2 X_i + \alpha_3 X_i^2 + \alpha_4 X_i^3 + v_i \quad (5)$$
where v is an error term with the usual properties.
4. For large-sample size, Engle has shown that n (the sample size) times the R^2 estimated from the (auxiliary) regression on the Equation (5) follows the chi-square distribution with degree of freedom (df) equal to the number of restrictions imposed by the restricted regression [14]. Symbolically, it can be written as Equation (6)
$$nR^2 \underset{\text{asy}}{\sim} \chi^2_{(\text{number of restrictions})} \quad (6)$$
where asy means asymptotically.
5. If the chi-square obtained from Equation (6) exceeds the critical chi-square value at the chosen level of significance, reject the restricted regression. Otherwise, do not reject it.

Two types of LM tests include the White test and Terasvirta test, which will be explained in the following sections.

White Test

The White test is a non-linearity detection test developed from a neural network model [15]. In general, the steps for the White test are the same as the LM test, but the unrestricted regression model involves interactions between predictor variables. To illustrate White test, consider the regression model on the Equation (7)
$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \quad (7)$$
The White test proceeds as follows [13]

1. Estimate regression model on the Equation (7) and obtain the residuals, $\hat{u}_i$.
2. Run the (auxiliary) regression model on the Equation (8)
$$\begin{aligned}\hat{u}_i^2 &= \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{2i}^2 + \\ &\alpha_5 X_{3i}^2 + \alpha_6 X_{2i} X_{3i} + v_i\end{aligned} \quad (8)$$
That is, the squared residuals from the original regression are regressed on the original predictor variables, their squared values, and the cross product(s) of the predictor variables. Obtain R^2 from this (auxiliary) regression model.
3. Under the null hypothesis that there is no heteroscedasticity, it can be shown that the sample size (n) times R^2 obtained from the auxiliary regression *asymptotically* follows the chi-square distribution with the df equal to the number of predictor variables (excluding the constant term) in the auxiliary regression. That is,
$$nR^2 \underset{\text{asy}}{\sim} \chi^2_{\text{df}} \quad (9)$$
4. If the chi-square value obtained on the Equation (9) exceeds the critical chi-square value at the chosen level of significance, the conclusion is that there is heteroscedasticity. If it is does not exceed the critical chi-square value, there is no heteroscedasticity, which is to say that in the auxiliary regression model on the Equation (8), $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$.

Terasvirta Test

In using the Terasvirta test, there are two ways of making decisions namely the χ^2 test and the F test. For procedures using the χ^2 test, the steps are carried out the same as the LM test step. The difference is in Step 2, where X_2 and X_3 are the results of the Taylor series expansion approach [16]. While the procedure for the F test in the Terasvirta test is as follows [16]

1. Do the estimation on the Equation (1), so we get residuals $\hat{u}_i$ and compute the sum of the squared residuals $SSR_0 = \sum_i^n \hat{u}_i^2$.
2. Regress $\hat{u}_i$ to the predictor variables as in Equation (5) where X_2 and X_3 are the results of the Taylor series expansion approach. Calculate residuals $\hat{v}_i = \hat{u}_i - \hat{\hat{u}}_i$ and sum of the squared residuals $SSR_1 = \sum_i^n \hat{v}_i^2$.
3. Compute

$$F = \frac{(SSR_0 - SSR_1)/\text{NR}}{SSR_1/(\text{NObs-NPNM})} \quad (10)$$

4. If the computed F value is significant, it can be accepted that the model in Equation (1) is mis-specified.

C. Type of Regression Models

To distinguish a linear model or not, it can be divided into four types, as in TABLE 1 below [13]

TABLE 1
LINEAR REGRESSION MODELS

Model linear in parameters?	Model linear in variables?	
	Yes	No
Yes	LRM	LRM
No	NLRM	NLRM

where LRM is the linear regression model and NLRM is the nonlinear regression model. Based on TABLE 1, four different models will be given. To make it easier to choose a model, the model provided by the Minitab software can be adopted. Therefore, four different models are obtained in TABLE 2. And, we multiply each model twice with different coefficients. So, the total number of models is eight.

TABLE 2
FOUR TYPES OF LINEAR REGRESSION MODELS ADOPTED FROM MINITAB SOFTWARE

Model linear in parameters?	Model linear in variables?	
	Yes	No
Yes	Linear: $Y_1 = X_1 + 2X_2$ $Y_2 = 2X_1 + 7X_2$	Power (Convex): $Y_3 = X_1^2 + 3X_2^4$ $Y_4 = X_1^3 + 2X_2^2$
No	Exponential: $Y_5 = e^{2X_1} + 3e^{4X_2}$ $Y_6 = e^{4X_1} + 2e^{6X_2}$	2-Parameter Sigmoid 1: $Y_7 = \left(1 - e^{-X_1^2}\right) + \left(1 - e^{-3X_2^4}\right)$ $Y_8 = \left(1 - e^{-2X_1^3}\right) + \left(1 - e^{-2X_2^2}\right)$

III. Applications

In this study, eight types of models are given which are described in TABLE 2. These models are Linear, Power (Convex), Exponential, and 2-parameter Sigmoid 1. For each of these models, we provide 2 formulas with different coefficients for each model. So, the total model is 8. Therefore, to analyze the robustness of the non-linearity test will be used simulation data. The variable consisted of one response variable and two continuous predictor variables. Predictor variables X_1 and X_2 are data generated following the Normal distribution with mean 0 and variance 1, can be written mathematically $X_1 \sim N(0,1)$ and $X_2 \sim N(0,1)$. The response variable Y is a function of X_1 and X_2. In general, the non-linearity test hypothesis can be written

$$H_0 : \text{model=LRM}$$
$$H_A : \text{model=NLRM}$$

A simulation study was conducted in this study using R software. The steps are as follows

1. Determine the functions that will later be used as a regression model, which has been explained in TABLE 2.
2. For Linear models, generate data for two continuous predictor variables, both of which are normally distributed with a mean of 0 and a variance of 1. Data will be raised 4 times, namely for $n_1 = 30$, $n_2 = 100$, $n_3 = 1,000$, and $n_4 = 10,000$.
3. For the first-generation data, i.e. $n_1 = 30$ give outliers with $o_1 = 0$ (no outliers), $o_2 = 1$, $o_3 = 3$ and $o_4 = 5$. For the second-generation data, namely $n_2 = 100$, give outliers with $o_1 = 0$ (no outliers), $o_2 = 1$, $o_3 = 3$ and $o_4 = 5$. For the third-generation data, namely $n_3 = 1,000$, give outliers with the number $o_1 = 0$ (no outliers), $o_2 = 10$, $o_3 = 30$ and $o_4 = 50$. For the fourth-generation data, namely $n_4 = 10,000$, give outliers with the number $o_1 = 0$ (no outliers), $o_2 = 10$, $o_3 = 30$ and $o_4 = 50$. Each generated data was replicated 1,000 times. Outlier data is given for the Y value that has been formed by the model.
4. Based on Step 3, there are sixteen data to be tested per model. The test used is
   - RESET test with nine types:
     - "regressor" with "power=3"
     - "regressor"
     - "fitted" with "power=2"
     - "fitted" with "power=3"
     - "fitted"
     - "princomp" with "power=2"
     - "princomp" with "power=3"
     - "princomp"
   - Terasvirta test with the F test, and
   - White test with the F test.

   In the RESET test, type "regressor" or "fitted" or "princomp" is a string indicating whether powers of the fitted response, the regressor variables (factors are left out), or the first principal component of the regressor matrix should be included in the extended model.
5. For sixteen data to be tested, the largest power will be determined. If the power is greatest, then the data is truly non-linear. Then it will be seen that the largest power is the power of which test. Power is the probability of making a correct decision (to reject the null hypothesis, H_0) when the null hypothesis is false.
6. The most robust test is a test with a power that is always large for each data generated.
7. Repeat Step 2 through Step 6 for the Power (Convex) model, Exponential, and 2-parameter Sigmoid 1 model.

So, the total number of scenarios tested was 128. Based on the steps previously described, the results obtained in TABLE 3 and TABLE 4 for the Linear model, TABLE 5and TABLE 6 for the Power (Convex) model, TABLE 7 and TABLE 8 for the Exponential model, and TABLE 9 and TABLE 10 for the 2-Parameter Sigmoid 1 model. A total of 128 scenarios in TABLE 3 to TABLE 10 are given information, namely in 1 scenario the largest power value is sought and marked in bold. If in 1 scenario there are equally large power values, then the scenario fails to show the performance of each test, and a gray block is given.

TABLE 3
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR LINEAR MODEL $Y_1 = X_1 + 2X_2$

			power							
Kind of Test	Type	Power	n=30; with outlier				n=100; with outlier			
			0	1	3	5	0	1	3	5
RESET	regressor	2	0.059	0.100	0.063	0.049	0.044	0.078	0.081	0.077
RESET	regressor	3	0.064	0.072	0.087	0.070	0.054	0.050	0.087	0.089
RESET	regressor		0.057	0.123	0.105	0.060	0.045	0.079	0.109	0.120
RESET	fitted	2	0.056	0.170	0.087	0.048	0.042	**0.170**	0.105	0.101
RESET	fitted	3	0.063	0.113	0.102	0.063	**0.056**	0.103	0.105	0.092
RESET	fitted		0.063	0.178	0.128	0.060	0.048	0.157	0.132	0.115
RESET	princomp	2	0.049	0.073	0.056	0.040	0.049	0.067	0.066	0.052
RESET	princomp	3	0.055	0.049	0.068	0.058	0.045	0.031	0.061	0.065
RESET	princomp		0.056	0.074	0.094	0.053	0.044	0.061	0.082	0.075
Terasvirta	regressor		0.067	**0.199**	**0.130**	**0.078**	0.052	0.125	**0.150**	**0.143**
White	princomp		**0.070**	0.077	0.080	0.051	0.052	0.062	0.062	0.053
the highest power			**0.070**	**0.199**	**0.130**	**0.078**	**0.056**	**0.170**	**0.150**	**0.143**

TABLE 3 (CONTINUED)
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR LINEAR MODEL $Y_1 = X_1 + 2X_2$

			power							
Kind of Test	Type	Power	n=1,000; with outlier				n=10,000; with outlier			
			0	10	30	50	0	10	30	50
RESET	regressor	2	0.039	0.052	0.047	0.037	**0.063**	0.050	0.049	0.048
RESET	regressor	3	**0.056**	0.073	0.056	**0.073**	0.053	0.072	0.079	0.079
RESET	regressor		0.046	0.079	0.065	0.064	0.054	0.079	0.080	0.070
RESET	fitted	2	0.048	0.071	0.050	0.038	0.058	0.034	0.040	0.044
RESET	fitted	3	0.039	0.063	0.046	0.062	0.046	0.038	0.057	0.057
RESET	fitted		0.041	0.087	0.06	0.061	0.043	0.046	0.053	0.063
RESET	princomp	2	0.037	0.035	0.043	0.045	0.048	0.051	0.047	0.053
RESET	princomp	3	0.048	0.045	0.045	0.050	0.050	0.050	0.063	0.054
RESET	princomp		0.041	0.063	0.049	0.044	0.052	0.065	0.052	0.066
Terasvirta	regressor		0.043	**0.113**	**0.081**	0.070	0.047	**0.096**	**0.088**	**0.094**
White	princomp		0.046	0.050	0.053	0.046	0.050	0.047	0.056	0.057
the highest power			**0.056**	**0.113**	**0.081**	**0.073**	**0.063**	**0.096**	**0.088**	**0.094**

TABLE 4
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR LINEAR MODEL $Y_2 = 2X_1 + 7X_2$

			power							
Kind of Test	Type	Power	n=30; with outlier				n=100; with outlier			
			0	1	3	5	0	1	3	5
RESET	regressor	2	0.059	0.100	0.063	0.049	0.044	0.078	0.081	0.077
RESET	regressor	3	0.064	0.072	0.087	0.070	**0.054**	0.050	0.087	0.089
RESET	regressor		0.057	0.123	0.105	0.060	0.045	0.079	0.109	0.120
RESET	fitted	2	0.054	0.078	0.081	0.052	0.042	**0.155**	0.098	0.094
RESET	fitted	3	0.064	0.046	0.088	0.054	0.052	0.094	0.098	0.090
RESET	fitted		0.064	0.084	0.115	0.055	0.048	0.145	0.119	0.116
RESET	princomp	2	0.049	0.073	0.056	0.040	0.049	0.067	0.066	0.052
RESET	princomp	3	0.055	0.049	0.068	0.058	0.045	0.031	0.061	0.065
RESET	princomp		0.056	0.074	0.094	0.053	0.044	0.061	0.082	0.075
Terasvirta	regressor		**0.067**	**0.199**	**0.130**	**0.078**	0.052	0.125	**0.150**	**0.143**
White	princomp		0.060	0.074	0.065	0.047	0.031	0.078	0.059	0.053
the highest power			**0.067**	**0.199**	**0.130**	**0.078**	**0.054**	**0.155**	**0.150**	**0.143**

TABLE 4 (CONTINUED)
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR LINEAR MODEL $Y_2 = 2X_1 + 7X_2$

			power							
Kind of Test	Type	Power	n=1.000; with outlier				n=10.000; with outlier			
			0	10	30	50	0	10	30	50
RESET	regressor	2	0.039	0.052	0.047	0.037	**0.063**	0.050	0.049	0.048
RESET	regressor	3	**0.056**	0.073	0.056	**0.073**	0.053	0.072	0.079	0.079
RESET	regressor		0.046	0.079	0.065	0.064	0.054	0.079	0.080	0.070
RESET	fitted	2	0.051	0.047	0.059	0.035	0.049	0.033	0.040	0.050
RESET	fitted	3	0.048	0.055	0.051	0.055	0.041	0.047	0.060	0.062
RESET	fitted		0.048	0.070	0.061	0.057	0.043	0.051	0.056	0.057
RESET	princomp	2	0.037	0.035	0.043	0.045	0.048	0.051	0.047	0.053
RESET	princomp	3	0.048	0.045	0.045	0.050	0.050	0.050	0.063	0.054
RESET	princomp		0.041	0.063	0.049	0.044	0.052	0.065	0.052	0.066
Terasvirta	regressor		0.043	**0.113**	**0.081**	0.070	0.047	**0.096**	**0.088**	**0.094**
White	princomp		0.040	0.058	0.044	0.044	0.062	0.051	0.059	0.062
the highest power			**0.056**	**0.113**	**0.081**	**0.073**	**0.063**	**0.096**	**0.088**	**0.094**

TABLE 5

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR POWER (CONVEX) MODEL $Y_3 = X_1^2 + 3X_2^4$

			power							
Kind of Test	Type	Power	n=30; with outlier				n=100; with outlier			
			0	1	3	5	0	1	3	5
RESET	regressor	2	1	**0.774**	**0.583**	**0.467**	1	**0.400**	**0.212**	0.180
RESET	regressor	3	0.714	0.471	0.372	0.305	0.747	0.171	0.121	0.128
RESET	regressor		1	0.709	0.541	0.411	1	0.316	0.190	0.191
RESET	fitted	2	0.895	0.641	0.500	0.388	0.921	0.394	0.210	0.185
RESET	fitted	3	0.700	0.410	0.334	0.255	0.722	0.174	0.133	0.116
RESET	fitted		0.891	0.611	0.456	0.366	0.914	0.333	0.194	0.173
RESET	princomp	2	0.679	0.505	0.433	0.337	0.798	0.287	0.164	0.129
RESET	princomp	3	0.411	0.305	0.270	0.232	0.507	0.114	0.086	0.085
RESET	princomp		0.645	0.494	0.419	0.324	0.779	0.236	0.150	0.135
Terasvirta	regressor		1	0.633	0.496	0.384	1	0.268	0.206	**0.202**
White	princomp		0.868	0.522	0.384	0.283	0.969	0.222	0.123	0.100
the highest power			1	**0.774**	**0.583**	**0.467**	1	**0.400**	**0.212**	**0.202**

TABLE 5 (CONTINUED)

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR POWER (CONVEX) MODEL $Y_3 = X_1^2 + 3X_2^4$

			power							
Kind of Test	Type	Power	n=1.000; with outlier				n=10.000; with outlier			
			0	10	30	50	0	10	30	50
RESET	regressor	2	1	1	**0.984**	**0.870**	1	1	1	1
RESET	regressor	3	0.804	0.432	0.233	0.200	0.835	0.767	0.649	0.620
RESET	regressor		1	1	0.955	0.789	1	1	1	1
RESET	fitted	2	0.978	0.802	0.615	0.504	1	0.998	0.987	0.970
RESET	fitted	3	0.778	0.308	0.152	0.141	0.813	0.696	0.566	0.488
RESET	fitted		0.974	0.777	0.572	0.463	1	0.995	0.977	0.961
RESET	princomp	2	0.939	0.696	0.572	0.480	1	0.997	0.970	0.951
RESET	princomp	3	0.544	0.253	0.118	0.114	0.602	0.526	0.434	0.366
RESET	princomp		0.925	0.677	0.519	0.434	1	0.994	0.960	0.928
Terasvirta	regressor		1	1	0.902	0.686	1	1	1	1
White	princomp		0.998	0.878	0.598	0.402	1	0.999	1	0.999
the highest power			1	1	**0.984**	**0.870**	1	1	1	1

TABLE 6

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR POWER (CONVEX) MODEL $Y_4 = X_1^3 + 2X_2^2$

			power							
Kind of Test	Type	Power	n=30; with outlier				n=100; with outlier			
			0	1	3	5	0	1	3	5
RESET	regressor	2	0.989	0.130	0.098	0.064	1	0.092	0.082	0.082
RESET	regressor	3	0.886	0.077	0.101	0.079	0.997	0.050	0.089	0.091
RESET	regressor		1	0.138	0.138	0.084	1	0.085	0.115	0.129
RESET	fitted	2	0.464	0.179	0.125	0.069	0.504	**0.180**	0.109	0.101
RESET	fitted	3	0.658	0.114	0.113	0.075	0.956	0.104	0.098	0.096
RESET	fitted		0.707	0.182	**0.157**	0.074	0.978	0.160	0.129	0.120
RESET	princomp	2	0.704	0.101	0.088	0.065	0.813	0.078	0.070	0.058
RESET	princomp	3	0.632	0.065	0.071	0.075	0.743	0.031	0.060	0.067
RESET	princomp		0.860	0.090	0.123	0.079	0.990	0.066	0.084	0.080
Terasvirta	regressor		1	**0.205**	0.155	**0.103**	1	0.131	**0.153**	**0.147**
White	princomp		0.805	0.109	0.088	0.066	0.932	0.077	0.062	0.057
the highest power			1	**0.205**	**0.157**	**0.103**	1	**0.180**	**0.153**	**0.147**

