H_infinity Control for 2-D Markov Jump Systems Based on Asynchronous Observers

Xiaoshuai Xu, Feng Li, and Qingkai Kong

***Abstract*—This study focuses on the issue of asynchronous observer-based H_∞ control for Roesser model-based Two-dimensional Markov jump systems. The goal is to develop an asynchronous controller based on observer so that the system maintains asymptotic mean square stability while exhibiting H_∞ disturbance attenuation performance. The asynchrony between the observer and the control plant is described by a hidden Markov process. Furthermore, applying the the Lyapunov function method, a prerequisite for achieving asymptotic mean square stability in closed-loop systems, while maintaining a predefined level of H_∞ disturbance attenuation performance, is derived. Based on this criterion, a methodology for designing the intended controller is constructed through the utilization of linear matrix inequalities. The applicability of this scheme is illustrated by the thermal process model with Markov jump parameters.**

***Index Terms*—2-D System, Markov jump systems, asynchronous control, observer-based controller.**

I. INTRODUCTION

TWo-dimensional (2-D) systems have garnered massive attention owing to their significant role in multiple subject areas, such as image processing, circuits, thermal processes, signal transmission, and multi-dimensional digital filtering [1–5]. Generally, the most common models of two-dimensional system are the Fornasini-Marchesini model and the Roesser model. The Fornasini-Marchesini model is employed to tackle the digital filter design problem, utilizing a single vector to represent functions that are composed of two unrelated variables. The Roesser model is mostly used to solve image processing problems. The key difference lies in the fact that the Roesser model possesses two orthogonal components: one state is horizontal, whereas the other is vertical. Nowadays, there is a lot of important research on the Fornasini-Marchesini model and the Roesser model in [6–8]. The issue of H_∞ filtering for the 2-D system based on the Fornasini-Marchesini model was resolved in [9] through the application of a linear matrix inequality (LMI) method. The H_∞ control issue and stability analysis for the 2-D system based on the Roesser model were studied in [10].

On the other hand, Markov jump systems (MJSs) have recently become a popular research area for multi-modal systems [11–14], because they are widely used in fields like aerospace, communications, financial engineering, and biomedical engineering [15–17]. The MJSs serve as a tool to depict the transitions among various operational modes of the system, thereby offering significant assistance in the analysis and control of the system. For instance, the stability analysis of the MJSs was studied in [18–22], and the problem of controller design of MJSs was studied in [23–26]. However, the problem of structural and parameter changes in 2-D systems is difficult to handle. Some scholars have done a lot of research on this issue. In [27–30], the issues pertaining to stability analysis of 2-D MJSs were resolved. The authors discussed the challenge of detecting faults within 2-D MJSs in [31]. In addition, the difficulty of controller design within 2-D MJSs was discussed in [32–35].

As a matter of fact, it is significant to design the controller of MJSs. Up to now, the controller of MJSs is designed as the synchronous controller in most papers [36–38]. This control method requires the controller to match the mode of the system. Nevertheless, this assumption is challenging to accomplish in practical situations. For example, the system's inability to fully receive all information stems from a range of uncertain factors, including but not limited to communication delays, data quantization, and data loss that occur in the network control systems. It results in the operation of controller mode and system mode being out of synchronous. In addition, many controllers were designed based on state feedback in [32, 33]. This approach requires each state to be accurately measurable in order to achieve better system performance. However, in practical applications, it is often impossible to directly obtain state variables due to reasons such as unmeasurable state, high measurement cost or expensive measurement sensors. Thus, some solutions are proposed so that the above problems could be solved. Firstly, using asynchronous control can solve the problem of mode mismatch between controller and system. The asynchronous controller-based system is structured using a hidden Markov model. By solving a LMI, the controller gain matrix is derived, ensuring the system remains asymptotically mean square stable (AMSS). Secondly, it is necessary to use known information such as input and output to design a state observer and estimate the state variables. In this way, one can ensure that the controller is serviceable even if the state of the system remains unmeasurable in practical applications. For instance, observer-based controllers are designed to ensure system stability under network attacks and communication delays in [39, 40].

This paper deals with the topic of asynchronous observer-based H_∞ control for Roesser model-based 2-D MJSs. An asynchronous observer-based controller is formulated to guarantee that the system achieves AMSS while maintaining a designated level of H_∞ disturbance attenuation capability. The asynchrony between the observer and the control plant is described by a hidden Markov process. Through the Lyapunov function method, a sufficient criterion for the

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asymptotic stability of the systems with a specified level of H_∞ disturbance attenuation performance is derived. Drawing upon this criterion, a design method for the desired controller is formulated through the LMI. The applicability of the design method is illustrated by means of a thermal process model with Markov jump parameters.

Notation. In this paper, the notation N^m represents the m-dimensional vector space and $N^{n\times m}$ means a real matrix with n rows and m columns. The symbol $\aleph > 0$ indicates that the matrix $\aleph$ possesses both symmetric and positive definite properties. The inverse of a matrix $\aleph$ is represented by the symbol $\aleph^{-1}$ and the symbol $\aleph^T$ shows the transpose matrix. The notation $diag\{\cdots\}$ expressions the diagonal block matrix and $E\{\aleph\}$ represents the mathematical expectation of $\aleph$.

II. Preliminaries

A. System model

The present investigation considers the 2-D MJSs expressed in the Roesser model, which are outlined as follows:

$$\begin{cases} \bar{x}(\mathfrak{T},\mathfrak{H}) = A(r_{\mathfrak{T},\mathfrak{H}})\,x(\mathfrak{T},\mathfrak{H}) + B(r_{\mathfrak{T},\mathfrak{H}})\,u(\mathfrak{T},\mathfrak{H}) \\ \qquad +D(r_{\mathfrak{T},\mathfrak{H}})\,\omega(\mathfrak{T},\mathfrak{H}) \\ z(\mathfrak{T},\mathfrak{H}) = E(r_{\mathfrak{T},\mathfrak{H}})\,x(\mathfrak{T},\mathfrak{H}) + F(r_{\mathfrak{T},\mathfrak{H}})\,u(\mathfrak{T},\mathfrak{H}) \\ \qquad +G(r_{\mathfrak{T},\mathfrak{H}})\,\omega(\mathfrak{T},\mathfrak{H}) \\ y(\mathfrak{T},\mathfrak{H}) = C(r_{\mathfrak{T},\mathfrak{H}})\,x(\mathfrak{T},\mathfrak{H}) \end{cases} \tag{1}$$

with

$$\bar{x}(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} x_h(\mathfrak{T}+1,\mathfrak{H}) \\ x_v(\mathfrak{T},\mathfrak{H}+1) \end{bmatrix}, x(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} x_h(\mathfrak{T},\mathfrak{H}) \\ x_v(\mathfrak{T},\mathfrak{H}) \end{bmatrix}$$

where $x_h(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_h}$ and $x_v(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_v}$ denote the state of the horizontal and vertical; $u(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_u}$, $z(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_z}$, $y(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_y}$ show the control input, the measured output and control output, respectively. The exogenous disturbance is defined by $\omega(\mathfrak{T},\mathfrak{H}) \in \mathbb{R}^{n_w}$. $A(r_{\mathfrak{T},\mathfrak{H}})$, $B(r_{\mathfrak{T},\mathfrak{H}})$, $\cdots$, $C(r_{\mathfrak{T},\mathfrak{H}})$ are known matrices of suitable dimensions. $r_{\mathfrak{T},\mathfrak{H}}$ represents the Markov chain, which takes on values from the restricted set $\mathbb{M} = \{1,2,3,\ldots,s\}$ and its transition probability matrix is defined by the elements $\Pi = [\pi_{mn}]$.

