Benders Decomposition Approach on Adjustable Robust Counterpart Optimization Model for Multi-objective Supply Chain Problems in Sugar Distribution

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***Abstract*—This paper discusses the Benders decomposition approach on the multi-objective adjustable robust counterpart optimization model with polyhedral uncertainty set for sugar distribution supply chain problems. It focused on the optimization modeling of sugar distribution among producers, local food hubs, and consumers in sub-districts. This problem is considered a multi-objective mixed integer linear programming problem with two objective functions: to maximize demand fulfillment and minimize logistics costs. Uncertain data was collected from real-life scenarios and analyzed using the adjustable robust counterpart methodology with polyhedral uncertainty set assumption. The uncertain data consisted of the adjustable and non-adjustable variables. This research was carried out in northern Bandung City, Indonesia. The result shows that the evaluation of the two-stage supply chain optimization model with adjustable robust counterpart methodology is effectively solved using the benders decomposition approach.**

***Index Terms*—adjustable robust counterpart, benders decomposition, multi-objective optimization, supply chain.**

I. Introduction

IN this study, the main aim was to develop the Benders Decomposition Approach (BDA) for the multi-objective adjustable robust counterpart (ARC) optimization model with a polyhedral uncertainty set for supply chain problems, especially in the case of processed sugar distribution. This work includes numerical experiments to validate the result using R Software. The discussion focused on supply chain management issues, which, according to Irmansyah *et al.* [1], is essential to regularly meet consumer demand, ensure goods capacity, and maintain economic efficiency. The ARC methodology was adopted to overcome the problem of uncertainty in the multi-objective integer optimization model. This method was selected based on two kinds of variables: integer and continuous decision variables applied at the first and second stages, respectively. Multi-objective optimization modeling is used by Wihartiko *et al.* [2] to study the agricultural product price recommendation problem.

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Several studies have been conducted on ARC, each focusing on various problems. For example, Chaerani *et al.* [3] investigated the adjustable robust maximum flow problem with parametric ellipsoidal and polyhedral uncertainty sets. A study by Wei *et al.* [4] focused on the comprehensive Mixed Integer Linear Programming (MILP) model for the distribution of energy reserves using the ARC methodology. Cranmer *et al.* [5] conducted a study on household resource management with a profit-maximizing portfolio, using fundamental network models to track the wake effects through a series of wind farms. Chen *et al.* [6] minimizes operational cost for combined cooling, heating, and power-based microgrid (CCHP-MG) using a two-stage adjustable robust optimization. Ji *et al.* [7] focused on modern power systems by formulating a two-stage adjustable robust optimization model designed to address the uncertainties in photovoltaic outputs. Min *et al.* [8] concentrated on a robust two-stage omega portfolio optimization with cardinality constraints. Furthermore, Sun *et al.* [9] used a mixed integer programming method to analyze economic network design problems. Another study by Zhao *et al.* [10] focused on the robust optimization of mixed-load school bus routes using a multi-objective genetic algorithm. Chaerani *et al.* [11] also studied spatial land-use allocation problems and solved the problem as a robust optimization model using ellipsoidal and polyhedral uncertainty sets.

The BDA was considered a suitable method for solving ARC because it is similar to a two-stage robust optimization. According to Bisschop [12], BDA divided the problem into two parts, namely, linear or continuous and nonlinear or integer variables, which are challenging to solve. When applying BDA, the main concern is partitioning the variables into two sets, x and y, as continuous and integer variables. This optimization method effectively resolves issues with feasible subproblems. Previous studies have focused on developing mathematical methods for solving the robust counterpart (RC) multi-objective integer optimization model using the BDA approach. For example, Siddiqui *et al.* in [13] discusses how Benders decomposition can be used to solve mixed-integer robust optimization problems with interval uncertainty. Saito and Murota in [14] contribute to robust mixed integer programming by determining the benders decomposition approach. See also Karbowski [15] designed a large-scale MILP optimization model with both objective and convex constraint functions. Sha *et al.* [16] studied the MILP optimization model designed for distribution problems, implementing the Cutting Plane Method in the first stage. Meanwhile, Gong and Zhang [17] focused on the MILP optimization model containing inequalities, local branching, In-out variant method, and scenario-based aggregated cuts. In Liu *et al.* [18], a study on multi-objective mixed integer programming modeling for closed-loop supply chain network design is presented.

To claim the novelty, a literature search has been done for the last three years (2022-2024). By considering the keywords ”Benders Decomposition Approach,” ”Robust Optimization,” ”Mixed Integer Programming,” and ”Supply chains,” there are 57 articles as a result of literature searching. Based on the title, abstract, keywords, and paper content, only 15 articles are considered for determining the state of the art of the research, as presented in Table I.

TABLE I: Summary of the aspect covered in our article and existing articles.

Article	Robust Optimization	Uncertainty Set	MILP	Supply chains	Benders Decomposition
[19]	-	-	✓	-	✓
[20]	✓	Box	✓	✓	✓
[21]	✓	Stochastic	✓	✓	✓
[22]	✓	✓	✓	✓	✓
[23]	-	-	✓	✓	✓
[24]	✓	Interval	✓	✓	✓
[25]	✓	Interval	✓	✓	✓
[26]	✓	Stochastic	✓	✓	-
[27]	✓	Interval	✓	✓	-
[28]	✓	Interval	✓	✓	-
[29]	-	-	✓	✓	-
[30]	✓	Convex	✓	-	-
[31]	-	-	✓	✓	-
[32]	-	-	✓	✓	-
[33]	-	✓	✓	✓	-
Our Article	✓	Polyhedral	✓	✓	✓

Thus, in this paper, a new result is expected on how the ARC model and the numerical experiments are obtained to show the use of the BDA in addressing supply chain problems in sugar distribution. The uncertain selling price data is assumed to be polyhedral uncertainty. In addition, two objective functions were analyzed to maximize demand fulfillment and minimize logistics costs. The case of numerical experiment on sugar distribution was selected due to the significant influence as an agricultural processed product.