TABLE 6 (CONTINUED)

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR POWER (CONVEX) MODEL $Y_4 = X_1^3 + 2X_2^2$

			power							
Kind of Test	Type	Power	n=1.000; with outlier				n=10.000; with outlier			
			0	10	30	50	0	10	30	50
RESET	regressor	2	1	0.057	0.071	0.060	1	0.109	1	0.963
RESET	regressor	3	1	0.073	0.070	**0.091**	1	0.089	0.972	0.893
RESET	regressor		1	0.078	0.084	0.089	1	0.128	1	**0.998**
RESET	fitted	2	0.536	0.078	0.066	0.061	0.603	0.109	0.068	0.070
RESET	fitted	3	1	0.068	0.065	0.067	1	0.087	0.972	0.885
RESET	fitted		1	0.088	0.093	0.080	1	0.121	0.970	0.825
RESET	princomp	2	0.898	0.041	0.050	0.050	0.938	0.094	0.607	0.526
RESET	princomp	3	0.853	0.046	0.051	0.054	0.905	0.067	0.463	0.389
RESET	princomp		1	0.065	0.061	0.061	1	0.095	0.921	0.758
Terasvirta	regressor		1	**0.114**	**0.102**	0.085	1	**0.136**	1	0.992
White	princomp		0.992	0.050	0.062	0.056	1	0.081	0.801	0.662
the highest power			1	**0.114**	**0.102**	**0.091**	1	**0.136**	1	**0.998**

TABLE 7

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR EXPONENTIAL MODEL $Y_5 = e^{2X_1} + 3e^{4X_2}$

			power							
Kind of Test	**Type**	**Power**	**n=30; with outlier**				**n=100; with outlier**			
			0	**1**	**3**	**5**	**0**	**1**	**3**	**5**
RESET	regressor	2	1	0.998	0.998	0.994	1	1	1	0.999
RESET	regressor	3	0.626	0.594	0.601	0.580	0.876	0.887	0.874	0.866
RESET	regressor		1	**0.999**	0.998	0.999	1	1	1	0.999
RESET	fitted	2	0.998	0.996	0.994	0.989	1	1	1	1
RESET	fitted	3	0.728	0.719	0.724	0.716	0.942	0.948	0.942	0.931
RESET	fitted		0.998	0.995	0.994	0.992	1	1	1	1
RESET	princomp	2	0.554	0.553	0.599	0.572	0.639	0.613	0.629	0.617
RESET	princomp	3	0.337	0.314	0.361	0.341	0.467	0.460	0.465	0.464
RESET	princomp		0.534	0.538	0.591	0.552	0.624	0.599	0.617	0.596
Terasvirta	regressor		1	0.998	**0.998**	**0.999**	1	1	1	0.999
White	princomp		0.718	0.686	0.680	0.696	0.873	0.850	0.862	0.835
the highest power			1	**0.999**	**0.998**	**0.999**	1	1	1	1

TABLE 7 (CONTINUED)

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR EXPONENTIAL MODEL $Y_5 = e^{2X_1} + 3e^{4X_2}$

			power							
Kind of Test	**Type**	**Power**	**n=1.000; with outlier**				**n=10.000; with outlier**			
			0	**10**	**30**	**50**	**0**	**10**	**30**	**50**
RESET	regressor	2	1	1	1	1	1	1	1	1
RESET	regressor	3	1	1	1	1	1	1	1	1
RESET	regressor		1	1	1	1	1	1	1	1
RESET	fitted	2	1	1	1	1	1	1	1	1
RESET	fitted	3	1	1	1	1	1	1	1	1
RESET	fitted		1	1	1	1	1	1	1	1
RESET	princomp	2	0.749	0.719	0.702	0.728	0.812	0.817	0.797	0.790
RESET	princomp	3	0.641	0.618	0.614	0.618	0.727	0.744	0.741	0.729
RESET	princomp		0.727	0.704	0.682	0.711	0.796	0.799	0.795	0.779
Terasvirta	regressor		1	1	1	1	1	1	1	1
White	princomp		0.961	0.975	0.975	0.975	0.996	0.996	0.991	0.989
the highest power			1	1	1	1	1	1	1	1

TABLE 8

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR EXPONENTIAL MODEL $Y_6 = e^{4X_1} + 2e^{6X_2}$

			power							
Kind of Test	**Type**	**Power**	**n=30; with outlier**				**n=100; with outlier**			
			0	**1**	**3**	**5**	**0**	**1**	**3**	**5**
RESET	regressor	2	1	0.998	0.997	0.997	1	1	1	1
RESET	regressor	3	0.688	0.654	0.658	0.657	0.907	0.911	0.906	0.900
RESET	regressor		1	1	1	1	1	1	1	1
RESET	fitted	2	0.974	0.987	0.986	0.985	0.999	0.998	0.999	0.998
RESET	fitted	3	0.730	0.743	0.753	0.752	0.944	0.952	0.946	0.938
RESET	fitted		0.973	0.979	0.984	0.982	0.999	0.997	0.996	0.998
RESET	princomp	2	0.615	0.605	0.648	0.604	0.652	0.624	0.638	0.643
RESET	princomp	3	0.396	0.372	0.420	0.391	0.497	0.485	0.490	0.487
RESET	princomp		0.602	0.594	0.630	0.594	0.641	0.608	0.629	0.625
Terasvirta	regressor		1	1	1	1	1	1	1	1
White	princomp		0.680	0.673	0.658	0.634	0.814	0.839	0.822	0.825
the highest power			1	1	1	1	1	1	1	1

TABLE 8 (CONTINUED)

COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR EXPONENTIAL MODEL $Y_6 = e^{4X_1} + 2e^{6X_2}$

			power							
Kind of Test	**Type**	**Power**	**n=1.000; with outlier**				**n=10.000; with outlier**			
			0	**10**	**30**	**50**	**0**	**10**	**30**	**50**
RESET	regressor	2	1	1	1	1	1	1	1	1
RESET	regressor	3	1	1	1	1	1	1	1	1
RESET	regressor		1	1	1	1	1	1	1	1
RESET	fitted	2	1	1	1	1	1	1	1	1
RESET	fitted	3	1	1	1	1	1	1	1	1
RESET	fitted		1	1	1	1	1	1	1	1
RESET	princomp	2	0.716	0.700	0.669	0.693	0.764	0.777	0.767	0.756
RESET	princomp	3	0.616	0.606	0.587	0.611	0.689	0.714	0.722	0.690
RESET	princomp		0.698	0.684	0.652	0.672	0.750	0.760	0.758	0.744
Terasvirta	regressor		1	1	1	1	1	1	1	1
White	princomp		0.947	0.960	0.965	0.961	0.983	0.987	0.988	0.987
the highest power			1	1	1	1	1	1	1	1

TABLE 9
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR 2-PARAMETER SIGMOID 1 MODEL

$$Y_7 = \left(1-e^{-X_1^2}\right)+\left(1-e^{-3X_2^4}\right)$$

			power							
Kind of Test	**Type**	**Power**	**n=30; with outlier**				**n=100; with outlier**			
			0	**1**	**3**	**5**	**0**	**1**	**3**	**5**
RESET	regressor	2	**0.389**	0.106	0.069	0.051	**0.930**	0.078	0.082	0.078
RESET	regressor	3	0.052	0.071	0.087	0.071	0.059	0.050	0.088	0.089
RESET	regressor		0.336	0.128	0.110	0.060	0.894	0.079	0.110	0.122
RESET	fitted	2	0.267	**0.210**	0.101	0.046	0.730	**0.176**	0.107	0.102
RESET	fitted	3	0.072	0.129	0.103	0.056	0.075	0.101	0.097	0.094
RESET	fitted		0.205	0.206	0.128	0.055	0.636	0.159	0.128	0.120
RESET	princomp	2	0.309	0.081	0.058	0.045	0.764	0.070	0.067	0.053
RESET	princomp	3	0.069	0.050	0.069	0.059	0.058	0.031	0.061	0.065
RESET	princomp		0.258	0.075	0.100	0.054	0.693	0.061	0.086	0.076
Terasvirta	regressor		0.287	0.200	**0.134**	**0.086**	0.822	0.126	**0.150**	**0.143**
White	princomp		0.348	0.082	0.074	0.053	0.882	0.063	0.068	0.042
the highest power			**0.389**	**0.210**	**0.134**	**0.086**	**0.930**	**0.176**	**0.150**	**0.143**

TABLE 9 (CONTINUED)
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR 2-PARAMETER SIGMOID 1 MODEL

$$Y_7 = \left(1-e^{-X_1^2}\right)+\left(1-e^{-3X_2^4}\right)$$

			power							
Kind of Test	**Type**	**Power**	**n=1.000; with outlier**				**n=10.000; with outlier**			
			0	**10**	**30**	**50**	**0**	**10**	**30**	**50**
RESET	regressor	2	1	0.060	0.048	0.039	1	0.188	0.100	0.090
RESET	regressor	3	0.057	0.073	0.056	**0.072**	0.058	0.072	0.079	0.079
RESET	regressor		1	0.082	0.066	0.067	1	0.145	**0.123**	0.096
RESET	fitted	2	1	0.083	0.050	0.051	1	**0.200**	0.109	0.081
RESET	fitted	3	0.079	0.069	0.055	0.062	0.078	0.064	0.058	0.066
RESET	fitted		1	0.092	0.065	0.068	1	0.165	0.102	0.097
RESET	princomp	2	1	0.043	0.042	0.044	1	0.147	0.077	0.083
RESET	princomp	3	0.064	0.046	0.044	0.051	0.078	0.050	0.065	0.055
RESET	princomp		1	0.065	0.050	0.044	1	0.116	0.088	0.083
Terasvirta	regressor		1	**0.115**	**0.084**	0.071	1	0.156	0.122	**0.111**
White	princomp		1	0.056	0.050	0.041	1	0.165	0.089	0.087
the highest power			1	**0.115**	**0.084**	**0.072**	1	**0.200**	**0.123**	**0.111**

TABLE 10
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR 2-PARAMETER SIGMOID 1 MODEL

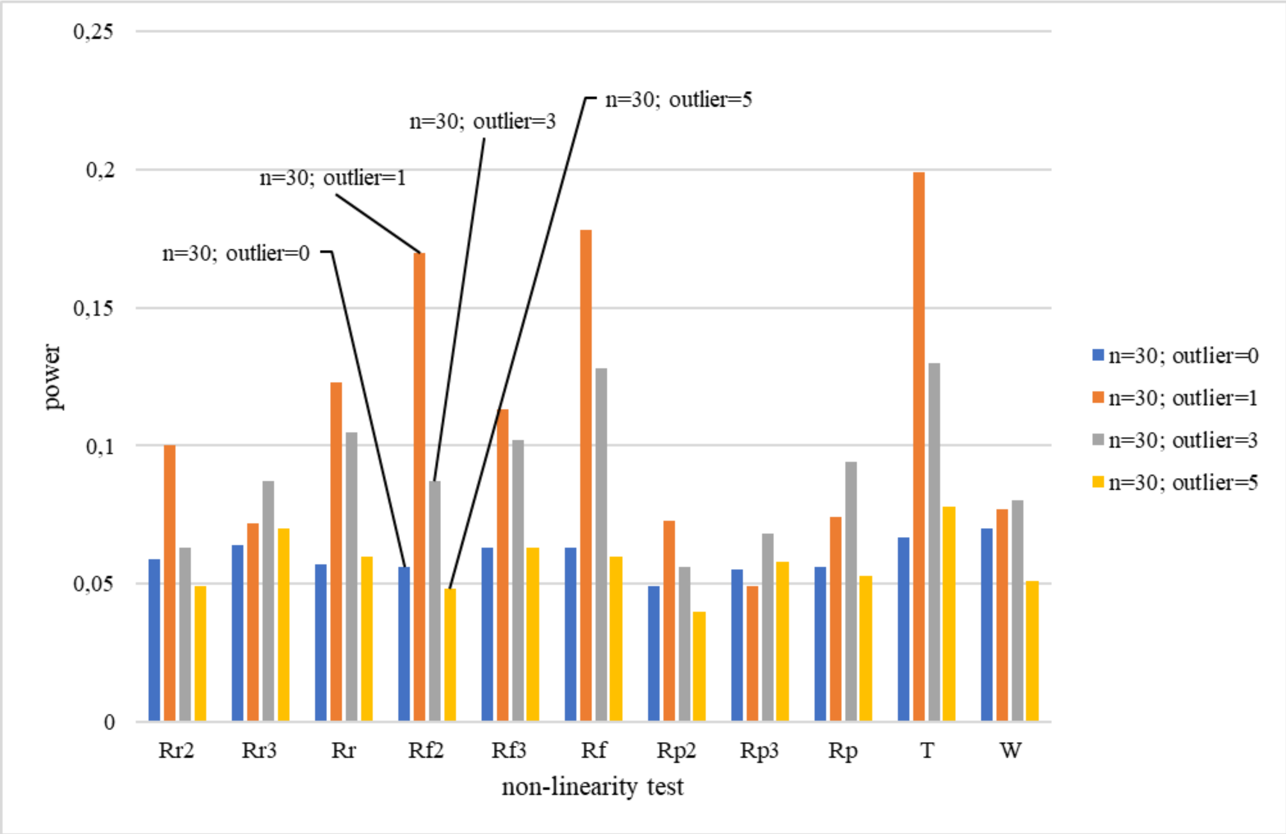
$$Y_8 = \left(1-e^{-2X_1^3}\right)+\left(1-e^{-2X_2^2}\right)$$

			power							
Kind of Test	**Type**	**Power**	**n=30; with outlier**				**n=100; with outlier**			
			0	**1**	**3**	**5**	**0**	**1**	**3**	**5**
RESET	regressor	2	0.905	0.888	0.859	0.876	1	1	0.998	0.997
RESET	regressor	3	0.575	0.607	0.573	0.592	0.810	0.826	0.853	0.852
RESET	regressor		**0.990**	0.954	**0.940**	**0.952**	1	1	1	0.998
RESET	fitted	2	0.987	**0.956**	0.935	0.937	1	1	0.999	0.998
RESET	fitted	3	0.737	0.757	0.725	0.740	0.939	0.942	0.959	0.943
RESET	fitted		0.985	0.952	0.940	0.942	1	1	0.999	0.998
RESET	princomp	2	0.475	0.489	0.470	0.477	0.513	0.545	0.545	0.556
RESET	princomp	3	0.310	0.345	0.294	0.327	0.378	0.426	0.428	0.415
RESET	princomp		0.483	0.497	0.481	0.482	0.501	0.540	0.532	0.539
Terasvirta	regressor		0.979	0.948	0.933	0.941	1	1	1	**0.998**
White	princomp		0.541	0.505	0.482	0.507	0.663	0.714	0.679	0.690
the highest power			**0.990**	**0.956**	**0.940**	**0.952**	1	1	1	**0.998**

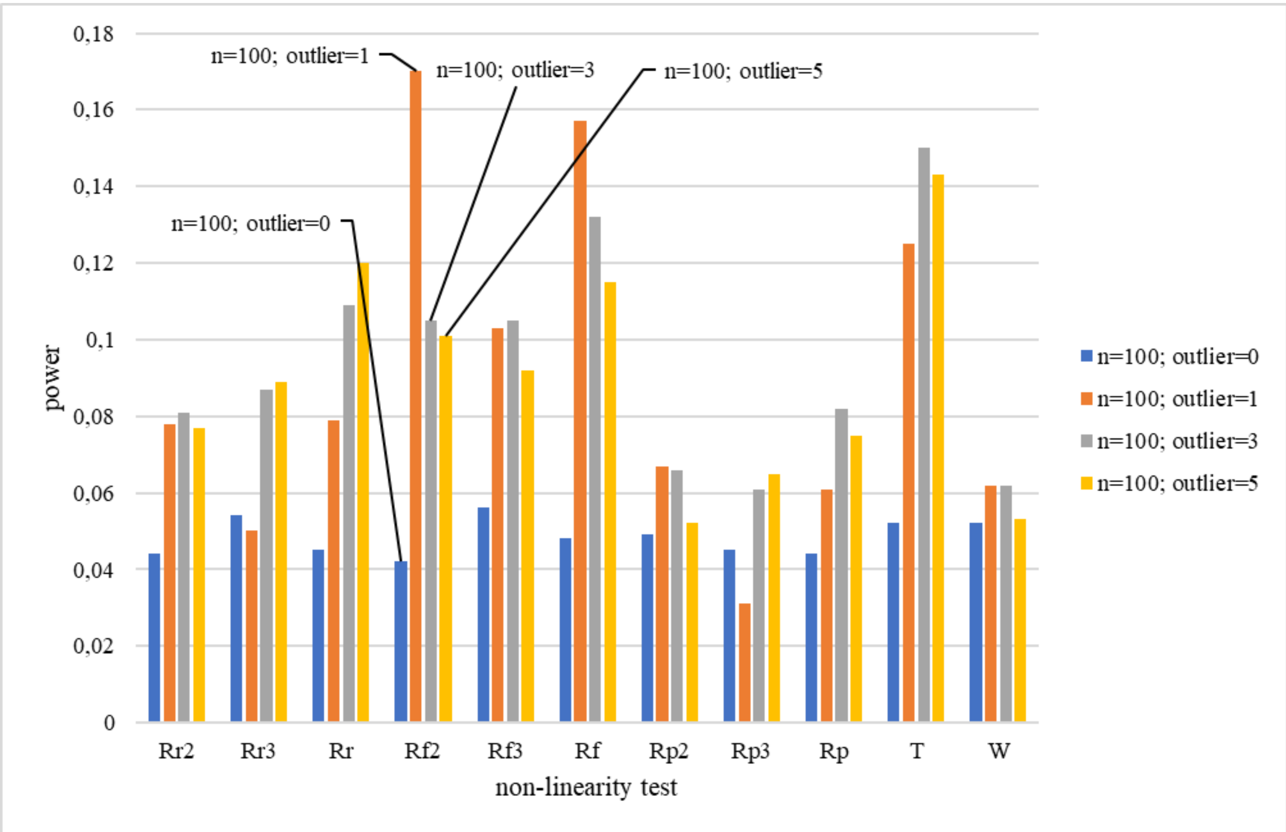
TABLE 10 (CONTINUED)
COMPARISON OF POWERS FOR SIXTEEN DATA GENERATION WITH OUTLIERS FOR 2-PARAMETER SIGMOID 1 MODEL

$$Y_8 = \left(1-e^{-2X_1^3}\right)+\left(1-e^{-2X_2^2}\right)$$

			power							
Kind of Test	**Type**	**Power**	**n=1.000; with outlier**				**n=10.000; with outlier**			
			0	**10**	**30**	**50**	**0**	**10**	**30**	**50**
RESET	regressor	2	1	1	1	1	1	1	1	1
RESET	regressor	3	1	1	1	1	1	1	1	1
RESET	regressor		1	1	1	1	1	1	1	1
RESET	fitted	2	1	1	1	1	1	1	1	1
RESET	fitted	3	1	1	1	1	1	1	1	1
RESET	fitted		1	1	1	1	1	1	1	1
RESET	princomp	2	0.647	0.647	0.634	0.637	0.711	0.652	0.683	0.675
RESET	princomp	3	0.567	0.572	0.558	0.561	0.640	0.586	0.615	0.621
RESET	princomp		0.625	0.629	0.613	0.617	0.688	0.633	0.667	0.659
Terasvirta	regressor		1	1	1	1	1	1	1	1
White	princomp		0.904	0.896	0.887	0.896	0.931	0.950	0.956	0.947
the highest power			1	1	1	1	1	1	1	1