$$\begin{aligned} &\text{Prob}\,(r_{\mathfrak{T}+1,\mathfrak{H}} = n \mid r_{\mathfrak{T},\mathfrak{H}} = m) \\ &= \text{Prob}\,(r_{\mathfrak{T},\mathfrak{H}+1} = n \mid r_{\mathfrak{T},\mathfrak{H}} = m) \\ &= \pi_{mn} \end{aligned} \tag{2}$$

where $\pi_{mn} \geq 0$ and $\sum_{n=1}^{s} \pi_{mn} = 1$ for all $m,n \in \mathbb{M}$.

B. Observer design

Then, the state observer system can be formulated in the manner outlined below:

$$\begin{cases} \tilde{\mathscr{R}}(\mathfrak{T},\mathfrak{H}) = A(\epsilon_{\mathfrak{T},\mathfrak{H}})\,\widetilde{x}(\mathfrak{T},\mathfrak{H}) + B(\epsilon_{\mathfrak{T},\mathfrak{H}})\,u(\mathfrak{T},\mathfrak{H}) \\ \qquad +L(\epsilon_{\mathfrak{T},\mathfrak{H}})\,[y(\mathfrak{T},\mathfrak{H}) - \widetilde{y}(\mathfrak{T},\mathfrak{H})] \\ \widetilde{y}(\mathfrak{T},\mathfrak{H}) = C(r_{\mathfrak{T},\mathfrak{H}})\widetilde{x}(\mathfrak{T},\mathfrak{H}) \end{cases} \tag{3}$$

with

$$\tilde{\mathscr{R}}(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} \widetilde{x}_h(\mathfrak{T}+1,\mathfrak{H}) \\ \widetilde{x}_v(\mathfrak{T},\mathfrak{H}+1) \end{bmatrix}, \widetilde{x}(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} \widetilde{x}_h(\mathfrak{T},\mathfrak{H}) \\ \widetilde{x}_v(\mathfrak{T},\mathfrak{H}) \end{bmatrix}$$

where $\widetilde{x}_h(\mathfrak{T},\mathfrak{H})$, $\widetilde{x}_v(\mathfrak{T},\mathfrak{H})$ denotes the horizontal and vertical state observer vectors, the estimated output is defined as $\widetilde{y}(\mathfrak{T},\mathfrak{H})$ and $L(\mathfrak{T},\mathfrak{H})$ is the observer gain. According to the above discussion, the asynchronous state feedback controller for 2-D MJSs (1) is considered as follows

$$u(\mathfrak{T},\mathfrak{H}) = K(\epsilon_{\mathfrak{T},\mathfrak{H}})\widetilde{x}(\mathfrak{T},\mathfrak{H}) \tag{4}$$

where the $K(\epsilon_{\mathfrak{T},\mathfrak{H}})$ represents the feedback gain to be designed. The correlative transition probability matrix is defined as $\Psi = [\psi_{mq}]$ and the random variable $\epsilon_{\mathfrak{T},\mathfrak{H}}$ takes on values within the finite set $\mathbb{S} = \{1,2,3,\ldots,c\}$. Then, the transitions of the $\epsilon_{\mathfrak{T},\mathfrak{H}}$ is associated with the $r_{\mathfrak{T},\mathfrak{H}}$ through the following conditional probability:

$$\text{Pr}\,(\epsilon_{\mathfrak{T},\mathfrak{H}} = q \mid r_{\mathfrak{T},\mathfrak{H}} = m) = \psi_{mq} \tag{5}$$

where $\psi_{mq} \geq 0$ for and $\sum_{q=1}^{c} \psi_{mq} = 1$ for all $m \in \mathbb{M}, q \in \mathbb{S}$.

Remark 1. The hidden Markov model is a general framework for synchronous, asynchronous, and mode-independent controllers. The controller has different working modes depending on the situation in $\mathbb{M}$ and $\mathbb{S}$. If $\mathbb{M} = \mathbb{S}$ and $\psi_{mq} = 1$ for $m = q$, the controller (4) is transformed into a synchronous controller and if $\mathbb{S} = \{1\}$, the controller is a mode-independent one. As a result, the conclusions drawn in this paper can be extended to encompass both synchronous scenarios and mode-independent.

C. Problem statement

The estimated error is defined as $e(\mathfrak{T},\mathfrak{H})$ and $e(\mathfrak{T},\mathfrak{H}) = x(\mathfrak{T},\mathfrak{H}) - \widetilde{x}(\mathfrak{T},\mathfrak{H})$. From the system (1) and (3), it is described by the following 2-D MJSs. In the follows, let's simplify some of the symbols and we define $r_{\mathfrak{T},\mathfrak{H}} = m$, $\epsilon_{\mathfrak{T},\mathfrak{H}} = q$ respectively. For instance, A_q represents $A(\epsilon_{\mathfrak{T},\mathfrak{H}})$ and A_m represents $A(r_{\mathfrak{T},\mathfrak{H}})$.

$$\begin{aligned} \overline{e}(\mathfrak{T},\mathfrak{H}) = &[A_m - A_q + (B_m - B_q)K_q]\,\widetilde{x}(\mathfrak{T},\mathfrak{H}) \\ &+ (A_m - L_q C_m)\,e(\mathfrak{T},\mathfrak{H}) + D_m\omega(\mathfrak{T},\mathfrak{H}) \end{aligned} \tag{6}$$

where

$$\overline{e}(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} e_h(\mathfrak{T}+1,\mathfrak{H}) \\ e_v(\mathfrak{T},\mathfrak{H}+1) \end{bmatrix}, e(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} e_h(\mathfrak{T},\mathfrak{H}) \\ e_v(\mathfrak{T},\mathfrak{H}) \end{bmatrix}$$

Then, the closed-loop 2-D MJSs is represented by

$$\begin{cases} \tilde{\mathscr{R}}(\mathfrak{T},\mathfrak{H}) = A_{mq}\widetilde{x}(\mathfrak{T},\mathfrak{H}) + L_q C_m e(\mathfrak{T},\mathfrak{H}) \\ \overline{e}(\mathfrak{T},\mathfrak{H}) = C_{mq}\widetilde{x}(\mathfrak{T},\mathfrak{H}) + D_{mq}e(\mathfrak{T},\mathfrak{H}) + D_m\omega(\mathfrak{T},\mathfrak{H}) \\ z(\mathfrak{T},\mathfrak{H}) = E_{mq}\widetilde{x}(\mathfrak{T},\mathfrak{H}) + E_m e(\mathfrak{T},\mathfrak{H}) + G_m\omega(\mathfrak{T},\mathfrak{H}) \end{cases} \tag{7}$$

where

$$\begin{aligned} A_{mq} &\triangleq A_q + B_q K_q \\ D_{mq} &\triangleq A_m - L_q C_m \\ E_{mq} &\triangleq E_m + F_m K_q \\ C_{mq} &\triangleq A_m - A_q + B_m K_q - B_q K_q. \end{aligned}$$