Some motivations were added to show the importance of the supply chain system, mainly agricultural processed goods. For instance, Gao *et al.* [34] and Liu *et al.* [35] focused on green agri-food blockchain technology. Winarno *et al.* [36] conducted a study focused on food hubs and brief supply chains.

According to Harrington *et al.* [37], establishing a food hub was considered the best and most efficient way to ensure smooth product delivery. Developing a food hub is essential for farmers aiming to expand their respective businesses by providing a comprehensive range of production, distribution, and marketing services. Matson *et al.* [38], [39] stated that a food hub plays a critical role in establishing regional and local connections between producers, including farmers and ranchers, as well as consumers, namely hospitals, schools, restaurants, etc.

Food hubs expand market opportunities on a larger scale by serving as a centralized pick-up location for distributors and consumers. Furthermore, it provides job opportunities and benefits to consumers and the general public. Mittal *et al.* [40] defined Local Food Hubs (LFH) as food distribution centers operating locally. In the case of Bandung City, LFH was constructed in every district (Perdana *et al.* [41]) or any area where it can function optimally. LFH served as a significant distribution center between the producer and consumer zones on a local scale.

Therefore, this study aimed to identify the most suitable location for LFH development in North Bandung and analyze how the supply chain is connected with product distribution from the producers to LFH and consumers. The numerical experiments relied on secondary data obtained in 2022 from various sources. The districts in the northern part of Bandung City, including Andir, Bandung Wetan, Cibeunying Kaler, Cibeunying Kidul, Cicendo, Cidadap, Coblong, Sukajadi, Sukasari, and Sumur Bandung, served as the locations for conducting these experiments. The Andir Market, the largest wholesale market for granulated sugar suppliers and sellers in the Andir District, influenced the choice of this area. Furthermore, this study is also an extension of the investigation on applying robust optimization to resolve supply chain problems (Chaerani *et al.* [42]).

The study is organized as follows: Section 2 discusses materials and methods. Section 3 shows the results, followed by the numerical experiment results and conclusions in Sections 4 and 5, respectively.

II. Materials and Methods

A. *Optimization Model Formulation for Supply Chain Problems*

The optimization model formulation for supply chain problems was adapted from the study by Perdana *et al.* [41] and Irmansyah *et al.* [1]. It focused on distributing agricultural processed products among producers, LFH, and consumers across all districts in Bandung. The model adopted several sets, including demand/consumer zone, LFH, production zones/producer, and commodity (types of processed agricultural products) denoted by $I, J, K,$ and C, respectively. This MILP, formulated in equations (1) to (10), addressed the complexities of the supply chain system. The parameters used in the model are d_{ci} which represents the product request for c in district i (tons/day), f_{ck} denotes the production capacity for product c in the district producer k (tons/day), v_{ci} is the selling price of product c in district i (Rp/tons), κ depicts the development costs of LFH (Rp/building), and q signifies the cost of handling health protocols (Rp/tons).

$$\max \sum_{c\in C}\sum_{i\in I}\sum_{j\in J} v_{ci}w_{cji}, \quad (1)$$

$$\min \left\{ \kappa \sum_{j\in J} x_j + q \sum_{c\in C}\sum_{j\in J} p_{cj} \right\}, \quad (2)$$

$$s.t. \sum_{k\in K} f_{ck}y_{ckj} = p_{cj}, \forall c \in C, j \in J, \quad (3)$$

$$\sum_{i\in I} d_{ci}w_{cji} = p_{cj}, \forall c \in C, j \in J, \quad (4)$$

$$\sum_{j\in J} y_{ckj} \le 1, \forall c \in C, k \in K, \quad (5)$$

$$\sum_{j\in J} w_{cji} \le 1, \forall c \in C, i \in I, \quad (6)$$

$$y_{ckj} \le x_j, \forall c \in C, k \in K, j \in J, \quad (7)$$

$$w_{cji} \le x_j, \forall c \in C, j \in J, i \in I, \quad (8)$$

$$p_{cj} = (p_1, p_2, ..., p_i) \ge 0, \quad (9)$$

$$y_{ckj}, w_{cji}, x_j \in \{0, 1\}. \quad (10)$$

The decision variables used were defined as x_j with the following descriptions. If the LFH was built in district j, then $x_j = 1$ and $x_j = 0$ if otherwise, for all $j \in J$. The LFH capacity for products c in district j (tons/day) is denoted by p_{cj}. Next, assume that $y_{ckj} = 1$ if the whole product c in the district producer k was sent to LFH in the district j and y_{ckj} = 0 if otherwise, for all $c \in C, k \in K, j \in J$. Also, suppose that $w_{cji} = 1$ if the entire product demand LFH fulfilled c in the district i in the district j and $w_{cji} = 0$ if otherwise, for all $c \in C, j \in J, i \in I$.

The complete optimization model of formulation (1) to (10) is a multi-objective optimization problem, with the first and second objective functions aimed to maximize the fulfilled demand and minimize logistics costs, respectively.

The model had eight constraints, and the first two ensured that the LFH capacity for product c was determined based on production demand. The third constraint prevented the production of product c sent from producers in district k to LFH in district j from exceeding the capacity. The fourth constraint ensured that the fulfillment of demand by LFH did not exceed the demand requested by district i. The fifth constraint guaranteed that no product c was sent to the red zone if the LFH was not built in the area. The sixth constraint guaranteed no demand was fulfilled, assuming the LFH was not built in the zone, while the last two constraints defined each decision variable.