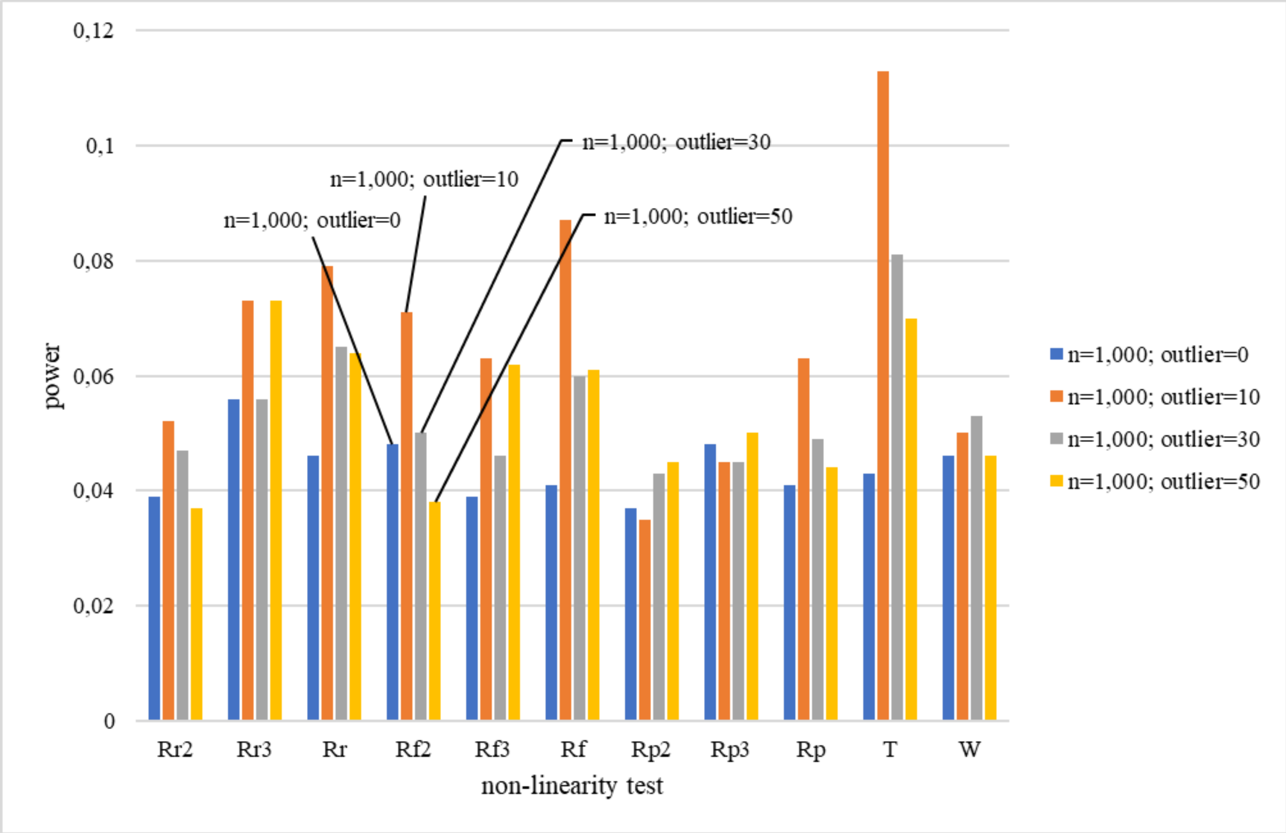


(a)

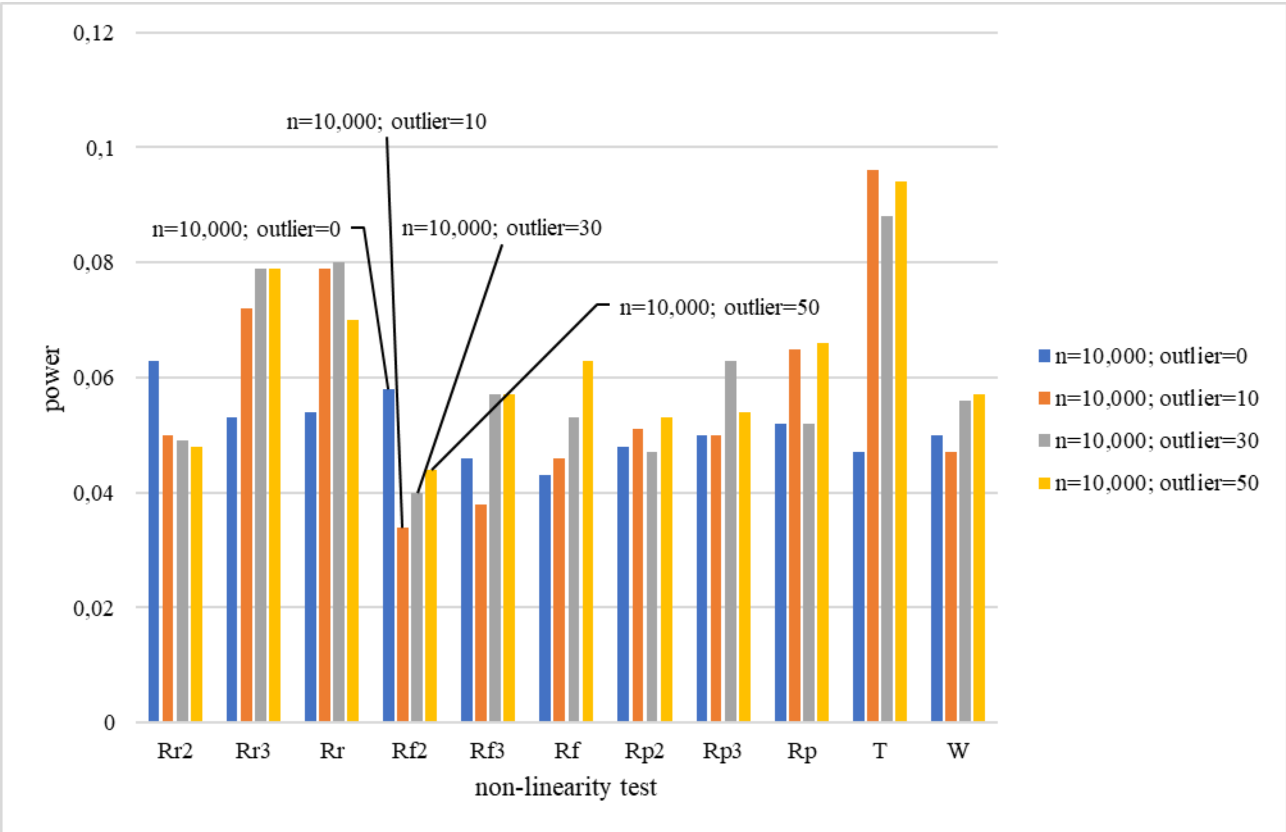


(b)

Fig. 1. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_1 = X_1 + 2X_2$

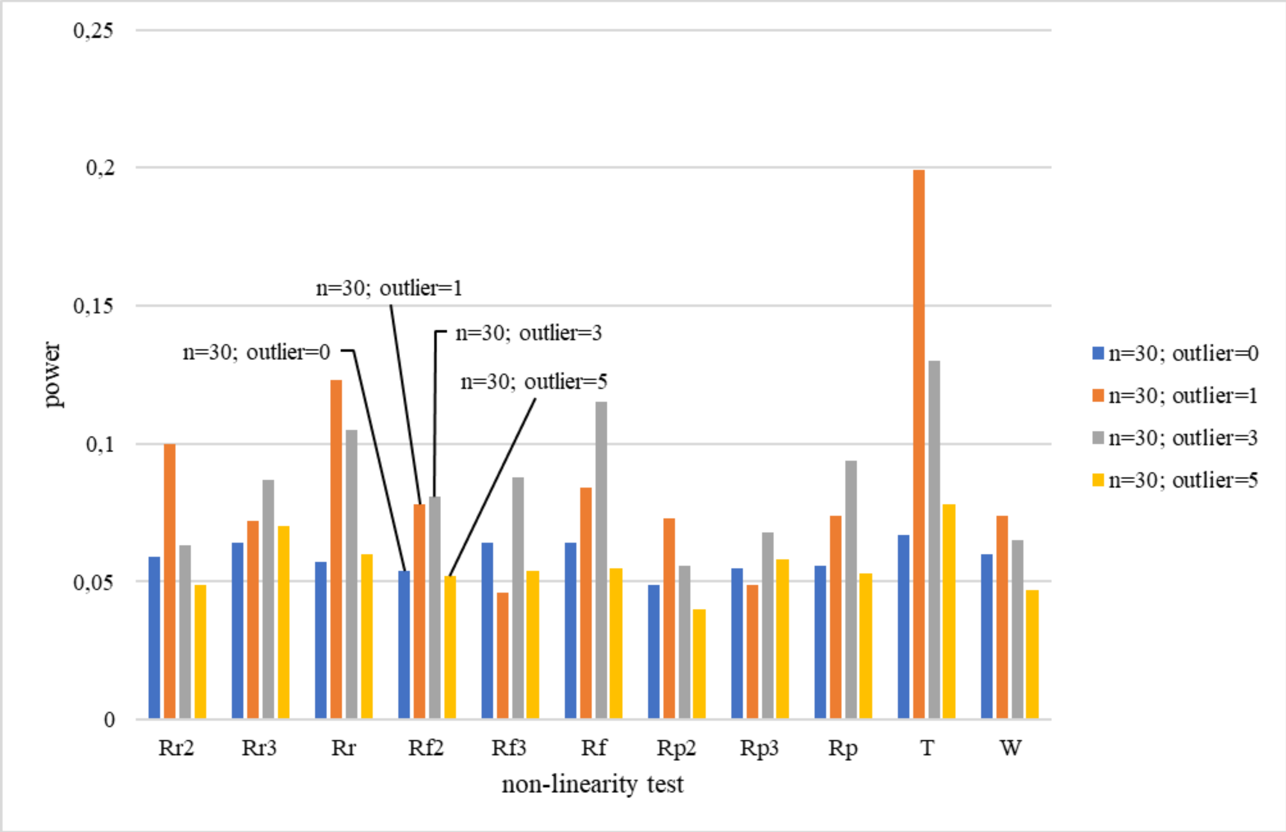


(c)

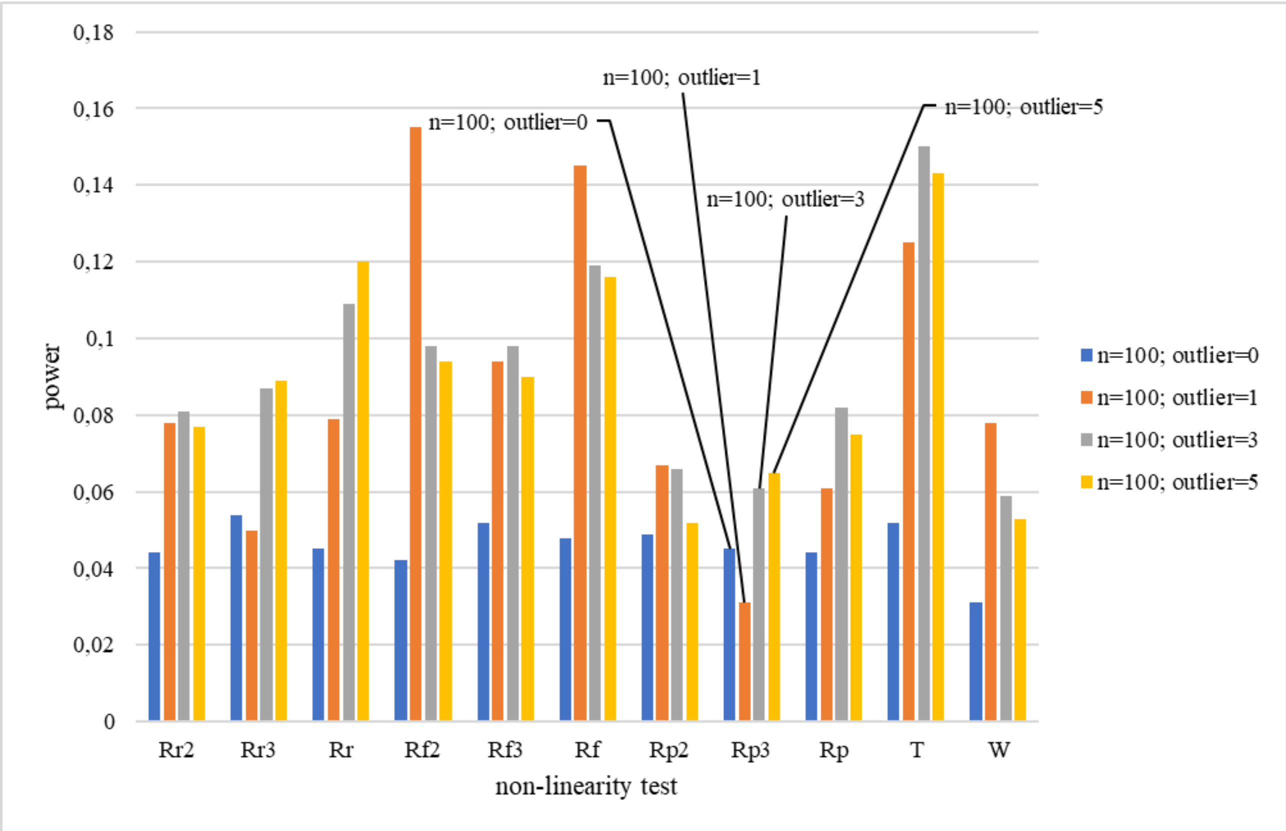


(d)

Fig. 1 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_1 = X_1 + 2X_2$

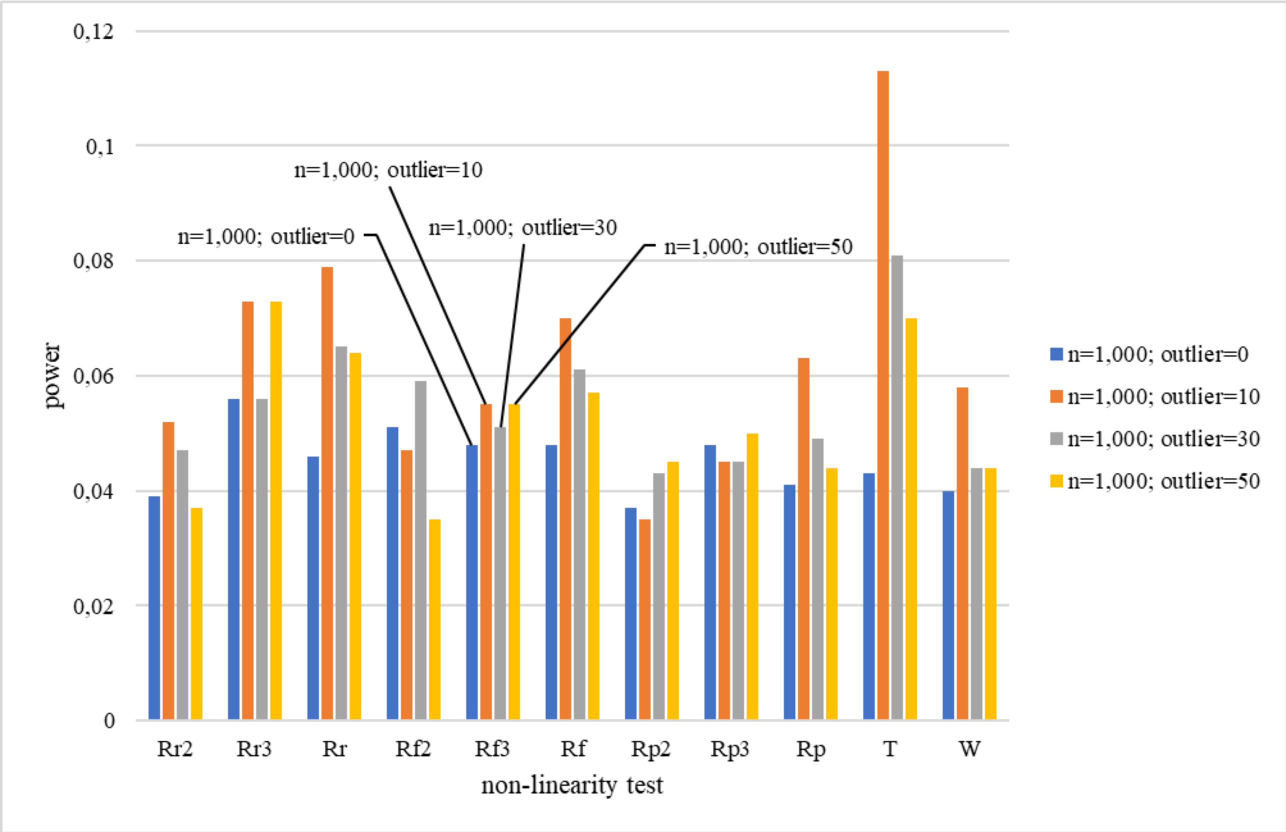


(a)