The boundary condition $(\mathfrak{Y}_0, \mho_0)$ pertaining to system (1) is described as follows:

$$\begin{cases} \mathfrak{Y}_0 = \{x_h(0,\mathfrak{H}), x_v(\mathfrak{T},0) \mid \mathfrak{T},\mathfrak{H} = 0,1,2,\cdots\} \\ \mho_0 = \{r_{0,\mathfrak{H}}, r_{\mathfrak{T},0} \mid \mathfrak{T},\mathfrak{H} = 0,1,2,\cdots\} \end{cases}$$

and $\mathfrak{Y}_0$ meets Assumption 1.

Assumption 1. [41] $\mathfrak{Y}_0$ satisfies

$$\lim_{\mathscr{T}\to\infty} E\left\{\sum_{\mathfrak{e}=0}^{\mathscr{T}} \| x_h(0,\mathfrak{e}) \|^2 + \| x_v(\mathfrak{e},0) \|^2\right\} < \infty. \tag{8}$$

Definition 1. [41] The 2-D MJSs(7) with $\omega(\mathfrak{T},\mathfrak{H}) \equiv 0$ is thought to be AMMS if

$$\lim_{\mathfrak{T},\mathfrak{H}\to\infty} E\left\{\| x(\mathfrak{T},\mathfrak{H}) \|^2\right\} = 0 \quad (9)$$

for any boundary standard $(\mathfrak{Y}_0, \mho_0)$.

Definition 2. [41] The system (7) qualifies as an AMSS with an H_∞ level of disturbance attenuation γ provided that the following boundary condition is satisfied:

$$\sum_{\mathfrak{T}=0}^{\infty}\sum_{\mathfrak{H}=0}^{\infty} E\left\{\|x(\mathfrak{T},\mathfrak{H})\|^2\right\} < \gamma^2 \sum_{\ell=0}^{\infty}\sum_{\hbar=0}^{\infty} \|\omega(\mathfrak{T},\mathfrak{H})\|^2 \quad (10)$$

holds all non-zero values $\omega(\mathfrak{T},\mathfrak{H}) \in l_2\{[0,\infty),[0,\infty)\}$

Lemma 1. [42] For any real matrix $A = A^T$, the follow LMI are eauivvalent

(1) $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^\top & A_{22} \end{bmatrix} < 0$

(2) $A_{11} < 0$ and $A_{22} - A_{12}^\top A_{11}^{-1} A_{12} < 0$

(3) $A_{22} < 0$ and $A_{11} - A_{12} A_{22}^{-1} A_{12}^\top < 0$

Lemma 2. [42] For given matrices $X = X^\top$, Y, and Z with appropriate dimensionst

$$X + YS(t)Z + Z^\top S^\top(t) Y^\top < 0$$

holds for all $S(t)$ satisfying $S^\top(t)S(t) \le I$ if and only if there exists a scalar ς such that

$$X + \varsigma^{-1} YY^\top + \varsigma Z^\top Z < 0$$

III. Main results

In this particular section, the main aim addresses the problem of designing an asynchronous observer-based controller for 2-D MJSs (7). The 2-D MJSs is proved to be AMSS through a sufficient condition:

Theorem 1. For scalar $\gamma > 0$, matrices Z_q and ψ_q satisfying $C_m Z_q = \psi_q C_m$, consider the system (7) on the basis of Assumption 1, if there have symmetric matrices $\bar{P}_m \triangleq diag\{\bar{P}_{mh}, \bar{P}_{mv}\}$, $\bar{Q}_m \triangleq diag\{\bar{Q}_{mh}, \bar{Q}_{mv}\}$ with $\bar{P}_{mh} > 0$, $\bar{P}_{mv} > 0$, $\bar{Q}_{mh} > 0$, $\bar{Q}_{mv} > 0$ for $\forall m \in \mathbb{M}$, $\bar{M}_{mq} > 0$, $\bar{N}_{mq} > 0$ for $\forall m \in \mathbb{M}$ and $\forall q \in \mathbb{S}$, and matrices $\bar{K}_q$, $\bar{L}_q$ such that the condition (11) and (12) hold for $\forall m \in \mathbb{M}$ and $\forall q \in \mathbb{S}$.

$$\begin{bmatrix} -\bar{P}_m & 0 & \bar{P}_m\Gamma_m & 0 \\ 0 & -\bar{Q}_m & 0 & \bar{Q}_m\Gamma_m \\ * & * & -\mathscr{P}_m & 0 \\ * & * & * & -\mathscr{Q}_m \end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} \Upsilon_1 & * & * & * & * & * \\ 0 & \Upsilon_2 & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{E}_{mq} & E_m Z_q & G_m & -I & * & * \\ \mathscr{M}_{mq}\bar{A}_{mq} & \mathscr{M}_{mq}\bar{L}_q C_m & 0 & 0 & \Upsilon_4 & * \\ \mathscr{M}_{mq}\bar{C}_{mq} & \mathscr{M}_{mq}\bar{D}_{mq} & \mathscr{M}_{mq} D_m & 0 & 0 & \Upsilon_5 \end{bmatrix} < 0 \quad (12)$$

where

$$\begin{aligned} \bar{A}_{mq} &= A_q Z_q + B_q \bar{K}_q \\ \bar{C}_{mq} &= A_m Z_q - A_q Z_q + B_m \bar{K}_q - B_q \bar{K}_q \\ \bar{D}_{mq} &= A_m Z_q - \bar{L}_q C_m, \quad \bar{E}_{mq} = E_m Z_q + F_m \bar{K}_q \\ \Upsilon_1 &= \bar{M}_{mq} - Z_q - Z_q^T, \quad \Upsilon_2 = \bar{N}_{mq} - Z_q - Z_q^T \\ \Upsilon_4 &= diag\{-\bar{P}_1, -\bar{P}_2, \cdots, -\bar{P}_s\} \\ \Upsilon_5 &= diag\{-\bar{Q}_1, -\bar{Q}_2, \cdots, -\bar{Q}_s\} \\ \mathscr{M}_{mq} &= [\sqrt{\pi_{m1}} I \quad \sqrt{\pi_{m2}} I \quad \cdots \quad \sqrt{\pi_{ms}} I] \\ \Gamma_m &= \left[\sqrt{\psi_{m1}} I \quad \sqrt{\psi_{m2}} I \quad \cdots \quad \sqrt{\psi_{mc}} I\right] \\ \mathscr{Q}_m &= diag\{\bar{N}_{m1}, \bar{N}_{m2}, \cdots \bar{N}_{mc}\} \\ \mathscr{P}_m &= diag\{\bar{M}_{m1}, \bar{M}_{m2}, \cdots \bar{M}_{mc}\} \end{aligned}$$