B. Robust Optimization

The theory of Robust Optimization is discussed. Refers to Bental and Nemirovski [43] and Gorissen *et al.* [44], let $c \in R^n, b \in R^m$ and $A \in R^m \times R^n$ are the parameters of the linear programming problems as formulated ini (11).

$$\min_x \left\{ c^T x : Ax \le b \right\} \quad (11)$$

In case the data (c, A, b) are uncertain but are known in an uncertain set U, a version of uncertain (11) becomes a focus problem of Robust Optimization. Noted that the (11) is a family of problems, one for each realization $(c, A, b) \in U$:

$$\min_{x\in R^n} \left\{ c^T x : Ax \le b \right\}_{(c,A,b)\in U} \quad (12)$$

Refers to Gorissen *et al.* [44], the decision environment is assuemed such that:

1) When the value of the actual parameters are taken as the here-and-now-decision. Thus the entire decision vector x is to be fixed before knowing
2) In uncertain LP (12), note that the variables $(x_1, \ldots, x_n)$ is needed to be determined as the here-and-now-decision. Some uncertain parameters become known (”wait and see” decision) after the rest may be determined.
3) The data (c, A, b) is represented by a compact uncertainty set U.
4) The hard constraint is the inequality constraints $Ax \le b$. This means that all constraints must be satisfied whenever the uncertain parameters reside in U.

Next, with all the above assumptions, the uncertain family of the problems (12) is converted into the following single deterministic problem using a robust optimization approach. The result is called robust counterpart (RC) as in (13).

$$\pi^* = \min_{x\in R^n} \left\{ c^T x : Ax \le b, (c, A, b) \in U \right\} \quad (13)$$

Whenfor all realizations $(c, A, b) \in U$, x^* is feasible, thus vector x^* is called a robust optimal solution. The π^* is claimed to be the best value of the objective function.

Problem (13) can be written equivalently as a problem with a linear specific objective function t and uncertain constraints as follows in (14).

$$\begin{aligned} &\min\ t \\ &s.t\ \ c^T x - t \le 0, \\ &\quad\ \ a_i^T x - b_i \le 0, i = 1, \ldots, m, \\ &\quad\ \ x \ge 0 \\ &\quad\ \ \forall (c, A, b) \in U. \end{aligned} \quad (14)$$

From Gorissen *et al.* [44], the first way of handling robust linear optimization (RLO) is making the objective $c^T x$ as a certain function. There is no uncertainty in the objective, and the uncertainty appears in a constraint, as can be seen in (14). Secondly, make sure that the right-hand side b is specific. In case b is uncertain, define a new variable $x_{n+1}v = 1$ and convert the problem into (15).

$$\begin{aligned} &\min\ t \\ &s.t\ \ c^T x - t \le 0, \\ &\quad\ \ a_i^T x - b_i x_{n+1} \le 0, \\ &\quad\ \ x_{n+1} = 1, i = \ldots, m, \\ &\quad\ \ \forall (A, b) \in U. \end{aligned} \quad (15)$$

Third, to achieve robustness concerning U, the uncertainty set U can be formulated constraint-wise and must be a closed and convex set. This means that the convex hull of U, i.e., the smallest convex set that includes U, can replace the uncertainty set U.

To reformulate the uncertain problem into a tractable optimization problem, thus finding a suitable U becomes a challenge. Each constraint that involves uncertain data is reformulated since the robustness based on U can be formulated constraint-wise. Refers to Gorissen *et al.* [44], see the following constraint

$$a^T x - b \le 0, \forall (a, b) \in U, \quad (16)$$

where a is a vector in R^n and b is the representatives of a_i and b_i, also U_i represents U. Thus, assuming that a, b, and U are uncertain parameters, reformulate the parameters in factor $\zeta \in R^L$. Namely,

$$a = \bar{a} + Q\zeta, b = \bar{b} + q^T\zeta \tag{17}$$

where $\bar{a} \in R^n, Q \in R^{n\times L}, \bar{b} \in R$ and $q \in R^L$ and

$$U = \left\{ \begin{pmatrix} a = \bar{a} + Q\zeta \\ b = \bar{b} + q^T\zeta \end{pmatrix} \; : \; \zeta \in Z \right\} \tag{18}$$

where $Z \subset R^L$ is the uncertainty set for the primitive factors. The fixed vector $\bar{a}$ and the scalar $\bar{b}$ will be called nominal. Thus (16) can be written as (19) as follows.

$$(\bar{a}^T x - \bar{b}) + (Q^T x - q)^T\zeta \leq 0, \forall\zeta \in Z. \tag{19}$$

In the case of the use of a polyhedral uncertainty set, formulation (16) can be written in the system of linear inequalities as in (20).

$$(\bar{a} + P\zeta)^T x \leq b, \forall\zeta \; : \; d - D\zeta \geq 0. \tag{20}$$

This (20) is equivalent with

$$\bar{a}^T x + \max_{\zeta \,:\, d - D\zeta \geq 0} (P^T x)\zeta \leq b. \tag{21}$$

For this formulation, the robust counterpart formulation can be obtained by defining its dual formulation as follows. The primal problem (22)

$$\max \;\; \left\{(P^T x)\zeta \; : \; d - D\zeta \geq 0\right\} \tag{22}$$

is equivalent to its dual (23).

$$\min\left\{d^T y \; : \; D^T y = P^T x, y \geq 0\right\}. \tag{23}$$

Hence x satisfies (21) if and only if x satisfy

$$\bar{a}^T x + \min_{y}\left\{d^T y \; : \; D^T y = P^T x, y \geq 0\right\} \leq b. \tag{24}$$

If a feasible y satisfy this constraint, i.e., that satisfies $D^T y = P^T x, y \geq 0$ then the minimum over y certainly satifsfy the constraint. Further, the minimum can be deleted. This means that x satisfies (21) if and only if there exists a y such that (x, y) satisfies

$$\bar{a}^T x + d^T y \leq b, D^T y = P^T x, y \geq 0. \tag{25}$$

Since it is clear that (25) is a system of linear (in) equalities, then it can be concluded that result is very tractable.

C. Adjustable robust counterpart (ARC) optimization

Yanıkoğlu *et al.* [45], introduced a special method for handling adjustable integer variables in the optimization problem of ARC. The method starts from the general RC problem, which was stated as follows.

$$\begin{aligned} \max_{x,y,z} \;\; & c(x,y,z), \\ s.t \;\; & A(\zeta)x + B(\zeta)y + C(\zeta)z \leq q, \\ & \forall\zeta \in Z. \end{aligned} \tag{26}$$

with $x \in R$ and $y \in Z^{n_2}$ representing the here-and-now variables, while $z \in Z^{n3}$ denotes a wait-and-see variable, $A(\zeta), B(\zeta)$ served as indeterminate the here-and-now variables coefficient matrices. However, the integer wait-and-see variable z had an indeterminate coefficient matrix $C(\zeta)$, enabling the method to address the problem of uncertainty in the wait-and-see integer variable coefficients. For simplicity, it was assumed that the indeterminate coefficient matrix was linear in ζ. Additionally, without omitting generalization, $c(x, y, z)$ was assumed to be a linearly indeterminate objective function.