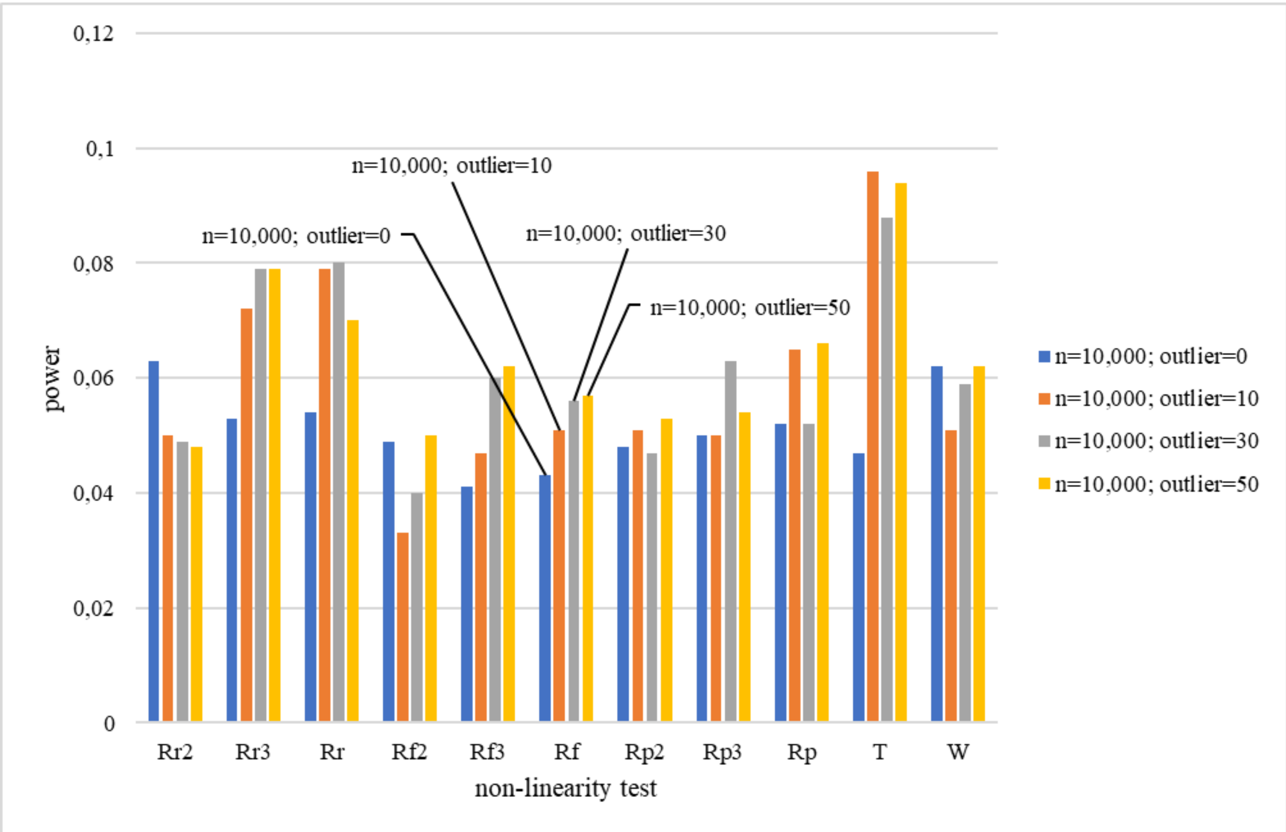


(b)

Fig. 2. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_2 = 2X_1 + 7X_2$

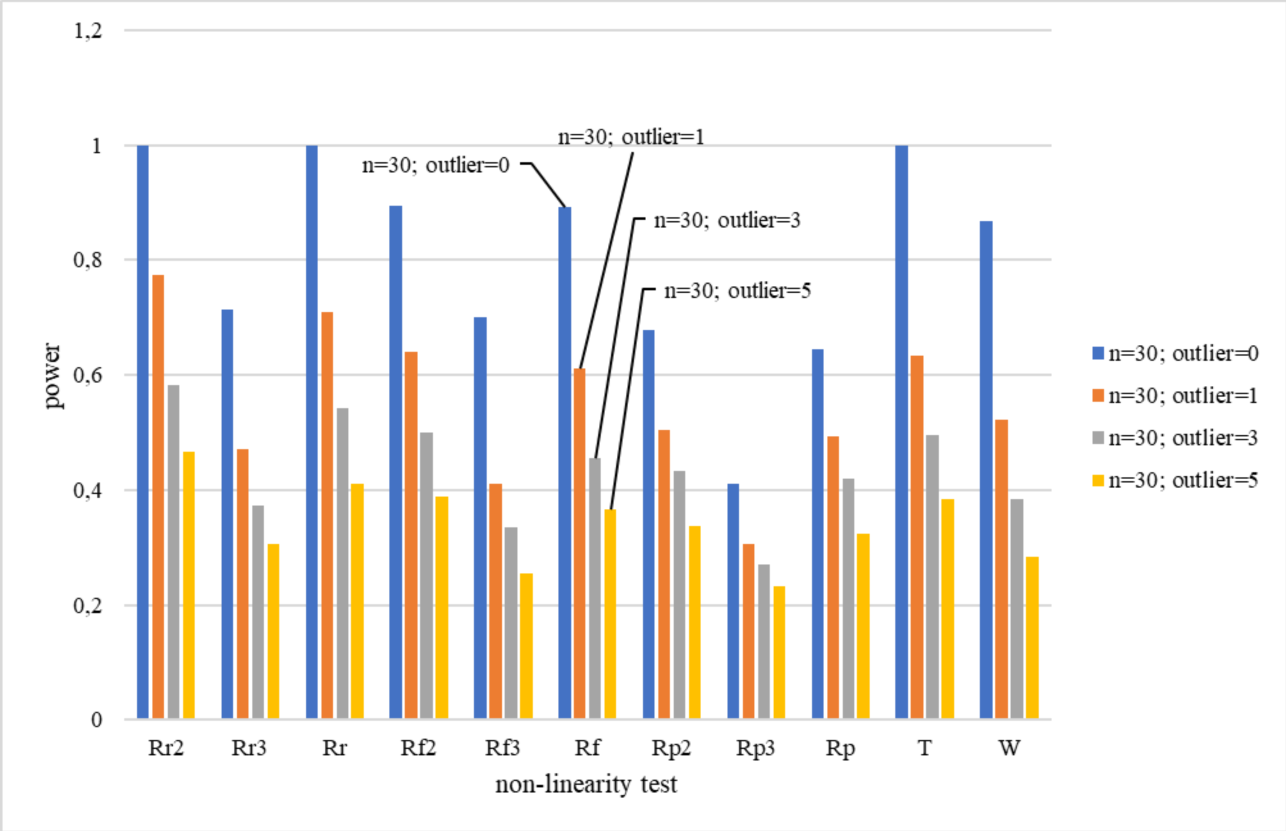


(c)

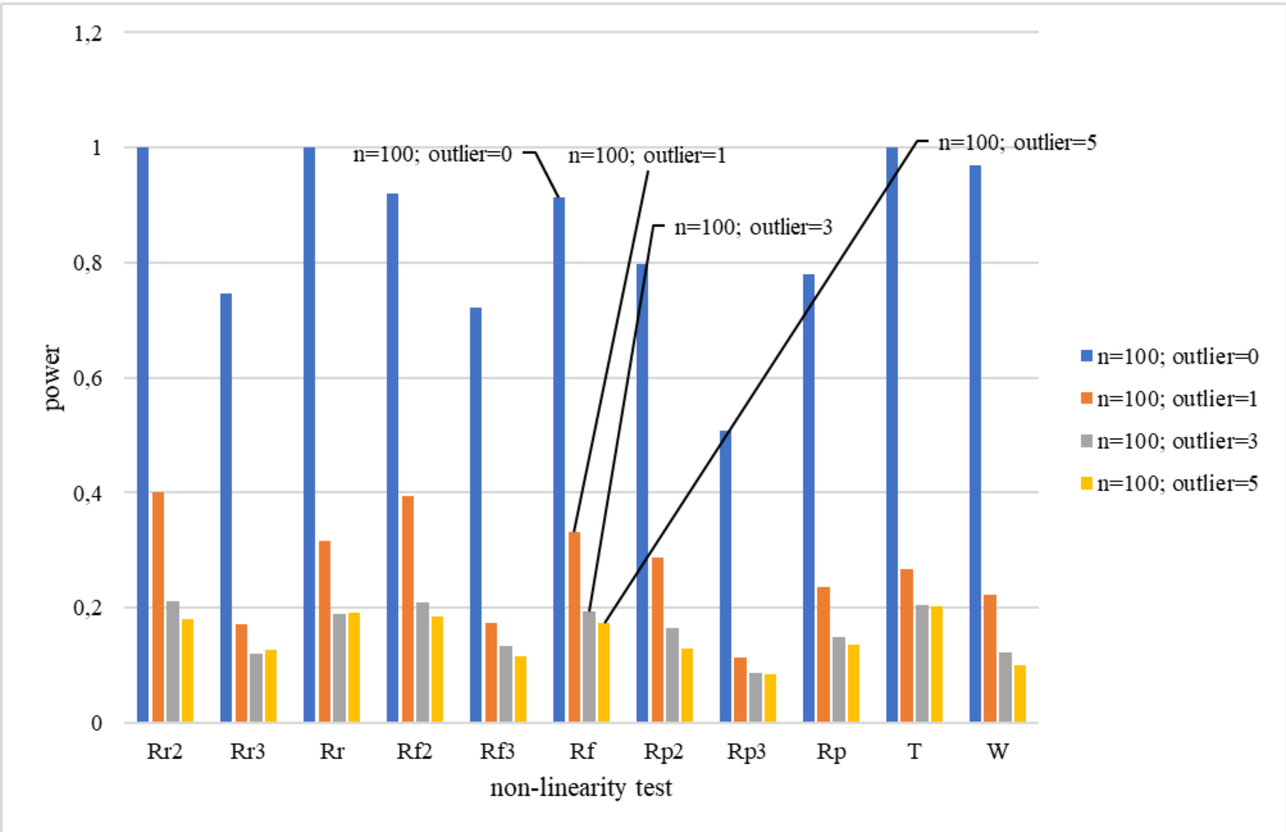


(d)

Fig. 2 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_2 = 2X_1 + 7X_2$

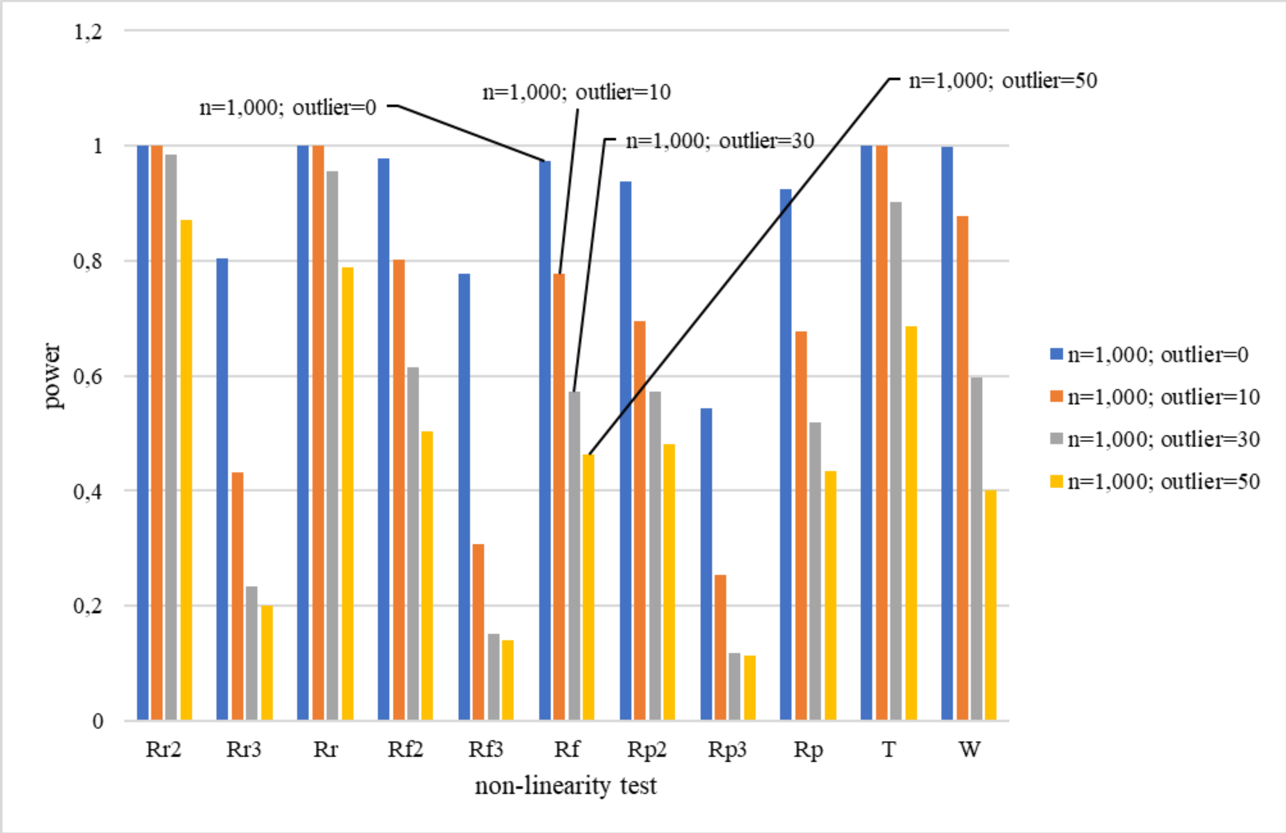


(a)

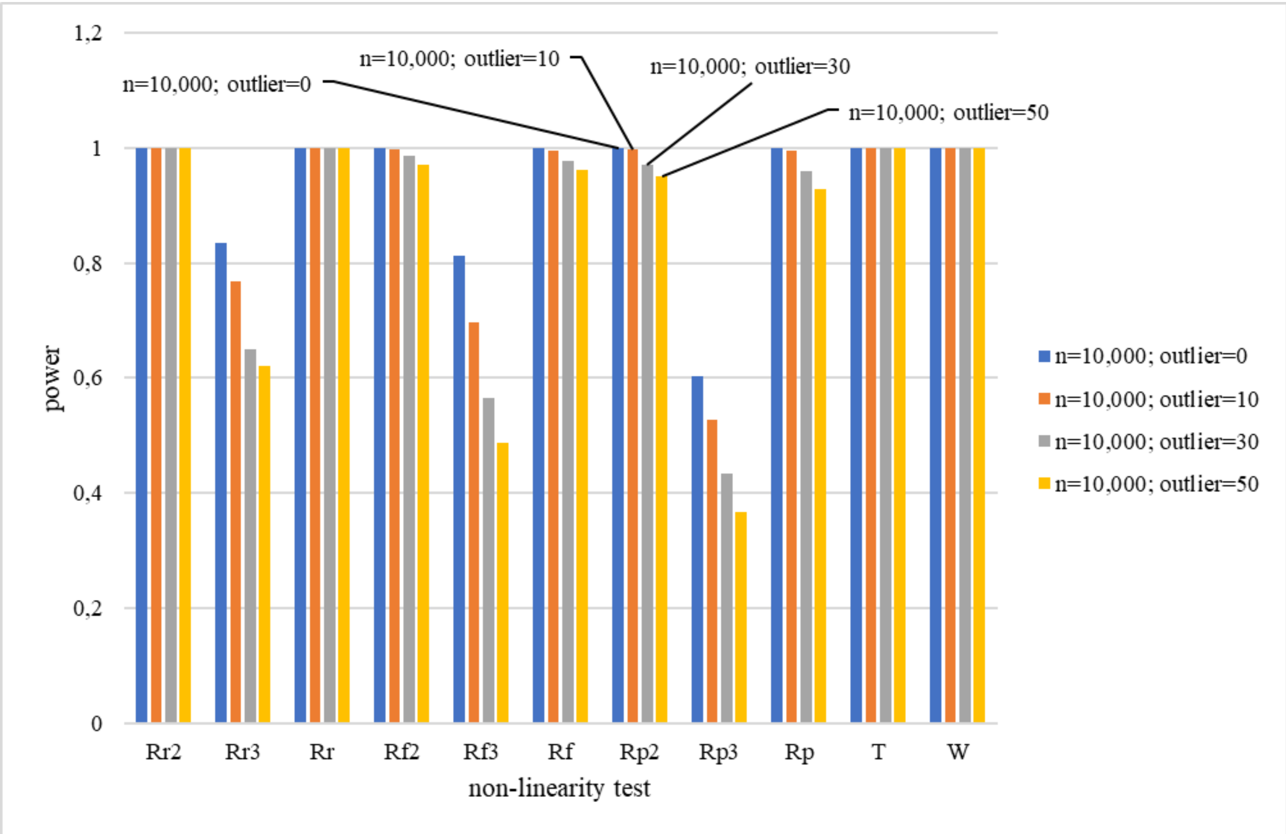


(b)

Fig. 3. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_3 = X_1^{\,2} + 3X_2^{\,4}$