Moreover, the controller and observer gains can be represented as the following:

$$K_q = \bar{K}_q Z_q^{-1}, \quad L_q = \bar{L}_q \psi_q^{-1} \quad (13)$$

Proof: The Lyapunov function can be defined as

$$V(\mathfrak{T},\mathfrak{H}) = V_1(\mathfrak{T},\mathfrak{H}) + V_2(\mathfrak{T},\mathfrak{H}) \quad (14)$$

with

$$\begin{aligned} V_1(\mathfrak{T},\mathfrak{H}) &= \widetilde{x}^T(\mathfrak{T},\mathfrak{H}) P_m \widetilde{x}(\mathfrak{T},\mathfrak{H}) \\ V_2(\mathfrak{T},\mathfrak{H}) &= e^T(\mathfrak{T},\mathfrak{H}) Q_m e(\mathfrak{T},\mathfrak{H}). \end{aligned}$$

Then, one can obtain that

$$\begin{aligned} E\{\triangle V_1(\mathfrak{T},\mathfrak{H})\} &= \textstyle\sum_{q=1}^c \psi_{mq} \tilde{\mathscr{R}}^T(\mathfrak{T},\mathfrak{H}) \Xi_{1m} \tilde{\mathscr{R}}(\mathfrak{T},\mathfrak{H}) \\ &\quad - \widetilde{x}^T(\mathfrak{T},\mathfrak{H}) P_m \widetilde{x}(\mathfrak{T},\mathfrak{H}) \\ &= \zeta^T(\mathfrak{T},\mathfrak{H}) \left(\textstyle\sum_{q=1}^c \psi_{mq} \mathcal{X}_{mq}^T \Xi_{1m} \mathcal{X}_{mq}\right) \zeta(\mathfrak{T},\mathfrak{H}) \\ &\quad - \widetilde{x}^T(\mathfrak{T},\mathfrak{H}) P_m \widetilde{x}(\mathfrak{T},\mathfrak{H}) \end{aligned}$$

where the variables $\mathcal{X}_{mq} = \begin{bmatrix} A_{mq} & L_q C_m & 0 \end{bmatrix}$ and $\zeta(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} \widetilde{x}^T(\mathfrak{T},\mathfrak{H}) & e^T(\mathfrak{T},\mathfrak{H}) & \omega^T(\mathfrak{T},\mathfrak{H}) \end{bmatrix}$.

Similarly, it is provable that

$$\begin{aligned} &E\{\triangle V_2(\mathfrak{T},\mathfrak{H})\} \\ &= \zeta^T(\mathfrak{T},\mathfrak{H}) \left(\textstyle\sum_{q=1}^c \psi_{mq} \mathcal{Y}_{mq}^T \Xi_{2m} \mathcal{Y}_{mq}\right) \zeta(\mathfrak{T},\mathfrak{H}) \\ &\quad - e^T(\mathfrak{T},\mathfrak{H}) Q_m e(\mathfrak{T},\mathfrak{H}) \end{aligned}$$

where $\mathcal{Y}_{mq} = \begin{bmatrix} C_{mq} & D_{mq} & D_m \end{bmatrix}$.

Consider the case of $\omega(\mathfrak{T},\mathfrak{H}) \equiv 0$, by using Schur complement Lemma 1 and Lemma 2 to (20), one can have

$$\Phi_{mq} - diag\{M_{mq}, N_{mq}\} < 0$$

where

$$\Phi_{mq} = \begin{bmatrix} A_{mq} & L_q C_m \\ C_{mq} & D_{mq} \end{bmatrix}^T diag\{\Xi_{1m}, \Xi_{2m}\} \begin{bmatrix} A_{mq} & L_q C_m \\ C_{mq} & D_{mq} \end{bmatrix}.$$

one can infer that

$$\triangle V_1(\mathfrak{T},\mathfrak{H}) = \widetilde{x}^T(\mathfrak{T},\mathfrak{H}) \left[\sum_{q=1}^c \psi_{mq} \mathfrak{A}^T \Xi_{1m} \mathfrak{A} - P_m\right] \widetilde{x}(\mathfrak{T},\mathfrak{H})$$

$$\triangle V_2(\mathfrak{T},\mathfrak{H}) = e^T(\mathfrak{T},\mathfrak{H}) \left[\sum_{q=1}^c \psi_{mq} \mathfrak{B}^T \Xi_{2m} \mathfrak{B} - Q_m\right] e(\mathfrak{T},\mathfrak{H})$$

$$\begin{aligned} E\{\triangle V(\mathfrak{T},\mathfrak{H})\} &= \xi^T(\mathfrak{T},\mathfrak{H}) \textstyle\sum_{q=1}^c \psi_{mq} \Phi_{mq} \xi(\mathfrak{T},\mathfrak{H}) \\ &\quad - \xi^T(\mathfrak{T},\mathfrak{H})\, diag\{P_m, Q_m\}\, \xi(\mathfrak{T},\mathfrak{H}) \\ &< 0 \end{aligned}$$

where

$$\xi(\mathfrak{T},\mathfrak{H}) = \begin{bmatrix} \widetilde{x}(\mathfrak{T},\mathfrak{H}) \\ e(\mathfrak{T},\mathfrak{H}) \end{bmatrix}, \mathfrak{A} = \begin{bmatrix} A_{mq} & L_q C_m \end{bmatrix}, \mathfrak{B} = \begin{bmatrix} C_{mq} & D_{mq} \end{bmatrix}.$$

Thus, one can get that

$$E\{\triangle V_1(\mathfrak{T},\mathfrak{H})\} \le -\varpi E\left\{\|\widetilde{x}(\mathfrak{T},\mathfrak{H})\|^2\right\} \quad (15)$$

where $\varpi = \lambda_{min}\left\{\sum_{q=1}^{c}\psi_{mq}\mathfrak{A}^T\Xi_{1m}\mathfrak{A} - P_m\right\}$, summing up on the both sides of (15), one can get

$$E\left\{\sum_{\mathfrak{T}=0}^{\mu_1}\sum_{\mathfrak{H}=0}^{\mu_2}\|\widetilde{x}(\mathfrak{T},\mathfrak{H})\|^2\right\}$$
$$\leq -\frac{1}{\varpi}E\left\{\sum_{\mathfrak{T}=0}^{\mu_1}\sum_{\mathfrak{H}=0}^{\mu_2}\triangle V_1(\mathfrak{T},\mathfrak{H})\right\}$$
$$\leq \frac{1}{\varpi}E\left\{\sum_{\mathfrak{T}=0}^{\mu_1}\widetilde{x}_h^\top(\mathfrak{T},0)P_{r(0,\mathfrak{H})}^h\widetilde{x}_h^\top(\mathfrak{T},\mathfrak{H},0)\right.$$
$$\left.+\sum_{\mathfrak{H}=0}^{\mu_2}\widetilde{x}_v^\top(0,\mathfrak{H})P_{r(\mathfrak{H},0)}^v\widetilde{x}_v^\top(0,\mathfrak{H})\right\}.$$