To model ARC with integer variables, the set of uncertainties Z was initially divided into many m disjoint subsets $(Z_i, i = 1, 2, 3, ..., m)$ as stated in (27).

$$Z = \bigcup_{i=1^m} Z_i. \tag{27}$$

An additional integer variable $z_i \in Z^{n_3}(i = 1, \ldots, m)$ which models the decision in Z_i was introduced. Next, the indeterminate constraint and objective function stated in equation (27) were reformulated for each z_i and the set of uncertainty Z_i as stated in (28).

$$\begin{aligned} \max_{x,y,z_i,t} \;\; & t, \\ s.t \;\; & c(x,y,z_i) \geq t, \\ & A(\zeta)x + B(\zeta)y + C(\zeta)z_i \leq q, \\ & \forall\zeta \in Z_i, \forall i = 1, \ldots, m. \end{aligned} \tag{28}$$

The integer ARC formulation (28) was more flexible than the non-adjustable (26) in selecting integer variable values. The ARC integer (28) had a special decision z_i for each subset Z_i. Therefore, the integer ARC formulation at (28) produced robust optimal results, which were as effective as the regular ARC formulation at (26).

D. Benders Decomposition Approach (BDA)

The BDA is a mathematical method aimed at partitioning or dividing a problem into linear and nonlinear parts, thereby simplifying the process. The initial problem $P(x, y)$ stated in (29) must be considered.

$$\begin{aligned} \min \;\; & c^T x + f(y), \\ s.t \;\; & Ax + F(y) = b, \\ & x \geq 0, y \in Y, \end{aligned} \tag{29}$$

where $A \in R^{m\times n}, x, c \in R^n, b \in R^m, y \in Y \subset R^p$, in this case, $f(y)$ and $F(y)$ are nonlinear, and Y representing a discrete or continuous range. Based on the studies by Bisschop [12], the steps for applying the BDA are as follows.

1) First, define a feasible subproblem $P(x|y)$ as a Linear Programming problem in the terms of x with a fixed value of $y \in Y$. Assume that $\forall\; y \in Y$, there is a finite optimal solution x for $P(x|y)$, Thus (29) is reformulated into a constraint equivalent to $P_1(x, y)$ using equation (30):

$$\min_{y}\left\{f(y) + \min_{x}\left\{c^T x : Ax = b - F(y), x \geq 0\right\}\right\}, \tag{30}$$

where

$$\min_{x}\left\{c^T x : Ax = b - F(y), x \geq 0\right\} \tag{31}$$

is called an inner optimization problem that is assumed to have an optimal solution x for every $y \in Y$.

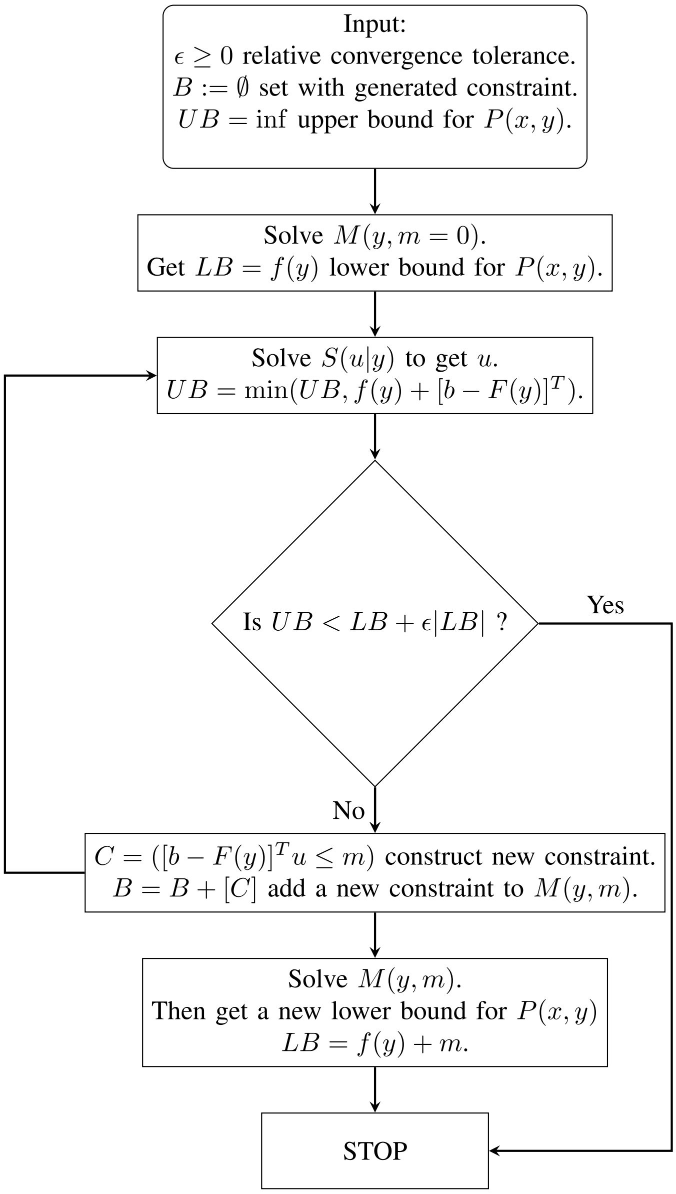

Fig. 1: Benders decomposition algorithm flowchart [12]

2) Second, for the inner optimization, find a dual problem formulation. Thus, formulation (29)can be rewritten as

$$\min_{y} \left\{ f(y) + \max_{u} \{ (b - F(y))^T u : A^T u \leq c \} \right\}. \tag{32}$$