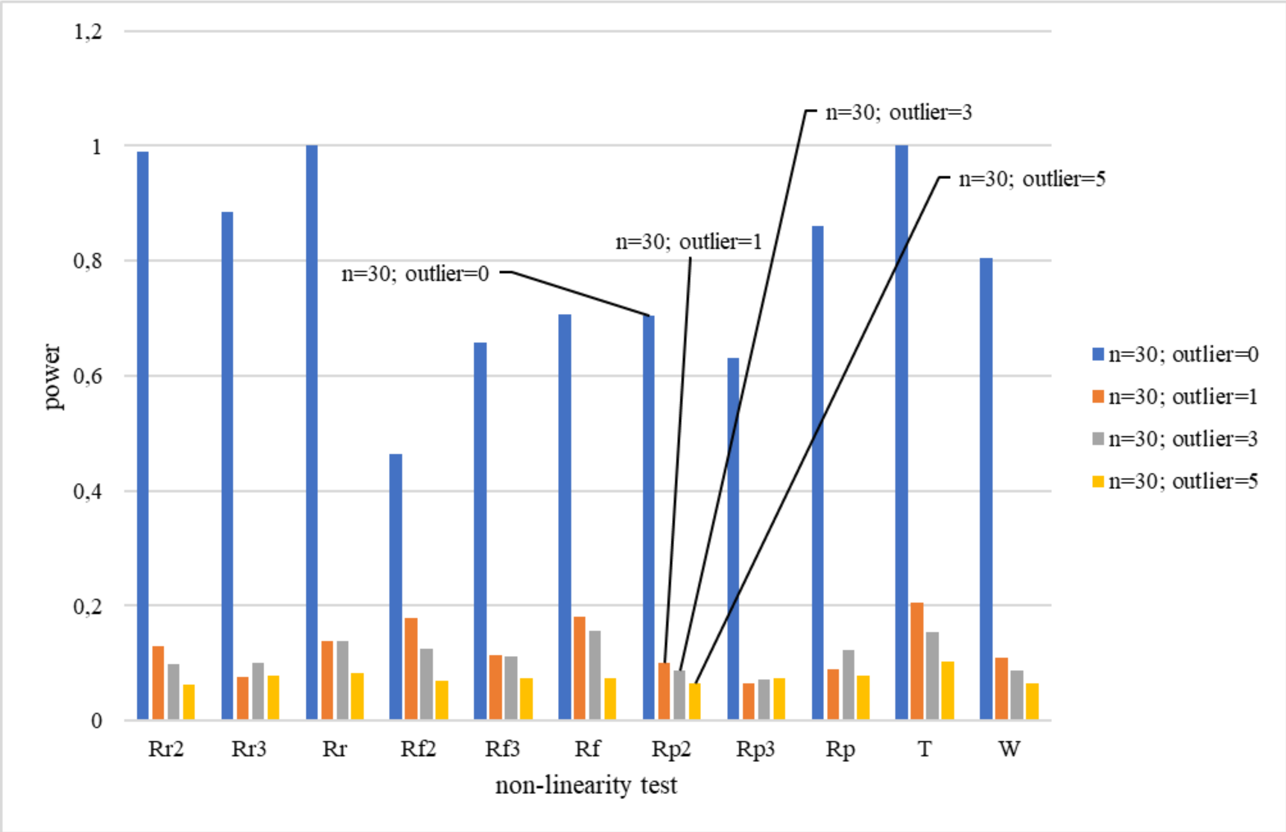


(c)

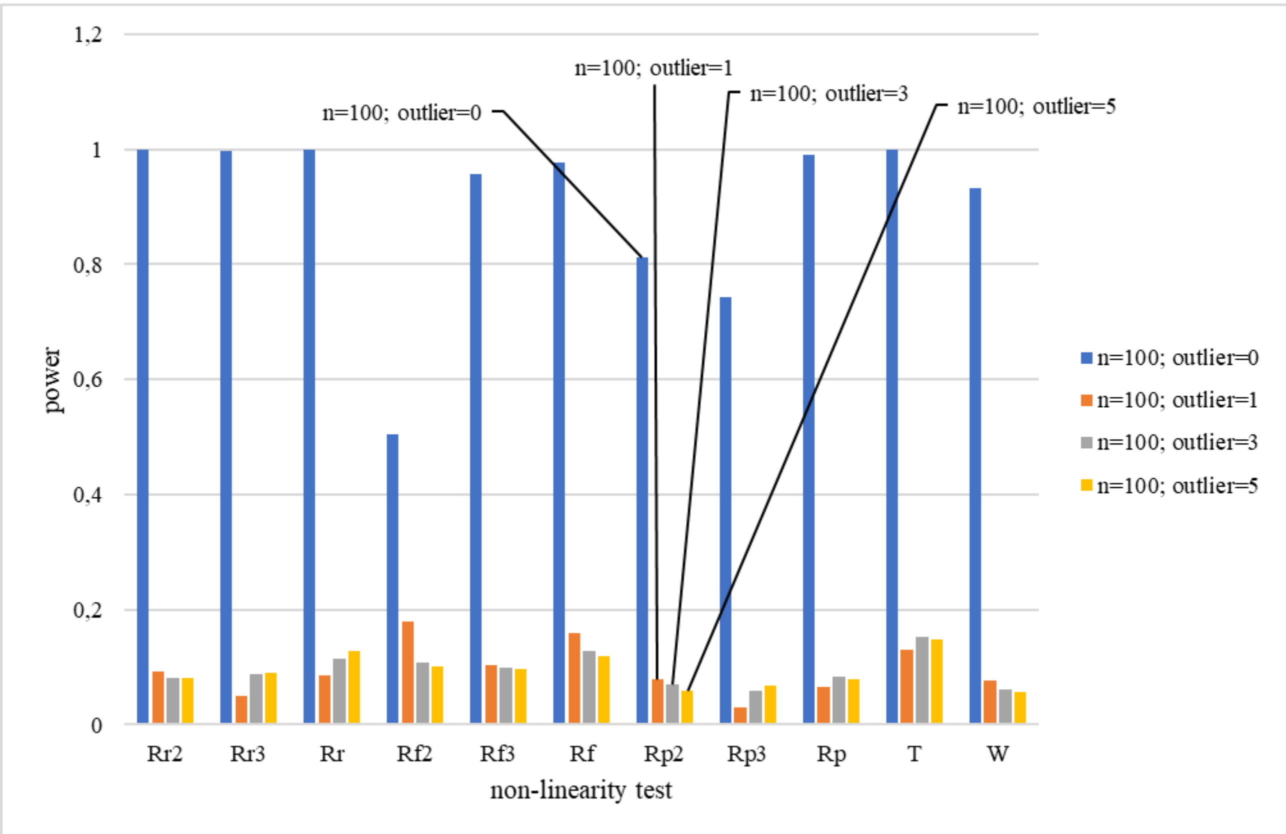


(d)

Fig.3 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_3 = X_1^2 + 3X_2^4$

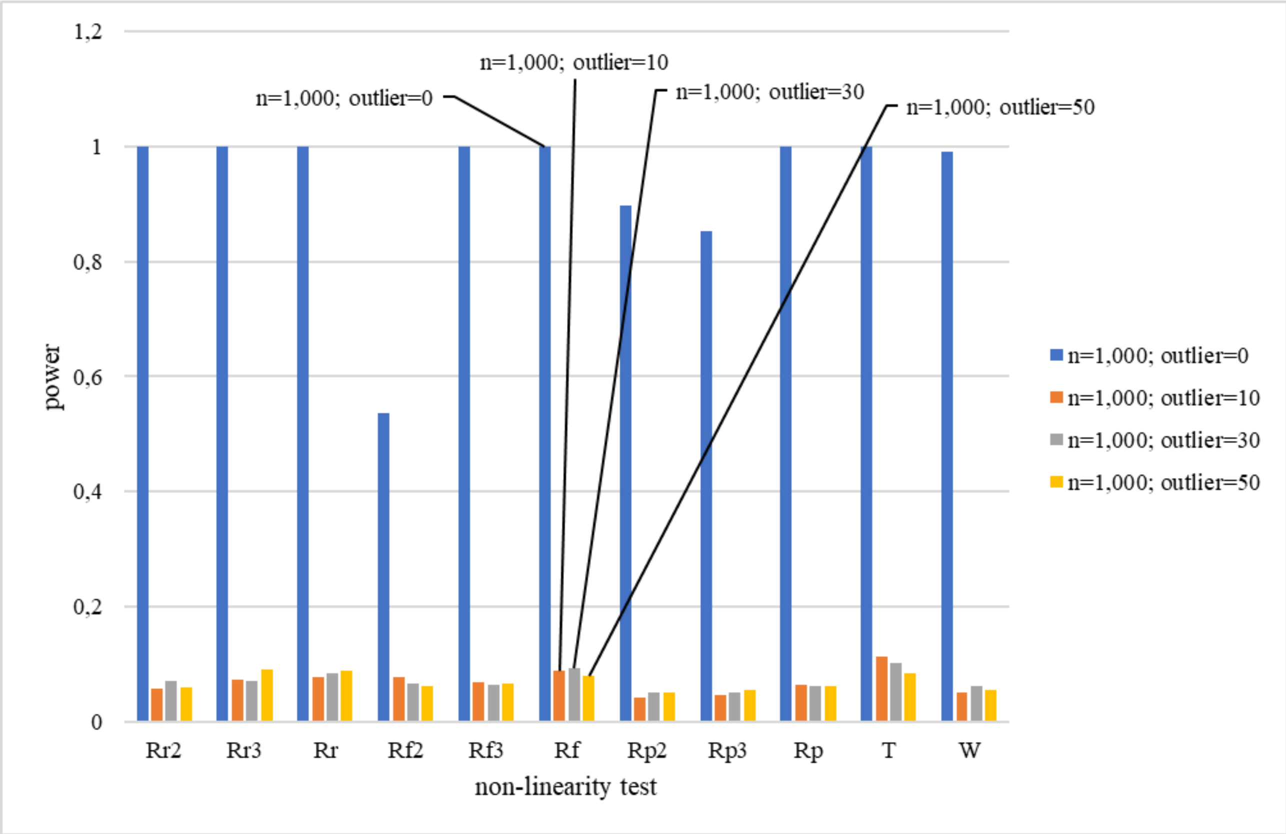


(a)

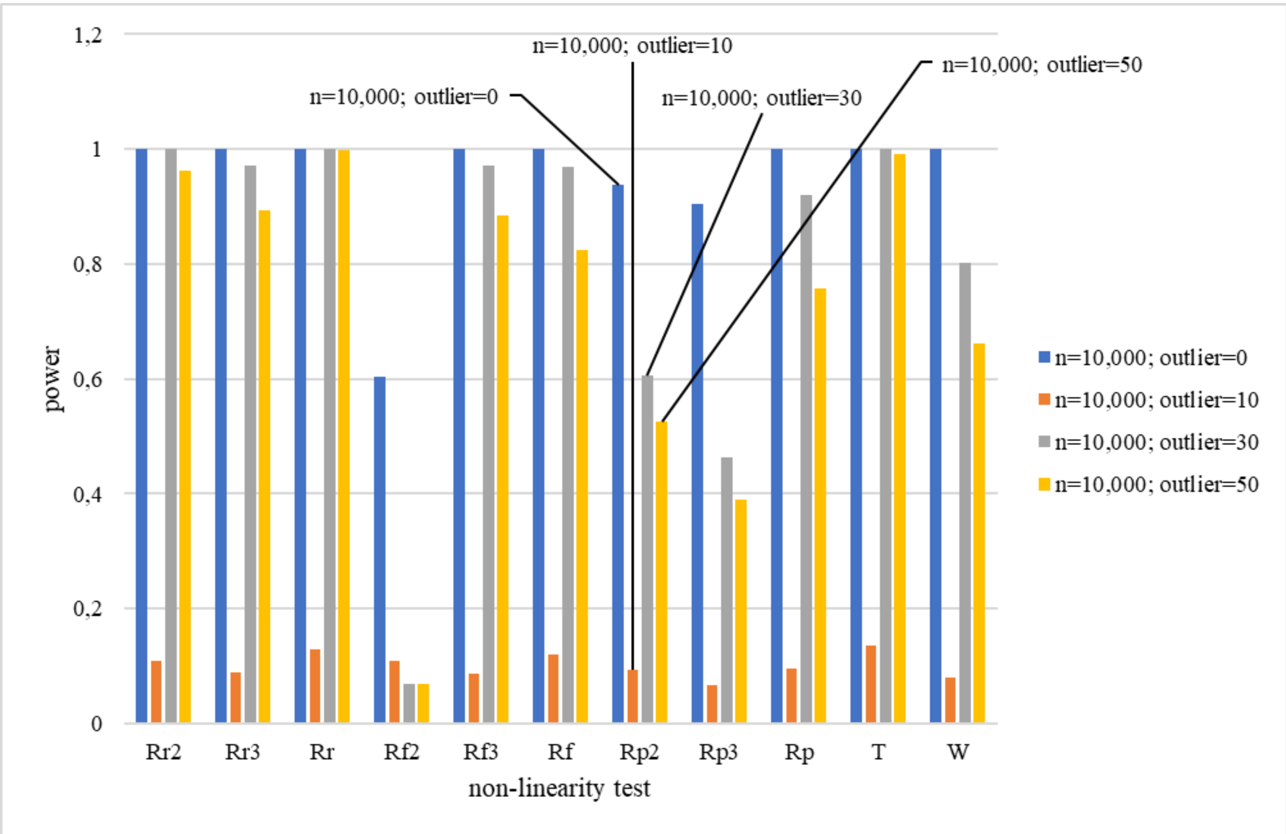


(b)

Fig. 4. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_4 = X_1^{\,3} + 2X_2^{\,2}$

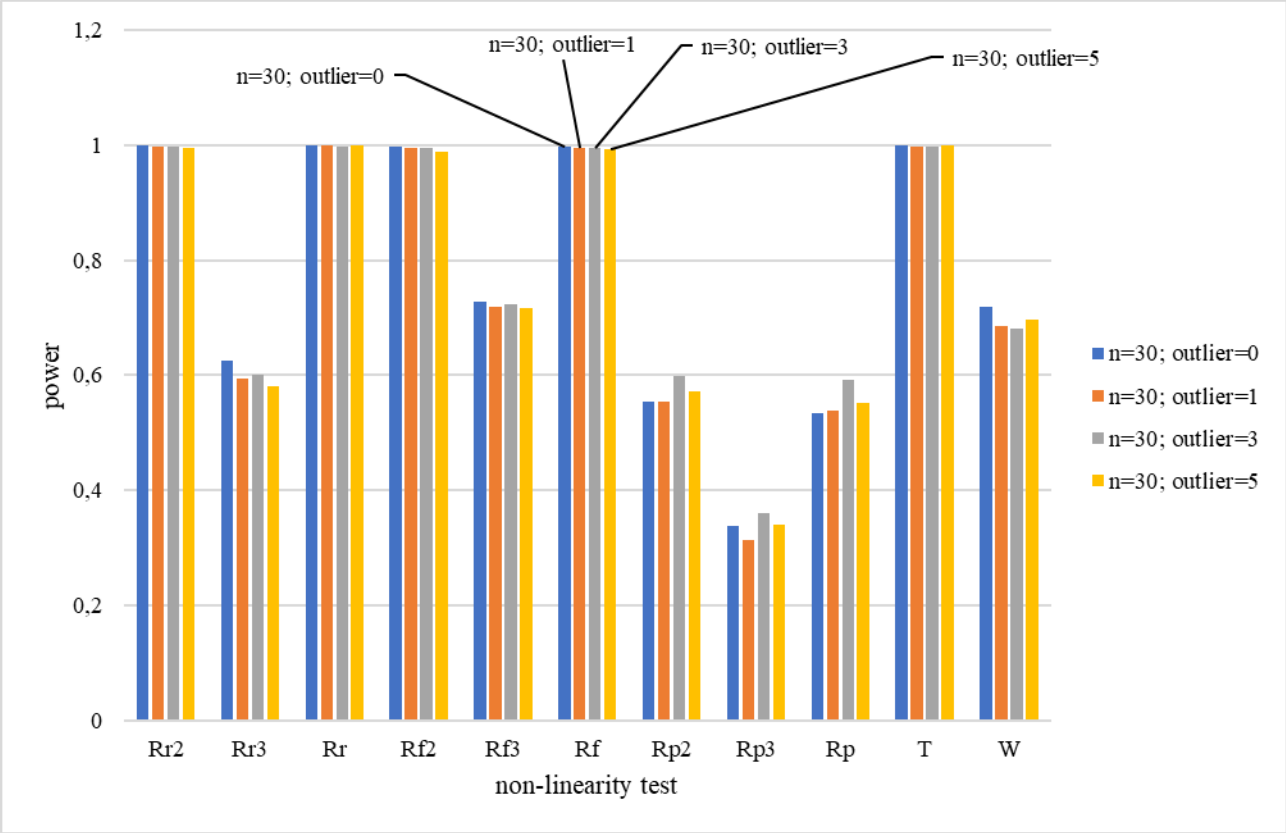


(c)