Let μ_1 and μ_2 tend to ∞ and ϑ_1 serve as the greatest eigenvalue of $P_{r(0,\mathfrak{H})}^h$ and $P_{r(\mathfrak{T},0)}^v$, one can get that

$$E\left\{\sum_{\mathfrak{T}=0}^{\mu_1}\sum_{\mathfrak{H}=0}^{\mu_2}\|\widetilde{x}(\mathfrak{T},\mathfrak{H})\|^2\right\}$$
$$\leq \frac{\vartheta_1}{\varpi}\sum_{t=0}^{\infty}\left(\|\widetilde{x}_h^\top(0,t)\|^2+\|\widetilde{x}_v^\top(0,t)\|^2\right)$$
$$<\infty.$$

Similarly, $E\{\triangle V_2(\mathfrak{T},\mathfrak{H})\} \leq -\varrho E\left\{\|e(\mathfrak{T},\mathfrak{H})\|^2\right\}$, which indicates that (9) holds. Therefore, the system (7) is AMMS.

Defining

$$\Im \triangleq \sum_{\mathfrak{T}=0}^{\infty}\sum_{\mathfrak{H}=0}^{\infty}E\left\{\|z(\mathfrak{T},\mathfrak{H})\|^2-\gamma^2\|\omega(\mathfrak{T},\mathfrak{H})\|^2\right\} \tag{16}$$

with

$$z^T(\mathfrak{T},\mathfrak{H})z(\mathfrak{T},\mathfrak{H}) = \zeta^T(\mathfrak{T},\mathfrak{H})\left(\sum_{q=1}^{c}\psi_{mq}\mathcal{Z}_{mq}^T\mathcal{Z}_{mq}\right)\zeta(\mathfrak{T},\mathfrak{H})$$
$$\mathcal{Z}_{mq} = \begin{bmatrix}E_{mq} & E_m & G_m\end{bmatrix}.$$

Due to the condition (19), it can be concluded that

$$\sum_{q=1}^{c}\psi_{mq}\left(diag\left\{M_{mq},N_{mq},\gamma^2 I\right\}\right) \leq diag\left\{P_m,Q_m,\gamma^2 I\right\}$$

then, letting $\triangle V(\mathfrak{T},\mathfrak{H}) \triangleq \triangle V_1(\mathfrak{T},\mathfrak{H})+\triangle V_2(\mathfrak{T},\mathfrak{H})$ and the expression of function $\Im$ can be rewritten in the following

$$\begin{aligned}\Im \leq \textstyle\sum_{\mathfrak{T}=0}^{\infty}\sum_{\mathfrak{H}=0}^{\infty}E\{&\|z(\mathfrak{T},\mathfrak{H})\|^2\\ &-\gamma^2\|\omega(\mathfrak{T},\mathfrak{H})\|^2+\triangle V(\mathfrak{T},\mathfrak{H})\}\end{aligned} \tag{17}$$

According Schur complement Lemma 1 and Lemma 2, we are able to infer from (20) that

$$\begin{aligned}&E\left\{\|z(\mathfrak{T},\mathfrak{H})\|^2-\gamma^2\|\omega(\mathfrak{T},\mathfrak{H})\|^2+\triangle V(\mathfrak{T},\mathfrak{H})\right\}\\ &=\zeta^T(\mathfrak{T},\mathfrak{H})\left[\sum_{q=1}^{c}\psi_{mq}\left(\mathcal{X}_{mq}^\top\Xi_{1m}\mathcal{X}_{mq}+\mathcal{Y}_{mq}^\top\Xi_{2m}\mathcal{Y}_{mq}\right.\right.\\ &\quad\left.\left.+\mathcal{Z}_{mq}^\top\mathcal{Z}_{mq}-diag\left\{M_{mq},N_{mq},\gamma^2 I\right\}\right)\right]\zeta(\mathfrak{T},\mathfrak{H})\\ &<0\end{aligned} \tag{18}$$

which, combining with (17), results in $\Im < 0$, i.e. (10) holds. The proof is completed. ■

The necessary prerequisites for ensuring the asymptotic stability of system (7) have been previously determined, but there are still some problems in solving the gain. The matrix of transition probability in inequality (20) is coupled. In order to solve this problem, Theorem 1 was processed, and the result is stated as follows:

Theorem 2. For scalar $\gamma > 0$, the 2-D MJSs (7) qualifies as AMSS with the desired H_∞ performance γ, if there are matrices $P_m \triangleq diag\{P_{mh},P_{mv}\}$, $Q_m \triangleq diag\{Q_{mh},Q_{mv}\}$ with $P_{mh}>0$, $P_{mv}>0$, $Q_{mh}>0$, $Q_{mv}>0$ for $\forall m\in\mathbb{M}$, $M_{mq}>0$, $N_{mq}>0$ for $\forall m\in\mathbb{M}$, $\forall q\in\mathbb{S}$ and K_q, L_q such that

$$\sum_{q=1}^{\mathbb{S}}\psi_{mq}\begin{bmatrix}M_{mq} & *\\ 0 & N_{mq}\end{bmatrix} < \begin{bmatrix}P_m & *\\ * & Q_m\end{bmatrix} \tag{19}$$

$$\begin{bmatrix}-M_{mq} & * & * & * & * & *\\ 0 & -N_{mq} & * & * & * & *\\ 0 & 0 & -\gamma^2 I & * & * & *\\ A_{mq} & L_q C_m & 0 & -\Xi_{1m}^{-1} & * & *\\ C_{mq} & D_{mq} & D_m & 0 & -\Xi_{2m}^{-1} & *\\ E_{mq} & E_m & G_m & 0 & 0 & -I\end{bmatrix} < 0 \tag{20}$$

where

$$\Xi_{1m} \triangleq \sum_{n=1}^{s}\pi_{mn}P_n, \Xi_{2m} \triangleq \sum_{n=1}^{s}\pi_{mn}Q_n \tag{21}$$

Proof: By useing the Schur complement, the inequality (19) can be reformulated as the following inequality:

$$\begin{bmatrix}-P_m & 0 & \Gamma_m & 0\\ 0 & -Q_m & 0 & \Gamma_m\\ * & * & -\mathscr{P}_m & 0\\ * & * & * & -\mathscr{Q}_m\end{bmatrix} < 0 \tag{22}$$

Then, pre-multiplying (22) with the transpose of $diag\left\{\bar{P}_m,\bar{Q}_m,I,\cdots,I\right\}$ and post-multiplying (22) with $diag\left\{\bar{P}_m,\bar{Q}_m,I,\cdots,I\right\}$, one has (11). Letting $\bar{P}_m = P_m^{-1}$, $\bar{Q}_m = Q_m^{-1}$, $\bar{M}_{mq} = M_{mq}^{-1}$, $\bar{N}_{mq} = N_{mq}^{-1}$ for $\forall m\in\mathbb{M}$ and $\forall q\in\mathbb{S}$, inequation (11) and (19) are equivalent.