In the inner optimization problem (32), the constraints depend on u and free from y. Using (32), the optimal solution obtained is finite because x in formulation (31) had an optimal solution x for every $y \in Y$. This implied that the optimal solution was at the extreme point $u \in U$. Formulation (32) was rewritten as follows

$$\min_{y} \left\{ f(y) + \max_{u} (b - F(y))^T u \right\}. \tag{33}$$

3) Third, present the Full Master Problem (33) into a simple minimization issue as follows :

$$\begin{aligned} \min\ & f(y) + m, \\ s.t\ & (b - F(y))^T u \leq m, u \in U, \\ & y \in Y. \end{aligned} \tag{34}$$

Next, the Relaxed Master Problem $M(y, m)$ of (34) becomes (35) where B is an empty set and m is initialized as 0.

$$\begin{aligned} \min\ & f(y) + m, \\ s.t\ & (b - F(y))^T u \leq m, u \in B, \\ & y \in Y. \end{aligned} \tag{35}$$

Benders subproblem refers to a problem that solves an u with a fixed value $y \in Y$, essentially considered a maximization problem.

$$S(u|y) = max \left\{ (b - F(y))^T u : A^T u \leq c \right\}, \tag{36}$$

where $u \in R, S(u|y)$ has a finite optimal solution. The algorithm of BDA can be seen in Figure 1.

E. Lexicographic Method for Multiobjective Optimization

Refers to Rao in [46], a multi-objective optimization problem is defined as follows. Find $x \in R^n$ which minimized $f_1(x), \ldots, f_k(x)$ subject to $g_j(x) \leq 0, j = 1, \ldots, m$, where

k is the number of objective functions to be minimized. The functions $f_i(x)$ and $g_j(x)$ can be nonlinear. According to Rao [46], the Lexicographic Method is one of the methods to solve multi-objective optimization problems. In this method, the objectives are ranked in order of importance. The priority scale for the minimization objective function was denoted by additional indexes i and l, with $l = (i-1)$.

$$\begin{aligned} &\min\ f_i(x), \\ &s.t.\ \ g_j(x) \leq 0, j = 1, 2, 3, \ldots, m, \\ &\qquad f_l(x) = f_l^*, l = 1, 2, 3, \ldots, (i-1). \end{aligned} \tag{37}$$

The optimal solution calculation process is done by sequentially minimizing the objective functions, starting from the most important and progressing according to the number and level of importance. The iterative approach continued until the optimum solution x was obtained. Multi-objective optimization problems can be effectively resolved using the Lexicographic Method. Rao [46] stated that this method sorts objective functions based on the interests or priorities determined by several studies.

III. Results

A. ARC Optimization Model Using Polyhedral Uncertainty Sets for Supply Chain Problems

The section focused on modeling ARC and the numerical experiments addressing supply chain problems. It included thoroughly explaining the model, using the BDA on the acquired data, and the calculation process performed using RStudio software. The general formulations of the ARC Optimization model were analyzed using polyhedral uncertainty sets and the BDA to address supply chain problems. Additionally, the data for numerical experimentation was reviewed using RStudio software.

The first step entailed determining the uncertainty parameter assumptions and adjustable decision variables to formulate the ARC optimization model using the polyhedral uncertainty set and referring to models (1) to (10), the study focused on the sixth and seventh constraint functions.

There are two parameters containing uncertainty that were assumed to be, i.e.,

1) The product selling price c in district i (Rp/tons), v_{ci}.
2) The y_{ckj} is considered as an adjustable decision variable representing wait-and-see variables.

Thus, the discrete values, x_j, w_{cji}, p_{cj} is considered as the non-adjustable variable denoting here-and-now.

Next, assumed that

$$v_{ci} = (v_1, v_2, ..., v_i) \in U_1, \tag{38}$$

and

$$y_{ckj} = (y_1, y_2, ..., y_i) \in U_2 \tag{39}$$

with $U_1, U_2 \in U$. Thus,

$$v_{ci}(\zeta_1) = \bar{v}_{ci} + P_{ci}\zeta_1, \tag{40}$$

$$y_{ckj}(\zeta_2) = \bar{y}_{ckj} + Q_{ckj}\zeta_2, \tag{41}$$

where

$$P_{ci} = (P_1, P_2, ..., P_i) \in R^{n \times L_1}, \tag{42}$$

$$\zeta_1 = \left(\zeta_1^{(1)}, \zeta_1^{(2)}, ..., \zeta_1^{(i)}\right) \in R^{L_1}, \tag{43}$$

$$\bar{y}_{ckj} = (\bar{y}_1, \bar{y}_2, ..., \bar{y}_i,) \in R^m \tag{44}$$

$$Q_{ckj} = (Q_1, Q_2, ..., Q_i) \in R^{m \times L_2}, \tag{45}$$

$$\bar{v}_{ci} = (\bar{v}_1, \bar{v}_2, ..., \bar{v}_i,) \in R^n, \tag{46}$$

$$\zeta_2 = \left(\zeta_2^{(1)}, \zeta_2^{(2)}, ..., \zeta_2^{(i)}\right) \in R^{L_2}. \tag{47}$$

Parameters $\bar{v}_{ci}$ and $\bar{y}_{ckj}$ denoted the nominal value vector, while P_{ci}, Q_{ckj} depicted the confounding matrix, and ζ_1, ζ_2 represented a primitive uncertainty vector. Due to the uncertainty in the objective function and fundamental assumptions related to the general model of the robust optimization problem, an uncertain model formulation was obtained and stated as follows.

$$\begin{aligned} &\max\ t, \\ &\min\ \left\{\kappa \sum_{j\in J} x_j + q \sum_{c\in C}\sum_{j\in J} p_{cj}\right\}, \\ &s.t\ \ (\bar{y}_{ckj} + Q_{ckj}\zeta_2) \leq x_j, \forall c \in C, k \in K, j \in J, \\ &\qquad w_{cji} \leq x_j, \forall c \in C, j \in J, i \in I, \\ &\qquad \sum_{c\in C}\sum_{i\in I}\sum_{j\in J} (\bar{v}_{ci} + P_{ci}\zeta_1) w_{cji} \leq t, \\ &\qquad \zeta_1 \in Z_1; \zeta_2 \in Z_2; Z_1, Z_2 \in Z, \\ &\qquad p_{cj} = (p_1, p_2, ..., p_i) \geq 0, \\ &\qquad y_{ckj}, w_{cji}, x_j \in \{0, 1\}, \\ &\qquad t \quad \text{unrestricted.} \end{aligned} \tag{48}$$