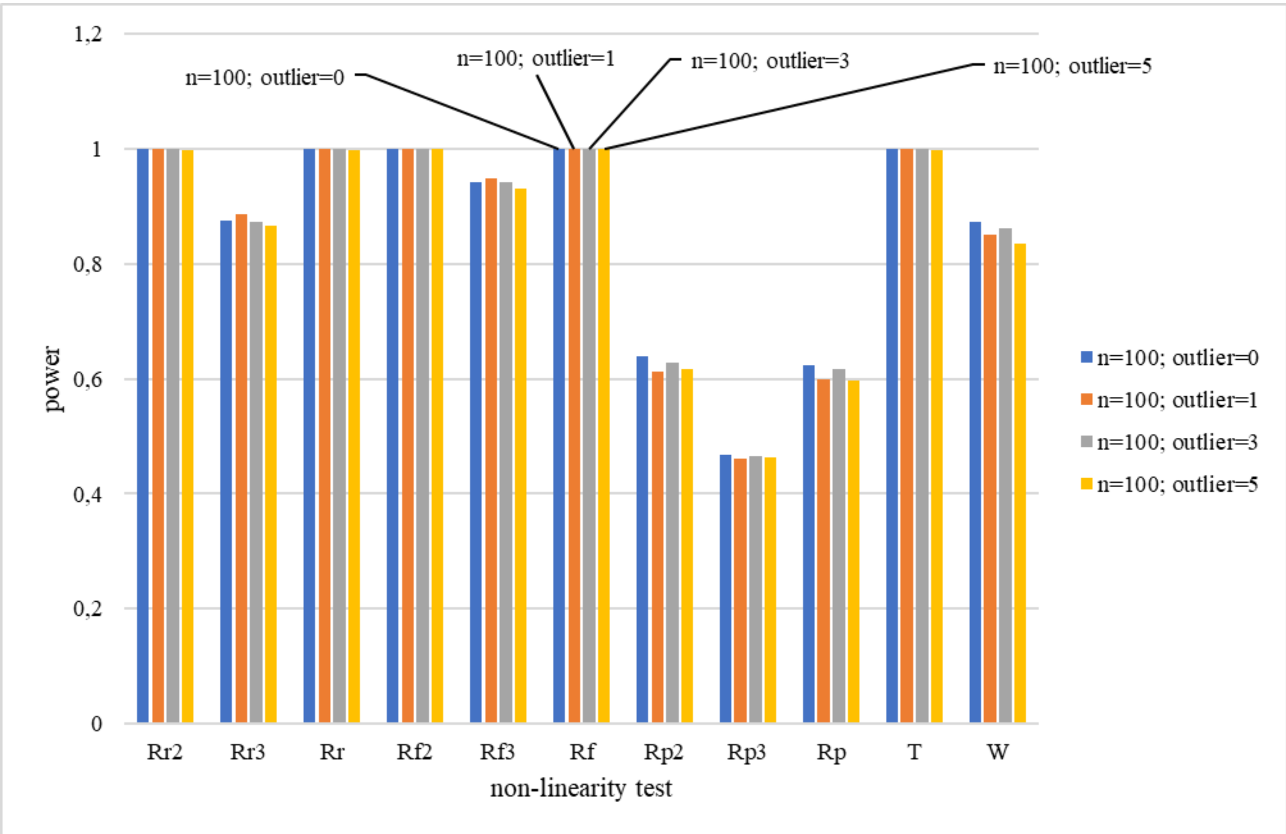


(d)

Fig.4 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_4 = X_1^3 + 2X_2^2$

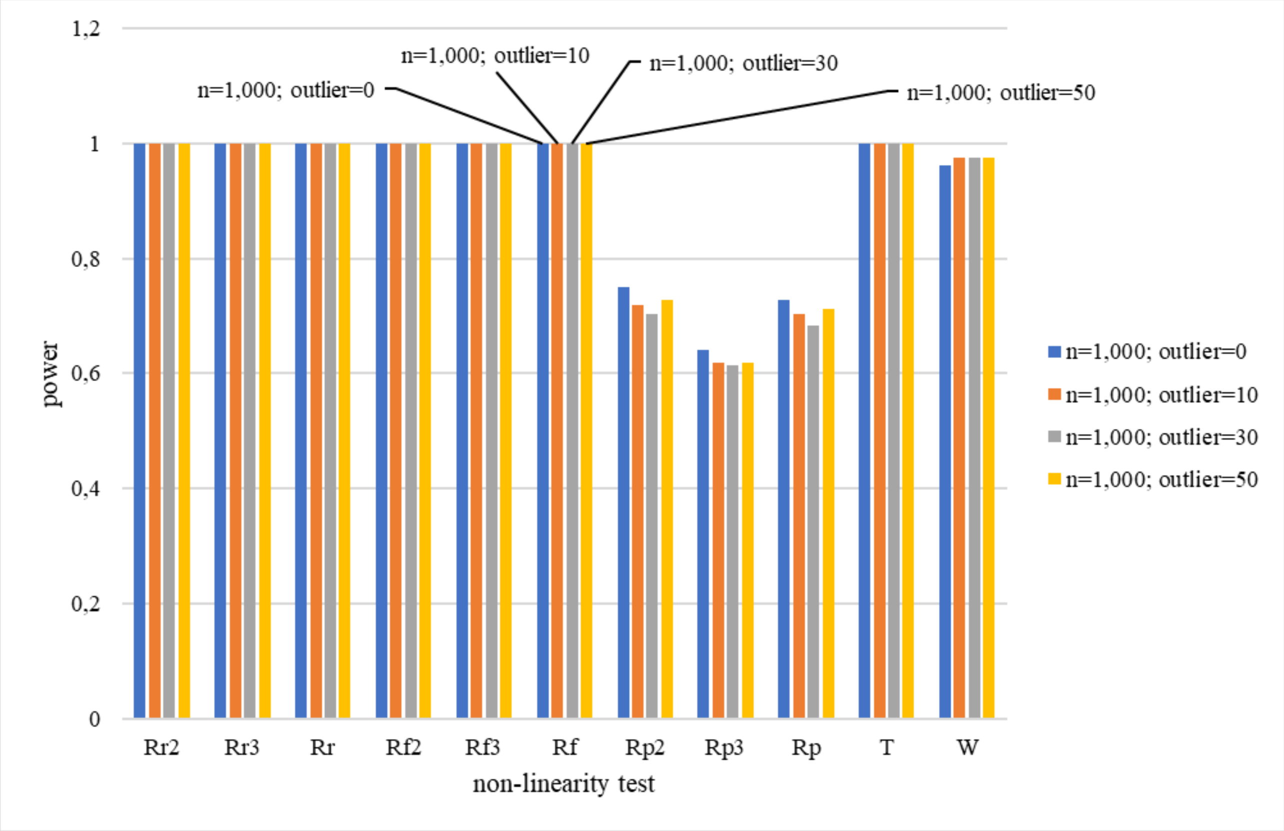


(a)

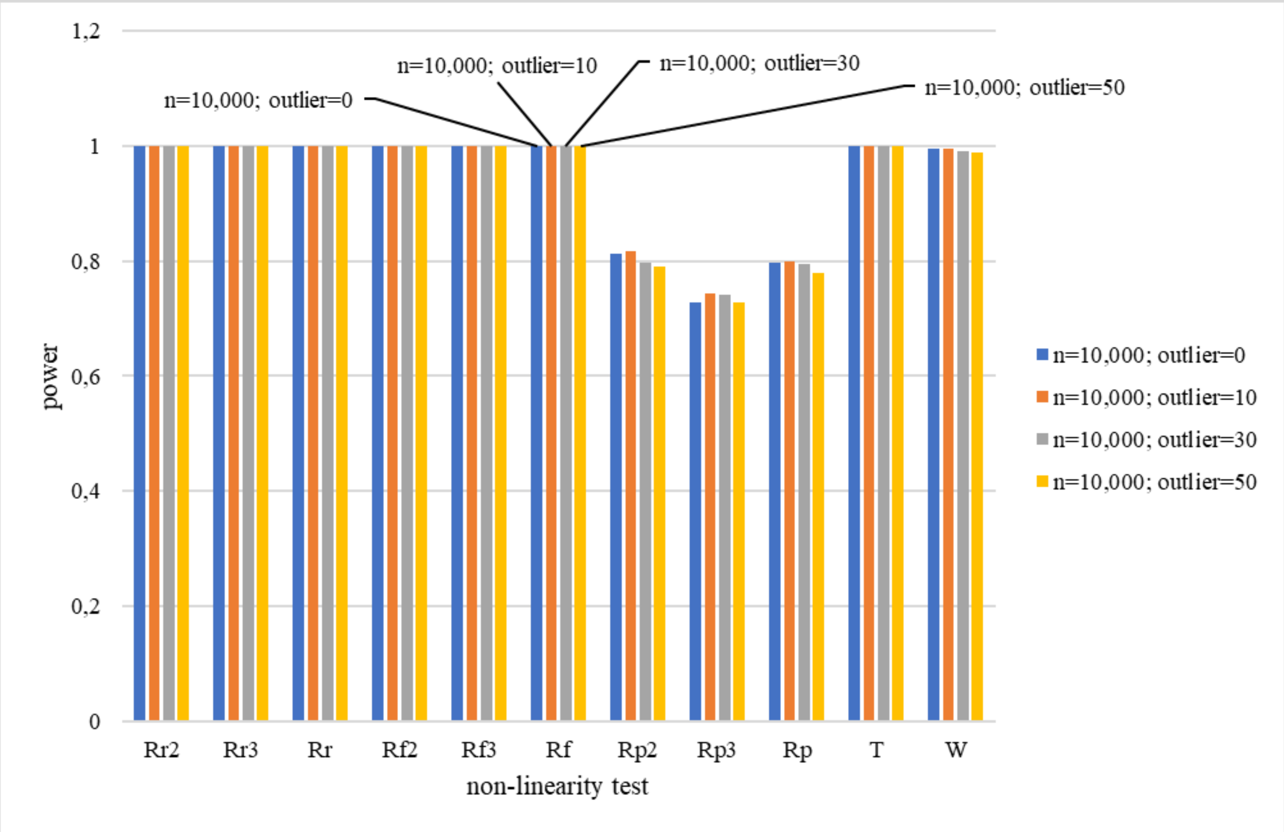


(b)

Fig. 5. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_5 = e^{2X_1} + 3e^{4X_2}$

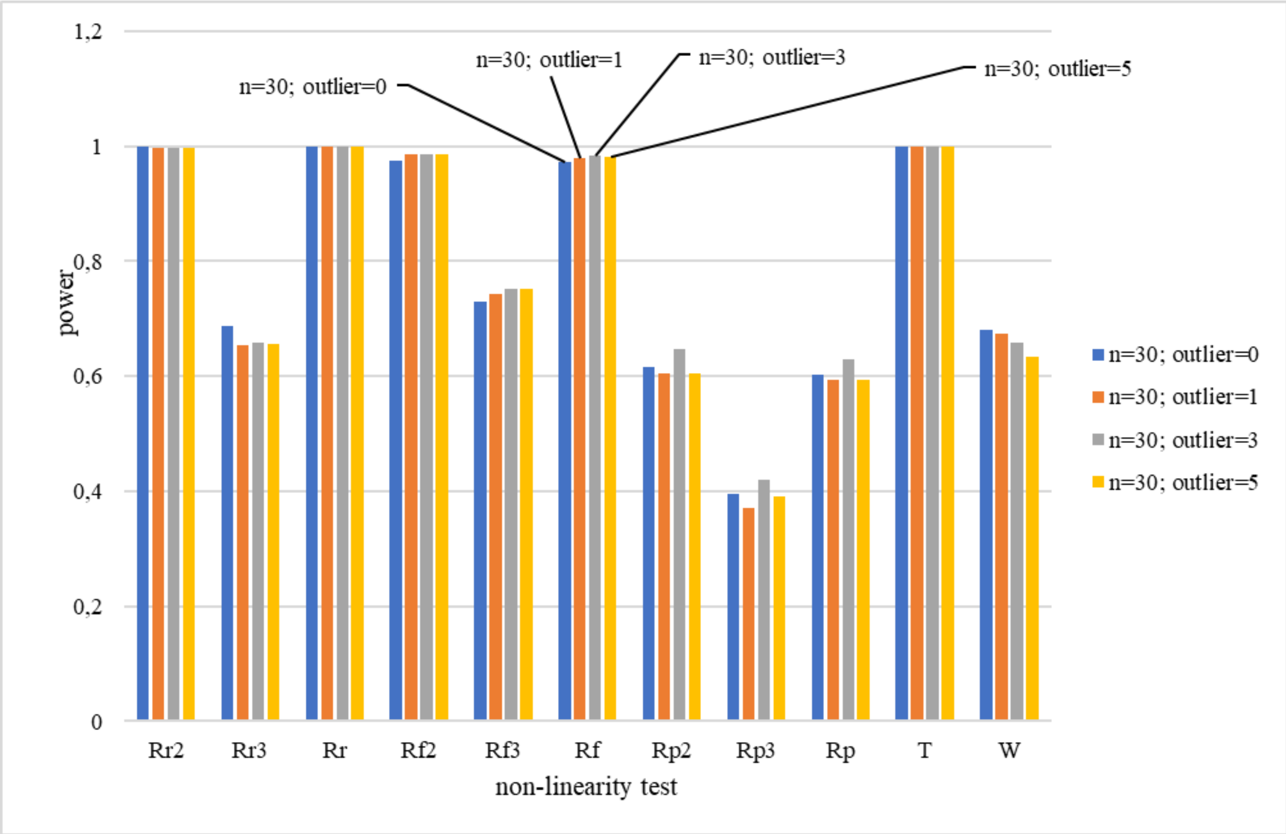


(c)

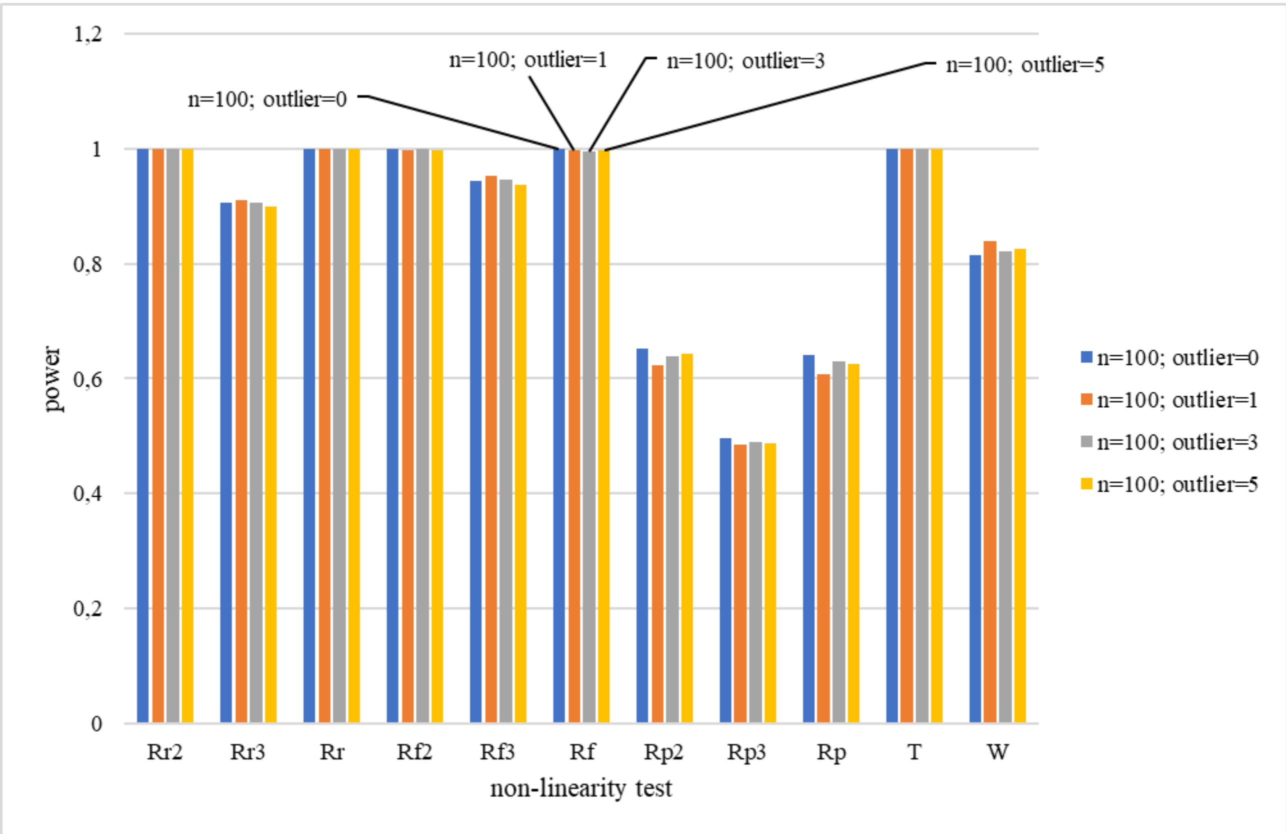


(d)

Fig. 5 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_5 = e^{2X_1} + 3e^{4X_2}$

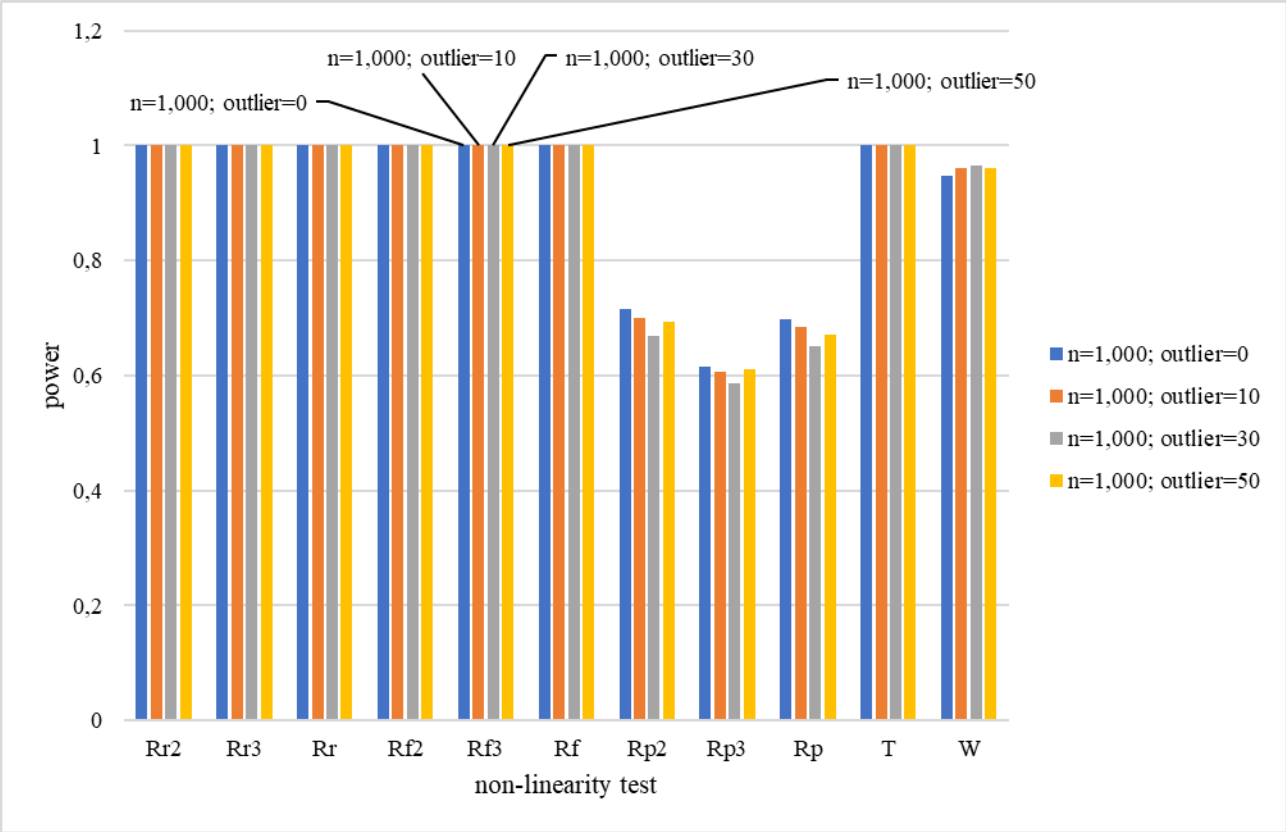


(a)