The following task is to prove that (12) and (20) are equivalent. It can be obtained that

$$-Z_q^T\bar{M}_{mq}^{-1}Z_q \leq \bar{M}_{mq}-Z_q-Z_q^T \tag{23}$$

Similarly

$$-Z_q^T\bar{N}_{mq}^{-1}Z_q \leq \bar{N}_{mq}-Z_q-Z_q^T \tag{24}$$

which, combining with (12), results in

$$\begin{bmatrix}-Z_q^T\bar{M}_{mq}^{-1}Z_q & * & * & * & * & *\\ 0 & -Z_q^T\bar{N}_{mq}^{-1}Z_q & * & * & * & *\\ 0 & 0 & -\gamma^2 I & * & * & *\\ \bar{E}_{mq} & E_m Z_q & G_m & -I & * & *\\ \mathscr{M}_{mq}\bar{A}_{mq} & \mathscr{M}_{mq}\bar{L}_q C_m & 0 & 0 & \Upsilon_4 & *\\ \mathscr{M}_{mq}\bar{C}_{mq} & \mathscr{M}_{mq}\bar{D}_{mq} & \mathscr{M}_{mq}D_m & 0 & 0 & \Upsilon_5\end{bmatrix} < 0 \tag{25}$$

Letting $\mathscr{R} = diag\left\{\left(Z_q^T\right)^{-1},\left(Z_q^T\right)^{-1},I,I,\cdots I\right\}$. Pre-multiplying (20) by $\mathscr{R}$ and post-multiplying (20) by $\mathscr{R}^T$,

TABLE I
PARAMETERS OF SYSTEM

m	1	2	3
a_m	1.25	1.58	0.86
b_m	1.0	0.5	1.0

and with in $K_q = \bar{K}_q Z_q^{-1}$, $L_q = \bar{L}_q \psi_q^{-1}$ mind, one has

$$\begin{bmatrix} -M_{mq} & * & * & * & * & * \\ 0 & -N_{mq} & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ E_{mq} & E_m & G_m & -I & * & * \\ \mathscr{M}_{mq}A_{mq} & \mathscr{M}_{mq}L_q C_m & 0 & 0 & \Upsilon_4 & * \\ \mathscr{M}_{mq}C_{mq} & \mathscr{M}_{mq}D_{mq} & \mathscr{M}_{mq}D_m & 0 & 0 & \Upsilon_5 \end{bmatrix} \quad (26)$$

Let $\bar{P}_m = P_m^{-1}$, $\bar{Q}_m = Q_m^{-1}$, $\bar{M}_{mq} = M_{mq}^{-1}$, $\bar{N}_{mq} = N_{mq}^{-1}$ for $\forall m \in \mathbb{M}$ and $\forall q \in \mathbb{S}$, condition (20) is equivalent to condition (12). This proof is accomplished. ■

IV. Example

In this part, consider the following partial differential equations with Markov jump parameters to describe the thermal processes in a heat exchanger.

$$\frac{\partial S(\mathfrak{v},\mathfrak{z})}{\partial \mathfrak{v}} = -\frac{\partial S(\mathfrak{v},\mathfrak{z})}{\partial \mathfrak{z}} - \partial_{r(\mathfrak{v},\mathfrak{z})} S(\mathfrak{v},\mathfrak{z}) + b_{r(\mathfrak{v},\mathfrak{z})} u(\mathfrak{v},\mathfrak{z}) \quad (27)$$

where $u(\mathfrak{v},\mathfrak{z})$ and $S(\mathfrak{v},\mathfrak{z})$ are, respectively, the control input and the temperature at space $\mathfrak{v} \in [0, \mathfrak{v}_f]$ and time $\mathfrak{z} \in [0, \infty)$. $a_{r(\mathfrak{v},\mathfrak{z})}$, $b_{r(\mathfrak{v},\mathfrak{z})}$ are ture coefficients, which $r(\mathfrak{v},\mathfrak{z})$ is the Markov paramete. Similar as [41], we define

$$S(\mathfrak{T},\mathfrak{H}+1) = \left(1 - \frac{\triangle\mathfrak{z}}{\triangle\mathfrak{v}} - a_{r_{\mathfrak{T},\mathfrak{H}}} \triangle\mathfrak{z}\right) S(\mathfrak{T},\mathfrak{H}) \quad (28)$$

$$+\frac{\triangle\mathfrak{z}}{\triangle\mathfrak{v}} S(\mathfrak{T}-1,\mathfrak{H}) + b_{r_{\mathfrak{T},\mathfrak{H}}} u(\mathfrak{T},\mathfrak{H})$$

then the (28) can be written into the form of (1) with the following parameters:

$$A(r_{\mathfrak{T},\mathfrak{H}}) = \begin{bmatrix} 0 & 1 \\ 1 & 1 - \frac{\triangle\mathfrak{z}}{\triangle\mathfrak{v}} - a_{r_{\mathfrak{T},\mathfrak{H}}} \triangle\mathfrak{z} \end{bmatrix}$$

$$B(r_{\mathfrak{T},\mathfrak{H}}) = \begin{bmatrix} 0 \\ b_{r_{\mathfrak{T},\mathfrak{H}}} \triangle\mathfrak{z} \end{bmatrix}$$

For this instance, we presume that the 2-D MJSs possesses three operational modes. Let $\triangle\mathfrak{v} = 0.17$ and $\triangle\mathfrak{z} = 0.05$, the table of system parameters is presented in Table I.

On the other hand, the switching signal $r_{\mathfrak{T},\mathfrak{H}}$ and $\epsilon_{\mathfrak{T},\mathfrak{H}}$ conform to the probability matrix $\Pi = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.36 & 0.54 & 0.1 \\ 0.2 & 0.05 & 0.75 \end{bmatrix}$ and $\Psi = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$. In this example, the other parameters of the system can be considered as

$$C_1 = \begin{bmatrix} 0 & 1 \\ 0.6 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 \\ 0.6 & 0.3 \end{bmatrix}, C_3 = \begin{bmatrix} 0 & 1 \\ 0.6 & 0.3 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, F_3 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}$$

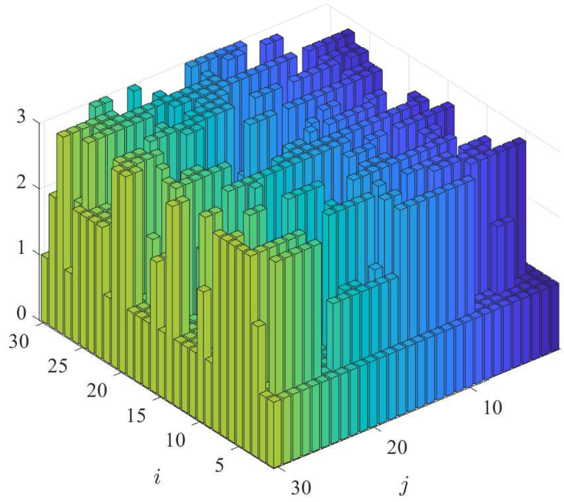

Fig. 1. Modes of 2-D MJSs.