The next step was to formulate the ARC model by applying the polyhedral uncertainty set, defined as follows:

$$Z_1 = \zeta_1 : \{b - B\zeta_1 \geq 0\}, \tag{49}$$

$$Z_2 = \zeta_2 : \{d - D\zeta_2 \geq 0\}. \tag{50}$$

Based on the formulation of the indeterminate model in (48), two constraint functions contained uncertainty vectors ζ_1 and ζ_2. The steps for determining the ARC formulation with the polyhedral uncertainty set for the first constraint are as follows.

$$(\bar{y}_{ckj} + Q_{ckj}\zeta_2) \leq x_j, \forall c \in C, k \in K, j \in J \tag{51}$$

which is equivalent to (52).

$$\begin{aligned} (\bar{y}_{ckj} + Q_{ckj}\zeta_2) &\equiv \bar{y} + Q\zeta_2 \\ &\equiv \bar{y} + \max_{\zeta_2 : \{d - D\zeta_2 \geq 0\}} Q\zeta_2 \\ &\equiv \bar{y} + \min_{\rho}\{d^T\rho : D^T\rho = Q, \rho \geq 0\} \\ &\equiv \bar{y} + d^T\rho \leq x, D^T\rho = Q, \rho \geq 0 \end{aligned} \tag{52}$$

with ρ denoting a dual variable. The final result (52) is converted to the following sigma form.

$$\begin{aligned} &\bar{y}_{ckj} + \sum_{h\in H} d_h\rho_h \leq x_j, \forall c \in C, k \in K, j \in J, \\ &\sum_{z\in Z} D_{zh}\rho_h = Q_{zckj}, \forall c \in C, k \in K, j \in J, z = 1, \ldots L_2, \\ &\rho_h = (\rho_1, \ldots, \rho_h) \geq 0, \forall h \in H, \end{aligned} \tag{53}$$

which is a substitute for the first constraint. Similarly, the ARC formulation and the polyhedral uncertainty set were determined for the third constraint of the uncertain model (48), which were stated as follows

$$\sum_{c\in C}\sum_{i\in I}\sum_{j\in J}(\bar{v}_{ci}+P_{ci}\zeta_1)w_{cji}\leq t, \tag{54}$$

is equivalent to

$$\begin{aligned}(\bar{v}+P\zeta_1)^T w &= \bar{v}^T w + (P\zeta_1)^T w\\ &= \bar{v}^T w + \underset{\zeta_1:\{b-B\zeta_1\geq 0\}}{max}(P^T w)^T\zeta_1\\ &= \bar{v}^T w + \underset{\varepsilon}{min}\{b^T\varepsilon : B^T\varepsilon = P^T w, \varepsilon\geq 0\}\\ &\leq t. \end{aligned}\tag{55}$$

This implies that

$$\bar{v}^T w + b^T\varepsilon\leq t, \tag{56}$$

$$B^T\varepsilon = P^T w, \tag{57}$$

$$\varepsilon\geq 0 \tag{58}$$

with ε denoting the dual variable. The final result (55) was converted to sigma form as stated in (59).

$$\begin{aligned}&\sum_{c\in C}\sum_{i\in I}\sum_{j\in J}\bar{v}_{ci}w_{cji}+\sum_{y\in Y}b_y\varepsilon_y\leq t,\\ &\sum_{y\in Y}B_{zy}\varepsilon_y=\sum_{c\in C}\sum_{i\in I}\sum_{j\in J}P_{zj}w_{cji}, \forall z=1,\ldots,L_1,\\ &\varepsilon_y=(\varepsilon_1,\ldots,\varepsilon_y)\geq 0, \forall y\in Y.\end{aligned}\tag{59}$$

which is a substitute for the third constraint, resulting in the overall formulation of the ARC supply chain model with the polyhedral uncertainty set obtained, as stated in the optimization model (60).

$$\begin{aligned}&\max\ t,\\ &\min\left\{\kappa\sum_{j\in J}x_j+q\sum_{c\in C}\sum_{j\in J}p_{cj}\right\},\\ &s.t\ \ \bar{y}_{ckj}+\sum_{h\in H}d_h\rho_h\leq x_j, \forall c\in C, k\in K, j\in J,\\ &\qquad\sum_{z\in Z}D_{zh}\rho_h=Q_{zckj},\\ &\qquad\qquad\forall c\in C, k\in K, j\in J, z=1,\ldots L_2,\\ &\qquad w_{cji}\leq x_j, \forall c\in C, j\in J, i\in I,\\ &\qquad\sum_{c\in C}\sum_{i\in I}\sum_{j\in J}\bar{v}_{ci}w_{cji}+\sum_{y\in Y}b_y\varepsilon_y\leq t,\\ &\qquad\sum_{y\in Y}B_{zy}\varepsilon_y=\sum_{c\in C}\sum_{i\in I}\sum_{j\in J}P_{zj}w_{cji}, \forall z=1,\ldots,L_1,\\ &\qquad p_{cj}=(p_1,p_2,...,p_i)\geq 0, \rho_h=(\rho_1,\ldots,\rho_h)\geq 0,\\ &\qquad\varepsilon_y=(\varepsilon_1,\ldots,\varepsilon_y)\geq 0, Q_{zckj}=(Q_1,\ldots,Q_i)\geq 0,\\ &\qquad y_{ckj}, w_{cji}, x_j\in\{0,1\},\\ &\qquad t\quad\text{unrestricted.}\end{aligned}\tag{60}$$

The resulting ARC Optimization Model Using Polyhedral Uncertainty Sets for Supply Chain Problems is the optimization model (60). This model is obtained by assuming the data uncertainties lie in a polyhedral uncertainty set. Thus, the robustness is guaranteed since the ARC is obtained as a mixed linear programming problem. The existence of discrete variables is handled using BDA.