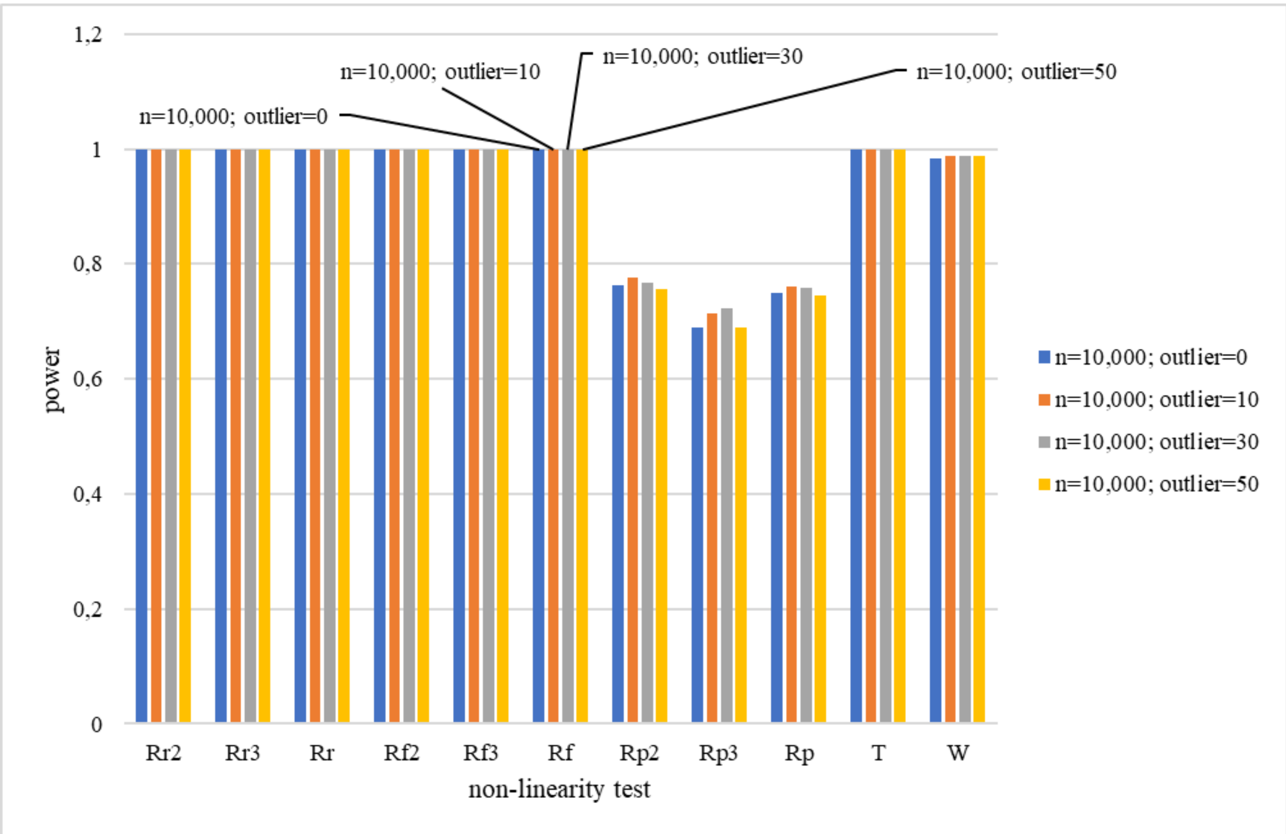


(b)

Fig. 6. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_6 = e^{4X_1} + 2e^{6X_2}$

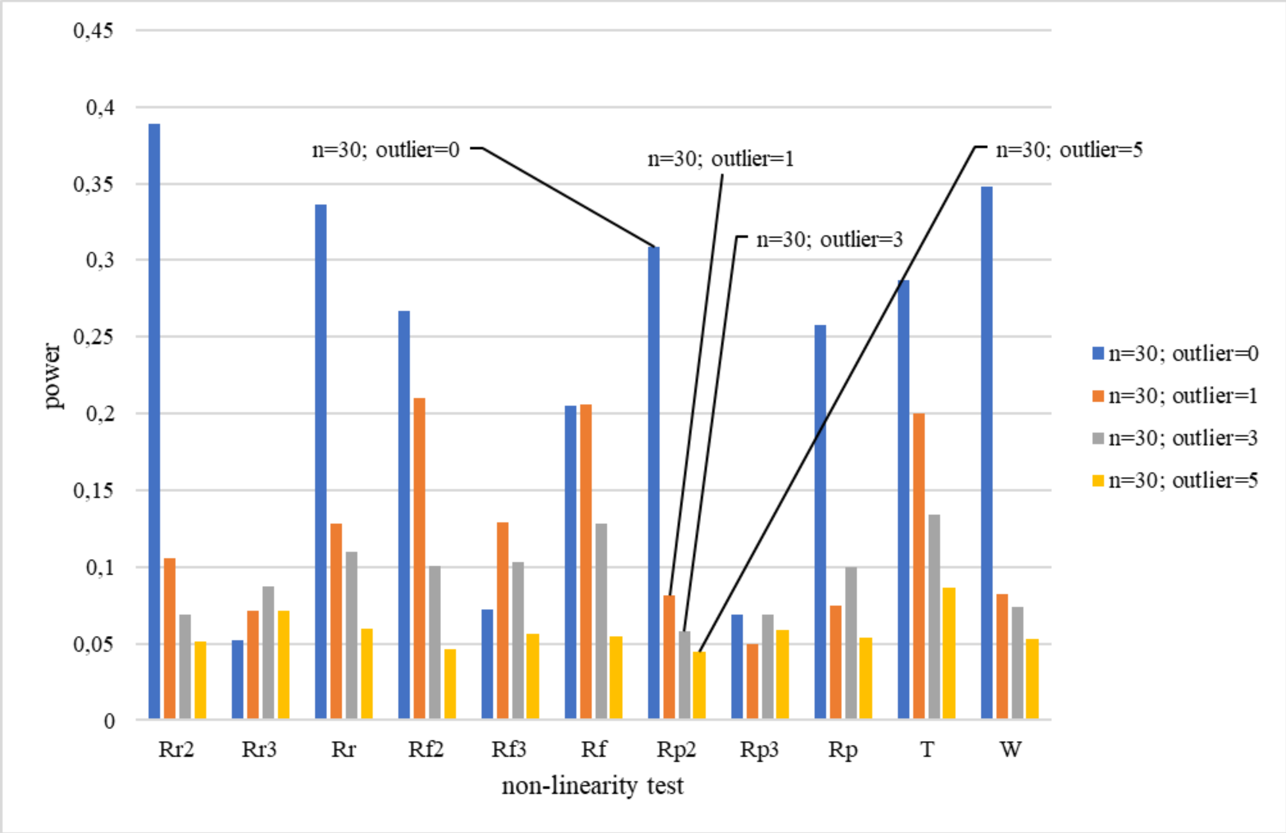


(c)

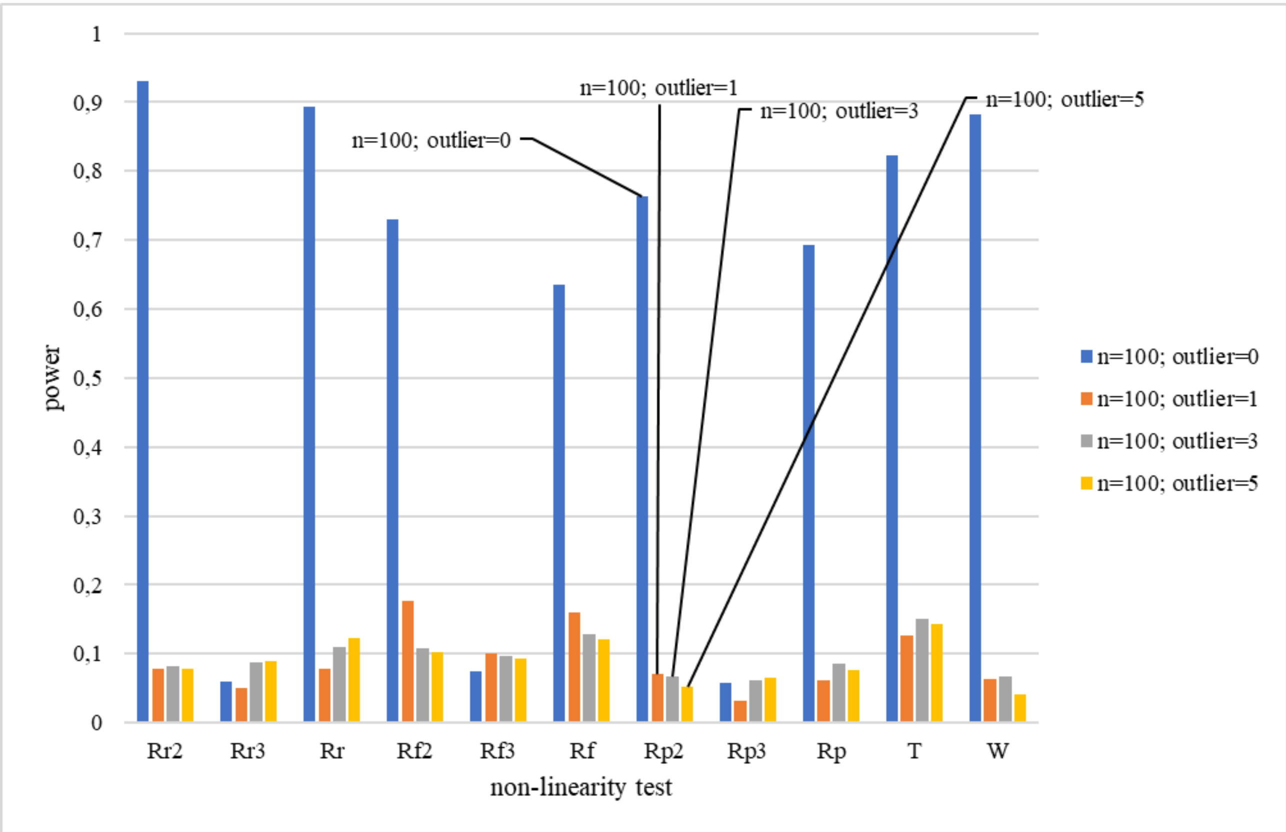


(d)

Fig. 6 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_6 = e^{4X_1} + 2e^{6X_2}$

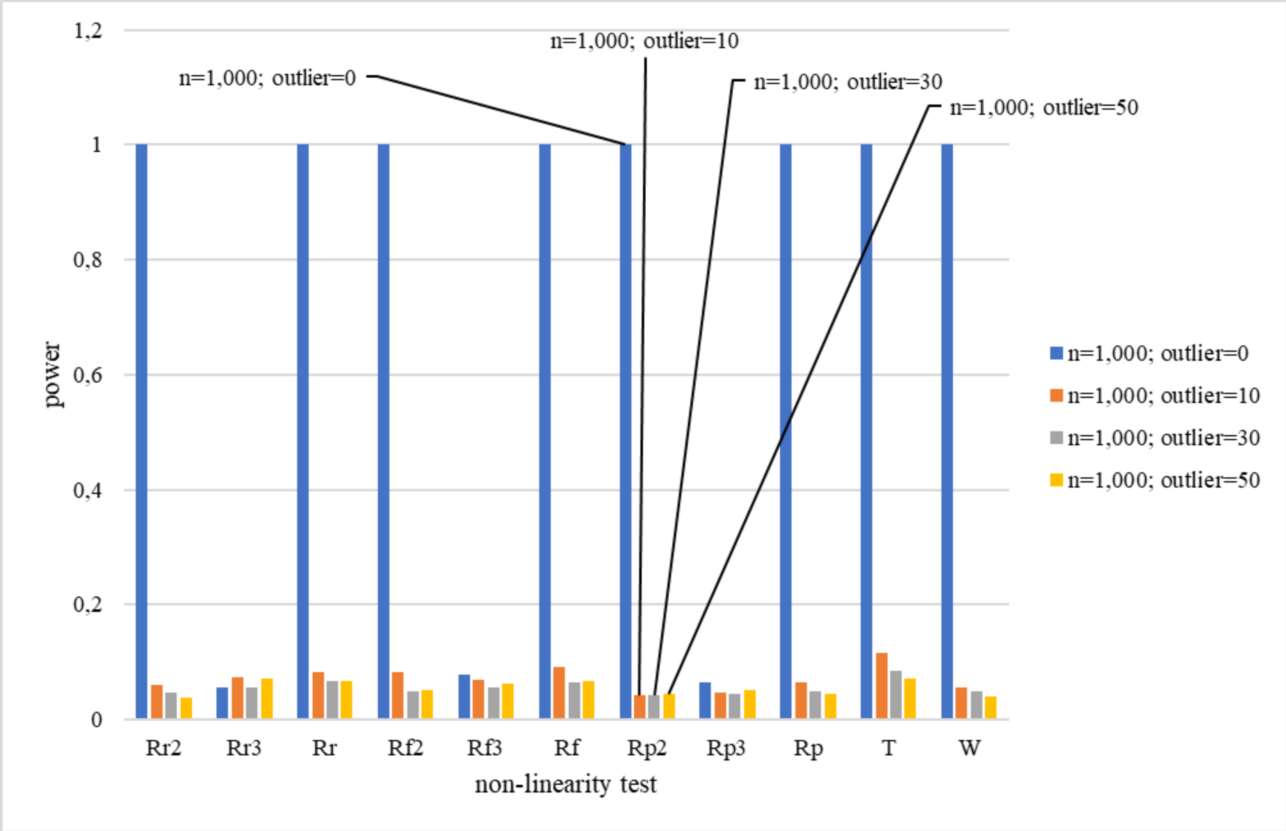


(a)

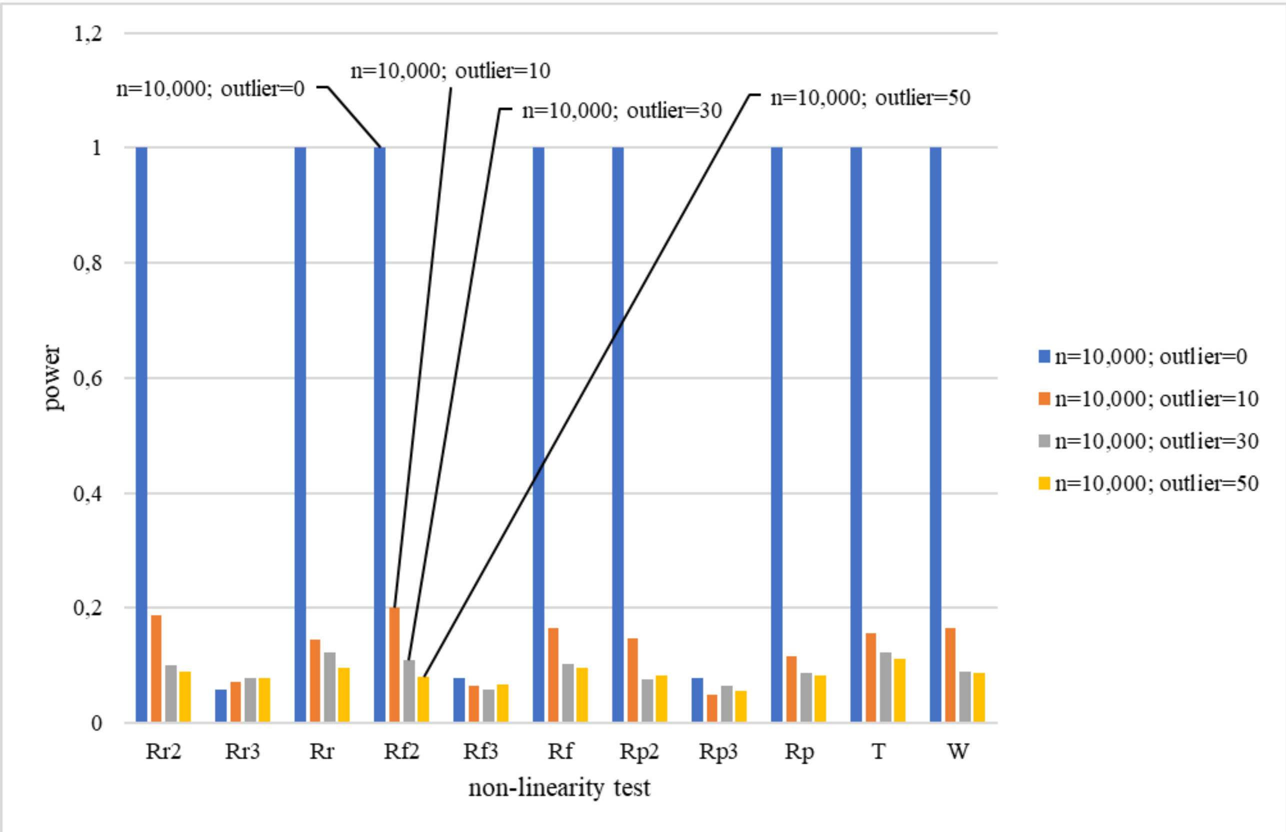


(b)

Fig. 7. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_7 = \left(1 - e^{-X_1^2}\right) + \left(1 - e^{-3X_2^4}\right)$