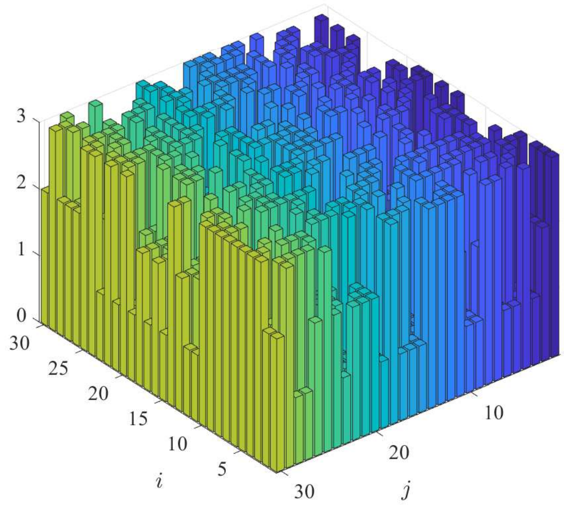

Fig. 2. Modes of observer-based controller.

$$D_m = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, G_m = \begin{bmatrix} 2.05 \\ 0.1 \end{bmatrix}, E_m = \begin{bmatrix} 1 & 0 \\ 1 & 0.6 \end{bmatrix}, m = 1, 2, 3$$

Subsequently, we postulate that the boundary condition of the system and the input of the disturbance are regarded as

$$x_h(0,\mathfrak{H}) = \begin{cases} 0.6 & 0 \le \mathfrak{H} \le 10 \\ 0 & otherwise \end{cases}$$

$$x_v(\mathfrak{T},0) = \begin{cases} 0.3 & 0 \le \mathfrak{T} \le 10 \\ 0 & otherwise \end{cases}$$

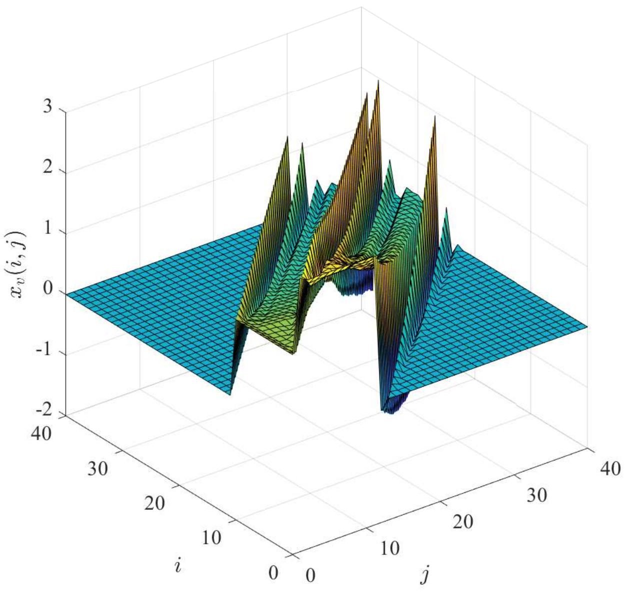

Fig. 3. Open-loop vertical state.

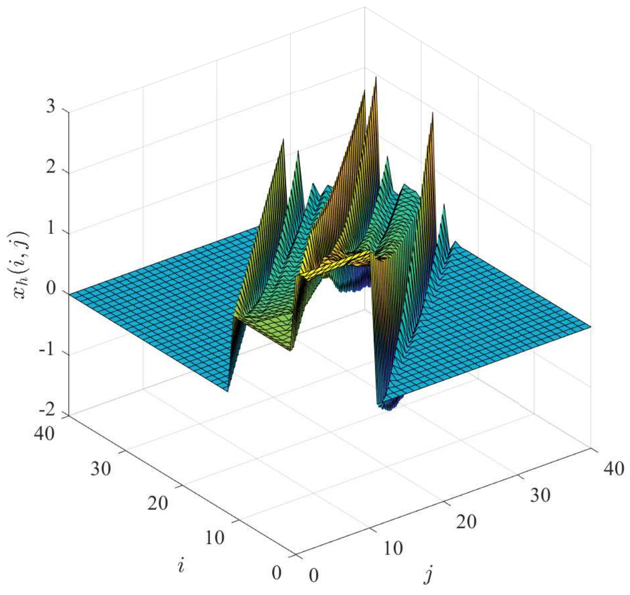

Fig. 4. Open-loop horizontal state.

$$\omega(\mathfrak{T},\mathfrak{H}) = \begin{cases} 0.2 & 2 \leq \mathfrak{T},\mathfrak{H} \leq 10 \\ 0 & otherwise \end{cases}$$

Setting $\gamma = 4.454$, $Z_q = diag\{1,1\}, \psi_q = Z_q, q = 1,2,3$, by solving LMIs (12) and (13), the controller gains as $K_1 = \begin{bmatrix} -3.0697 & -1.2630 \end{bmatrix}$, $K_2 = \begin{bmatrix} -2.9870 & -1.3128 \end{bmatrix}$, $K_3 = \begin{bmatrix} -3.0025 & -1.2897 \end{bmatrix}$ and the observer gains can be obtained as follows:

$$L_1 = \begin{bmatrix} -0.4753 & 0.9428 \\ 1.3759 & -3.0418 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.0849 & 0.2570 \\ 0.5356 & -1.2523 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -0.0911 & 0.2473 \\ 0.3686 & -20.8021 \end{bmatrix}$$

Figure 1 and Figure 2 represent the mode of the system (1) and the mode of the observer based controller (3), which presents that the modes of communication between the controller and the system occur asynchronously. The open-loop state trajectories pertaining to the horizontal $x_v(\mathfrak{T},\mathfrak{H})$ and vertical $x_h(\mathfrak{T},\mathfrak{H})$ are graphically represented in Figure 3 and Figure 4, from which it is evident that the open-loop system's state trajectories become unstable in the absence of controller gains. Figure 5 and Figure 6 present the closed-loop state trajectories pertaining to the horizontal $x_h(\mathfrak{T},\mathfrak{H})$ and the vertical $x_v(\mathfrak{T},\mathfrak{H})$, which displays the state trajectories pertaining to the closed-loop system tends to stabilize after a period of volatility. These findings demonstrate that the controller designed is capable of stabilizing the open-loop system effectively, and the proposed asynchronous observer-based controller design method is feasible.