B. BDA to the ARC Optimization Model Using Polyhedral Uncertainty Sets for Supply Chain Problems

Referring to the model as stated in formulation (60), the existence of discrete and continuous decision variables implied that BDA was required. As mentioned in Section 2.4, using the Lexicographic Method was essential for handling the multi-objective function. This entailed performing calculations on the model sequentially, according to the most crucial objective function. The model in formulation (60) comprised two types of objective functions: maximization and minimization. The maximization objective function was considered more important than the minimization type. Therefore, the BDA was applied twice for each function.

1) BDA for the first objective function: In the initial application of the BDA, the objective function used is the maximization of t. Referring to the model in formulation (61), there are three discrete-valued decision variables, namely y_{ckj}, w_{cji}, and x_j assumed to be v_1. Meanwhile, four continuous value decision variables, namely $\rho_h, \varepsilon_y, Q_{zckj}$, and t, were assumed to be v_2. Thus, we obtain the following results.

i. *Relaxed Master Problem* $M(v_1, m=0)$ is a separate model that solely contained decision variables v_1, obtained as follows

$$\begin{aligned}&\max\ 0\\ &s.t.\ \ w_{cji}\leq x_j, \forall c\in C, j\in J, i\in I,\\ &\qquad y_{ckj}, w_{cji}, x_j\in\{0,1\}.\end{aligned}\tag{61}$$

ii. *Inner Optimization Problem* $P(v_2, v_1)$ is a separate model containing decision variables v_2. Assuming the decision variable $v_1=1$, the formulation obtained was as follows.

$$\begin{aligned}&\max\ t,\\ &s.t\ \ \sum_{h\in H}d_h\rho_h\leq 0,\\ &\qquad\sum_{h\in H}D_{zh}\rho_h-Q_z=0, \forall z=1,\ldots L_2,\\ &\qquad\sum_{y\in Y}b_y\varepsilon_y-t\leq\sum_{c\in C}\sum_{i\in I}\bar{v}_{ci},\\ &\qquad\sum_{y\in Y}B_{zy}\varepsilon_y=\sum_{y\in Y}P_{zy}, \forall z=1,\ldots,L_1,\\ &\qquad\rho_h=(\rho_1,\ldots,\rho_h)\geq 0,\\ &\qquad\varepsilon_y=(\varepsilon_1,\ldots,\varepsilon_y)\geq 0,\\ &\qquad Q_z=(Q_1,\ldots,Q_z)\geq 0,\\ &\qquad t\quad\text{unrestricted.}\end{aligned}\tag{62}$$

iii. *Dual Subproblem* $S(\alpha,\beta,\gamma,\theta|v_1)$ *(first decomposition result)* is equivalent to the dual result $P(v_2, v_1)$ in (62). Since the model in formulation (62) had four decision variables, it defined four dual variables, namely α,β,γ, and θ. to each constraint sequentially. The formulation

$S(\alpha,\beta,\gamma,\theta|v_1)$ was obtained as follows

$$\begin{aligned}\min & \sum_{j\in J}\sum_{i\in I}\bar{v}_i\gamma_{ji}+\sum_{j\in J}P_{zj}\theta_j,\\ s.t. & \sum_{h\in H}d_h\alpha_h+\sum_{h\in H}D_{zh}\beta_h\leq 0,\forall z=1,\dots,L_2,\\ & \sum_{y\in Y}b_y\gamma_y+\sum_{y\in Y}B_{zy}\theta_y\leq 0,\forall z=1,\dots,L_1,\\ & \beta_h=(\beta_1,\dots,\beta_h)\geq 0,\\ & \alpha_h=(\alpha_1,\dots,\alpha_h)\leq 0,\\ & \gamma_{ji}=(\gamma_1,\dots,\gamma_i)\leq 0,\\ & \gamma_y=(\gamma_1,\dots,\gamma_y)\leq 0,\\ & \theta_j,\theta_y \quad \text{unrestricted.}\end{aligned} \tag{63}$$

iv. Benders' Cut formulated as an objective function constraint on $S(\alpha,\beta,\gamma,\theta|v_1)$ by m, was introduced as a new constraint function added to $M(v_1,m=0)$, expressed as follows.

$$\sum_{c\in C}\sum_{i\in I}\bar{v}_{ci}\gamma_{ci}+\sum_{j\in J}P_{zj}\theta_{zj}\leq m,\forall z=1,\dots,L_1. \tag{64}$$

v. *Full Master Problem $M(v_1,m)$(second decomposition result)* was obtained by adding the value of m on the objective function $M(v_1,m=0)$, which is stated as follows.

$$\begin{aligned}&\max m,\\ &s.t. \quad w_{cji}\leq x_j,\forall c\in C,j\in J,i\in I,\\ &\qquad w_{cji},x_j\in\{0,1\}.\end{aligned} \tag{65}$$

2) BDA for the second objective function: : In the second application of the BDA, the objective function used was minimization

$$\left\{\kappa\sum_{j\in J}x_j+q\sum_{c\in C}\sum_{j\in J}p_{cj}\right\}. \tag{66}$$

Referring to the model in formulation (59), three discrete-valued decision variables, namely y_{ckj}, w_{cji}, and x_j were assumed as v_3. While five continuous value decision variables, namely $p_{cj},\rho_h,\varepsilon_y,Q_{zckj}$, and t represented v_4. The same method was used to obtain the formulation of the Relaxed Master Problem or $M(v_3,m=0)$, which was stated as follows.

$$\begin{aligned}&\min \kappa\sum_{j\in J}x_j,\\ &s.t \quad w_{cji}\leq x_j,\forall c\in C,j\in J,i\in I,\\ &\qquad w_{cji},x_j\in\{0,1\},\end{aligned} \tag{67}$$

Inner Optimization Problem formulation $P(v_4,v_3)$, is stated by (68).