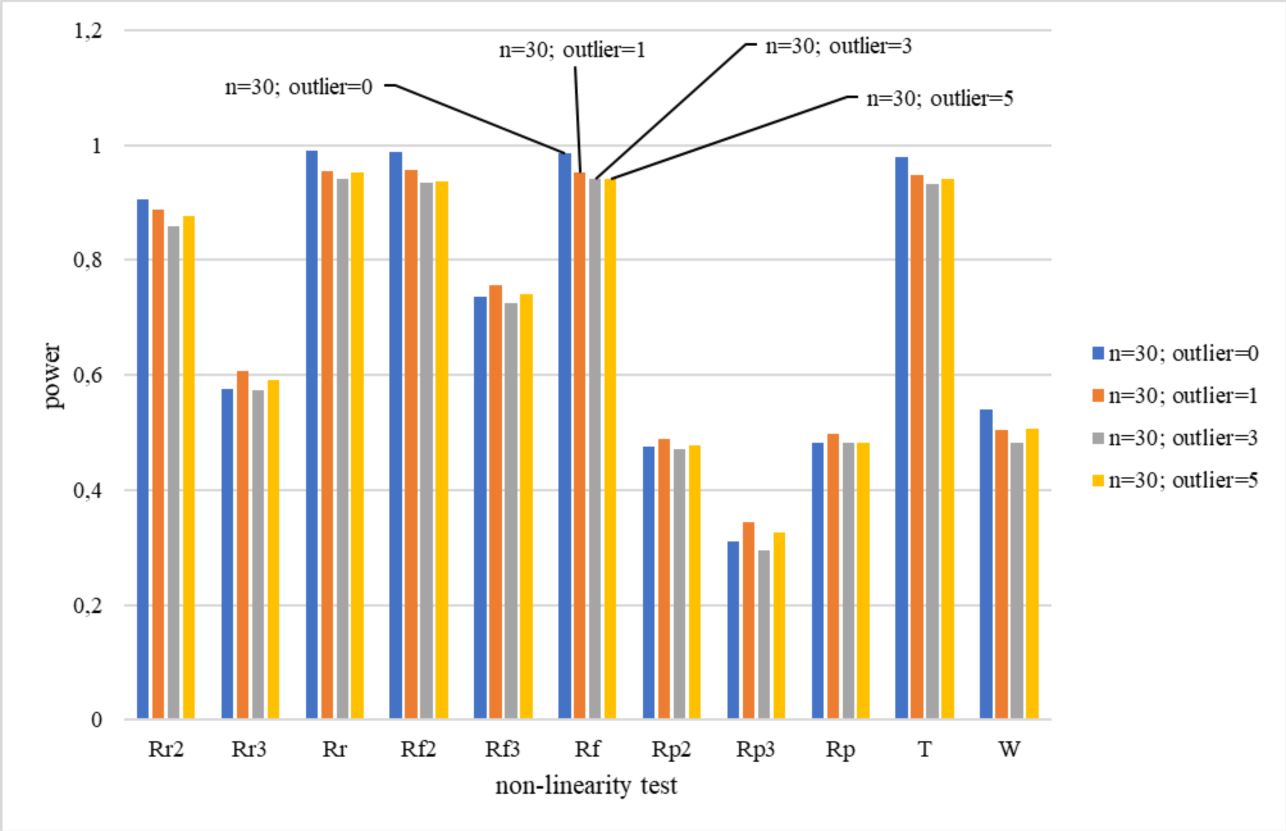


(c)

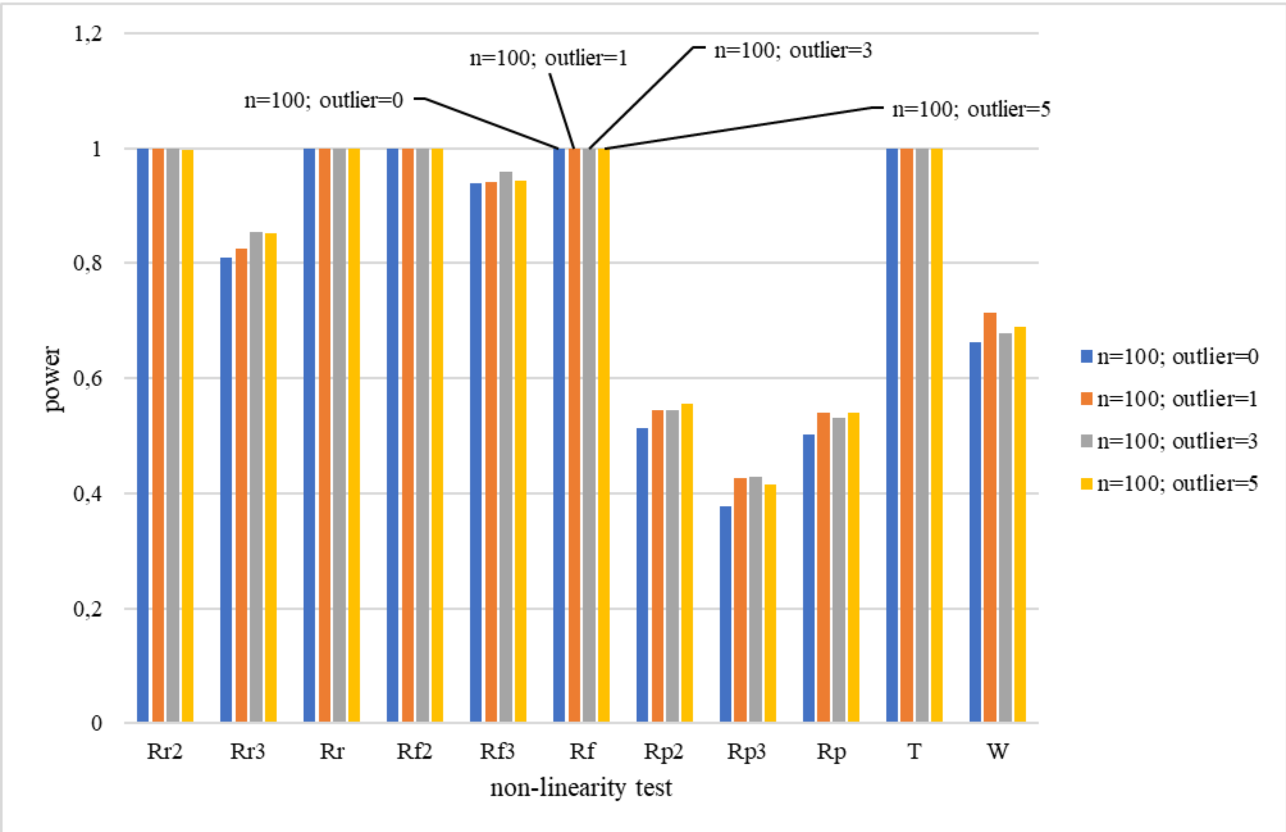


(d)

Fig. 7 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model
$Y_7 = \left(1 - e^{-X_1^2}\right) + \left(1 - e^{-3X_2^4}\right)$

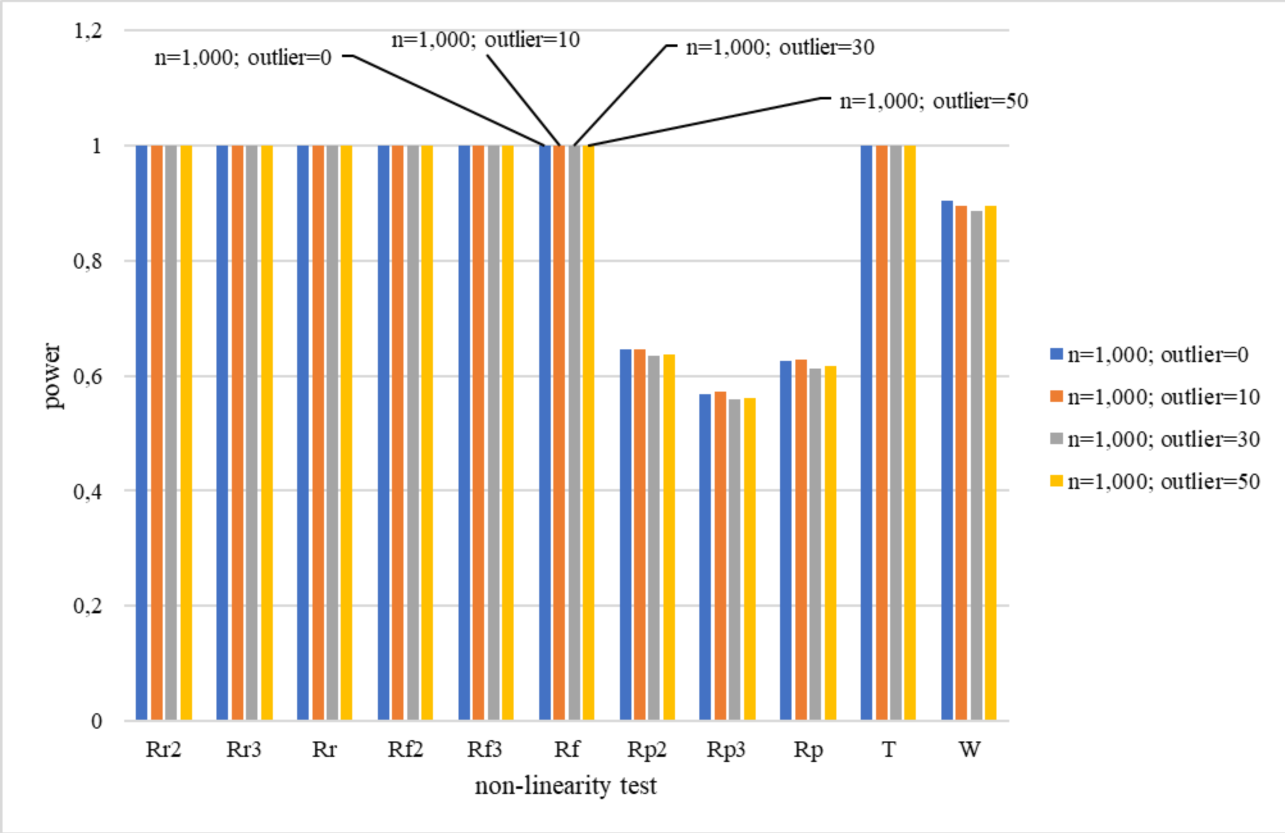


(a)

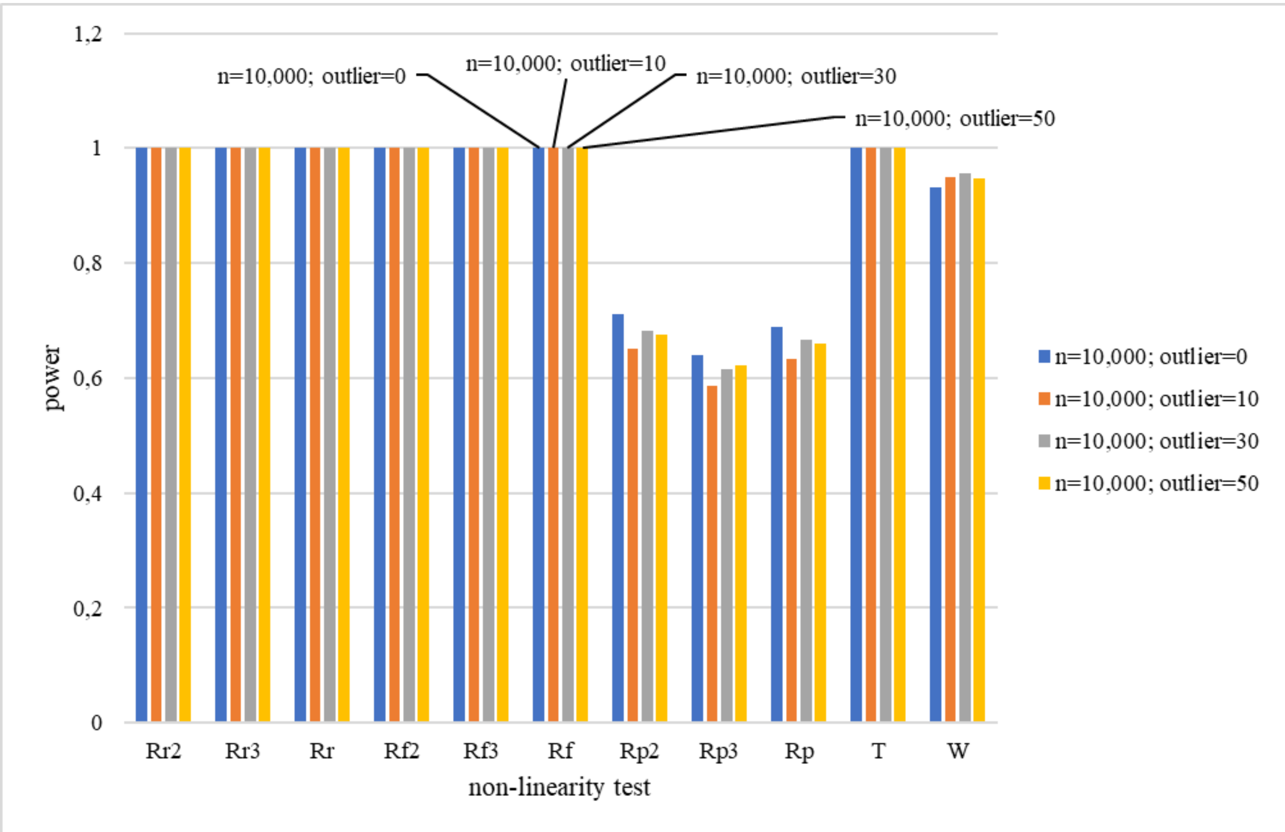


(b)

Fig. 8. Power comparison graph for (a) 30 random data sets and (b) 100 random data sets for the linear model $Y_8 = \left(1 - e^{-2X_1^3}\right) + \left(1 - e^{-2X_2^2}\right)$



(c)



(d)

Fig. 8 (continued). Power comparison graph for (c) 1,000 random data sets and (d) 10,000 random data sets for the linear model $Y_8 = \left(1 - e^{-2X_1^3}\right) + \left(1 - e^{-2X_2^2}\right)$

TABLE 11
COMPARISON OF TESTS BASED ON POWER VALUE WINS ON EACH SCENARIO

Kind of Test	Type	Power	The Number Has the Biggest Power Value	Percentage
RESET	regressor	2	11	15.07
RESET	regressor	3	7	9.59
RESET	regressor		6	8.22
RESET	fitted	2	7	9.59
RESET	fitted	3	1	1.37
RESET	fitted		1	1.37
RESET	princomp	2	0	0.00
RESET	princomp	3	0	0.00
RESET	princomp		0	0.00
Terasvirta	regressor		39	53.42
White	princomp		1	1.37
TOTAL			**73**	**100**

IV. CONCLUSIONS AND DISCUSSION

Based on Fig. 1 (a), (c), and (d) show that the largest power value is the non-linearity test when using Terasvirta. These results indicate that Terasvirta performs best among other tests in detecting outliers in linear models. This performance is tested in scenarios with small and large sample sizes. In addition to being tested on the number of samples, Terasvirta is also tested on the number of outliers. With outliers of 0, 1, 3, and 5 on the sample sizes of 30 and 100, Terasvirta dominates the results of the simulation. Likewise with the number of outliers of 0, 10, 30, and 50 on the sample sizes of 1,000 and 10,000.

Similar to Fig. 1, Fig. 2 also shows that (a), (c), and (d) are still dominated by the Terasvirta test. These results support the previous statement that Terasvirta has the best performance for linear models compared to other non-linearity tests.

In Fig. 3, some scenarios cannot be compared. These results are marked with gray columns in the TABLE 5. The power in these scenarios cannot be compared because the highest power is owned by more than 1 test. An example of this case is a scenario with n=30 and outliers=0. The highest power is 1, owned by RESET and Terasvirta test. Based on TABLE 5 and Fig. 3 obtained in the simulation of the power (convex) model, the best performance for non-linearity is the RESET test. However, Terasvirta has the highest power in scenario n = 100 and outlier = 5.

As explained previously, there are several scenarios that cannot be compared if the highest power value is obtained from several tests. Based on TABLE 6 , the scenarios that cannot be compared are n = 30 with outlier = 0, n = 100 with outlier = 0, n = 1,000 with outlier = 0, n = 10,000 with outlier = 0 and outlier = 30. In addition to these scenarios, other scenarios have results that Terasvirta has the highest power compared to other tests. Thus, to test non-linearity in the power (convex) model, Terasvirta is the best.

In the exponential model, there are more scenarios that cannot be compared than those that can be compared. Based on TABLE 7, the number of scenarios that can be compared is three. Of the three scenarios, two of them Terasvirta has the highest power value. Thus, Terasvirta has the best performance in detecting non-linearity in the exponential model.

In the second exponential model, all scenarios cannot be compared. This result can be seen in TABLE 8, it can be seen that all columns are gray. However, the fact that we can observe is that for each scenario, Terasvirta always has a power value of 1. This means that for all scenarios in the second exponential model, Terasvirta can always be relied on to detect non-linearity.

In the 2-parameter sigmoid 1 model, two scenarios cannot be compared, as can be seen in TABLE 9. Thus, fourteen scenarios can be compared. Of the fourteen scenarios, seven of them have the highest power value, and the other seven are spread across other non-linearity tests. This shows that Terasvirta also has the best performance for testing non-linearity for the 2-parameter sigmoid 1 model. Based on TABLE 10, only five scenarios can be compared. Of the five scenarios, four of them have the highest power values in the RESET test and one scenario in Terasvirta.

Based on TABLE 3 to TABLE 10, there are 55 failed scenarios. Thus, there are 73 scenarios left that can be used to compare the performance of the tests. The summary results are given in TABLE 11. Based on the research objectives to see which tests are more robust, the conclusion can be drawn is the Terasvirta test. From a total of 73 scenarios created, the Terasvirta test has a goodness of 53.42% compared to the RESET and White tests. This is done by sharing the number of simulation data. starting from 30; 100; 1,000; and 10,000. This scenario has also been applied to non-linear models, either non-linear in parameters, variables, or both.

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