To demonstrate that the obtained theoretical results are suitable for 2-D MJSs under an asynchronous control mechanism, the following scheme is considered: the probability matrix $\Pi = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.36 & 0.54 & 0.1 \\ 0.2 & 0.05 & 0.75 \end{bmatrix}$ and $\Psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

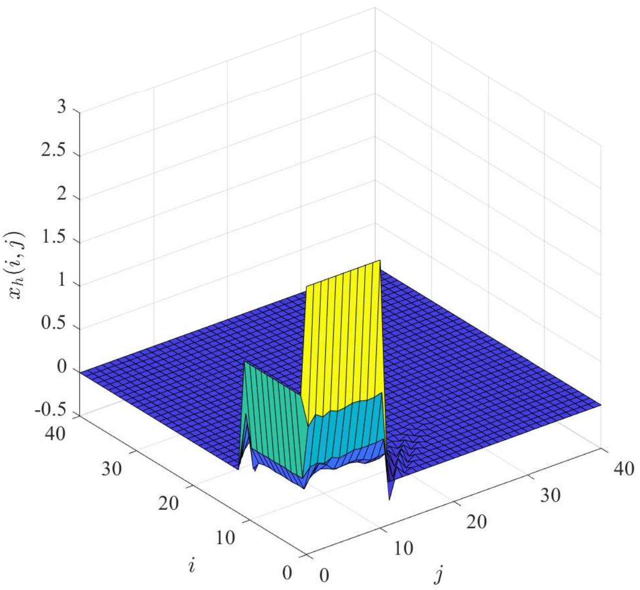

Fig. 5. Colsed-loop horizontal state.

Under the condition that other system parameters are the same, the control gains as $K_1 = \begin{bmatrix} -3.4286 & -0.8992 \end{bmatrix}$, $K_2 = \begin{bmatrix} -3.1691 & -1.2117 \end{bmatrix}$, $K_3 = \begin{bmatrix} -3.2032 & -1.1202 \end{bmatrix}$ and the observer gains can be obtained as follows:

$$L_1 = \begin{bmatrix} -0.0849 & 0.1281 \\ 0.5423 & -0.8391 \end{bmatrix}$$

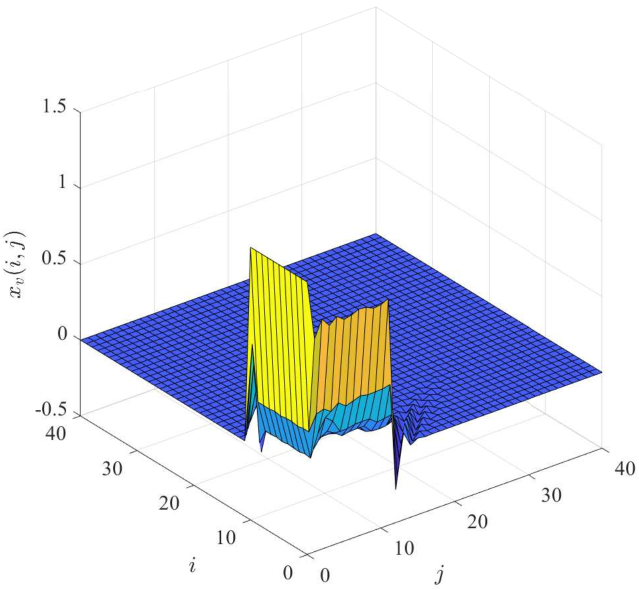

Fig. 6. Colsed-loop vertical state.

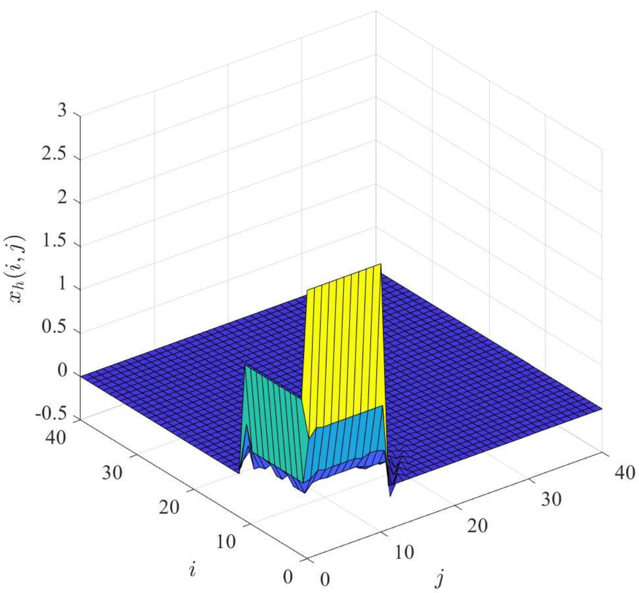

Fig. 7. Colsed-loop horizontal state of synchronous.

$$L_2 = \begin{bmatrix} -0.0482 & 0.0432 \\ 0.3642 & -0.7289 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -0.0569 & 0.0104 \\ 0.2995 & -0.5298 \end{bmatrix}$$

The responses of the state trajectories responses for the synchronous case are shown in Figure 7 and Figure 8. From Figure 5 to Figure 8, It is evident that both the asynchronous and synchronous control mechanisms function effectively.

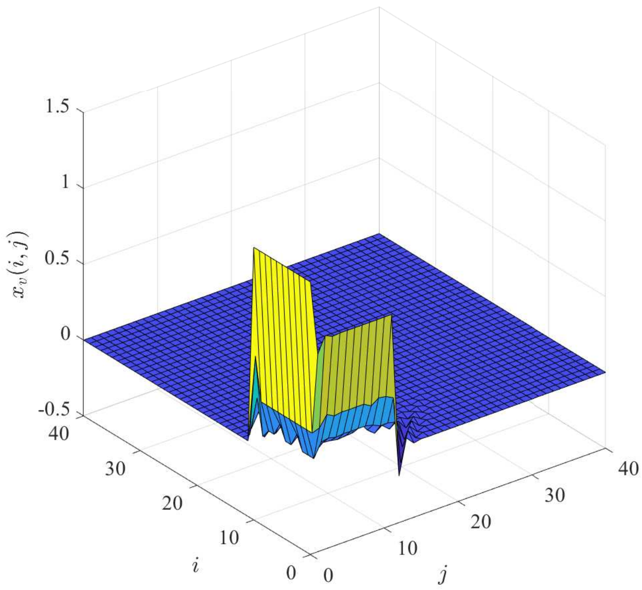

Fig. 8. Colsed-loop vertical state of synchronous.

V. Conclusion

In this work, the problem of asynchronous observer-based H_∞ control for Roesser model-based 2-D MJSs was studied. In order to address the asynchronous phenomenon of system mode and controller mode, an observer-based asynchronous controller was constructed. The asynchrony between the controller and the control plant was described by a hidden Markov process. Then, using method of the Lyapunov function, a sufficient criterion for achieving asymptotic stability in closed-loop systems, while maintaining a predefined level of H_∞ disturbance attenuation performance, was established. In addition, the correctness of the above method and the effectiveness of the designed controller were verified by a practical example of thermal processes.

In future work, we will focus on addressing the problem of asynchronous control for 2-D MJSs with general probabilities information where the transition probability information of the system and controller are partially known. It is well known that the transition probability information are precisely known, which is difficult in actual circumstances. Therefor, the asynchronous observer-based control of 2-D MJSs with general probabilities information deserves to be studied.

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