$$\begin{aligned}\min & \; q\sum_{c\in C}\sum_{j\in J}p_{cj},\\ s.t. & \sum_{h\in H}d_h\rho_h\leq 0,\\ & \sum_{h\in H}D_{zh}\rho_h-Q_z=0,\forall z=1,\dots,L_2,\\ & \sum_{y\in Y}b_y\varepsilon_y-T^*\leq\sum_{c\in C}\sum_{i\in I}\bar{v}_{ci},\\ & \sum_{y\in Y}B_{zy}\varepsilon_y=\sum_{j\in J}P_{zj},\forall z=1,\dots,L_1,\\ & p_{cj}=(p_1,\dots,p_i)\geq 0,\\ & \rho_h=(\rho_1,\dots,\rho_h)\geq 0,\\ & \varepsilon_y=(\varepsilon_1,\dots,\varepsilon_y)\geq 0,\\ & Q_{zckj}=(Q_1,\dots,Q_i)\geq 0,\end{aligned} \tag{68}$$

where T^* is an optimal value of the first objective function t.

Dual subproblem formulation $S(\delta,\sigma,\mu,\lambda|v_3)$ (first decomposition result) was obtained using (69).

$$\begin{aligned}\min & \sum_{j\in J}\sum_{i\in I}\bar{v}_i\mu_{ji}+\sum_{j\in J}P_j\lambda_j, \qquad (69)\\ s.t. & \sum_{h\in H}d_h\delta_h+\sum_{h\in H}D_{zh}\sigma_h\leq 0,\forall z=1,2,3,...,L_2,\\ & \sum_{y\in Y}b_y\mu_y+\sum_{y\in Y}B_{zy}\lambda_y\leq 0,\forall z=1,2,3,...,L_1,\\ & \sigma_h=(\sigma_1,\sigma_2,...,\sigma_h)\geq 0,\\ & \delta_h=(\delta_1,\delta_2,...,\delta_h)\leq 0,\\ & \mu_{ji},\mu_y,\lambda_y,\lambda_j \quad \text{unrestricted.}\end{aligned}$$

Benders cut formulation was stated in (70).

$$\sum_{c\in C}\sum_{i\in I}\bar{v}_{ci}\mu_{ci}+\sum_{j\in J}P_{zj}\lambda_{zj}\leq m_2,\forall z=1,2,3,...,L_1, \tag{70}$$

Complete Master Problem formulation $M(v_3,m)$ (second decomposition result) was stated in (71).

$$\begin{aligned}&\min \kappa\sum_{j\in J}x_j+m_2,\\ &s.t. \quad w_{cji}\leq x_j,\forall c\in C,j\in J,i\in I,\\ &\qquad w_{cji},x_j\in\{0,1\}.\end{aligned} \tag{71}$$

The algorithm for solving the optimization model obtained with the BDA was explained in the numerical experiments section using the RStudio software.

C. Research Data and Numerical Experiment

The numerical experiments conducted in this study used secondary data obtained from various sources in 2022. The study specifically focused on districts in the northern part of Bandung City, including Andir, Bandung Wetan, Cibeunying Kaler, Cibeunying Kidul, Cicendo, Cidadap, Coblong, Sukajadi, Sukasari, and Sumur Bandung. These districts were selected as the objects for numerical experiments due to their significance in the supply chain. Meanwhile, the northern city of Bandung was chosen because it had the largest wholesale market, Andir Market, which served as a center for sugar suppliers and sellers. Based on the supply chain optimization model (59), the ten selected districts acted as producers, consumers, and potential zones for LFH construction. Based on the index $i, j, k = \{1, 2, 3, ..., 10\}$, the sequence symbolized these. Table II shows population data for the ten districts.

The agricultural processed products used were solely granulated sugar, rendering the index c irrelevant. Total consumer demand for commodities was calculated by multiplying the average consumption per capita multiplied by the number of residents in each district. In the northern city of Bandung, the predicted average consumption of granulated sugar per capita in 2022 is 13.6 kg/capita. Moreover, the production capacity of granulated sugar exceeded the total demand by 46%, with a maximum distribution capacity of 20 tons per line. Data on consumer demand and production capacity for granulated sugar are shown in Table III.

TABLE II: Population Data by District in Bandung City in 2022 (Disdukcapil Bandung City, 2022)

Index	District	Total Population (people/year)
1	Andir	99,288
2	Bandung Wetan	28,686
3	Cibeunying Kaler	70,261
4	Cibeunying Kidul	112,583
5	Cicendo	95,826
6	Cidadap	53,992
7	Coblong	3,947
8	Sukajadi	102,352
9	Sukasari	77,384
10	Sumur Bandung	37,469

TABLE III: Consumer Demand Data and Sugar Production Capacity

Index	District	Demand (tons/year)	Capacity (tons/year)
1	Andir	1,350	1,971
2	Bandung Wetan	390	570
3	Cibeunying Kaler	956	1,395
4	Cibeunying Kidul	1.531	2,235
5	Cicendo	1,303	1,903
6	Cidadap	734	1,072
7	Coblong	54	78
8	Sukajadi	1,392	2,032
9	Sukasari	1,052	1,537
10	Sumur Bandung	510	744

The selling price of granulated sugar in each district was assumed to be the same, approximately Rp. 12,833/kg or equivalent to Rp.12,833,000/ton (for all districts) based on PIHPS [47]. Additionally, the construction cost for LFH was estimated to be Rp. 300,000,000 per building, while the cost of implementing health protocols was approximately Rp. 200,000/ton.

As it is assumed that the product selling price c in district i (Rp/tons), v_{ci} is uncertain, thus in Table IV and Figure 2, the polyhedral of v_{ci} can be presented as the convex hulls of the uncertain data of selling price of granulated sugar in each district.

TABLE IV: Determination of uncertainty parameters of the selling price of granulated sugar

	Line equation	Form in matrix:
Selling Price of Sugar	$y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$ $y - 0x = 12,833,000$	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \leq \begin{bmatrix} 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, b = \begin{bmatrix} 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \\ 12,833,000 \end{bmatrix}$

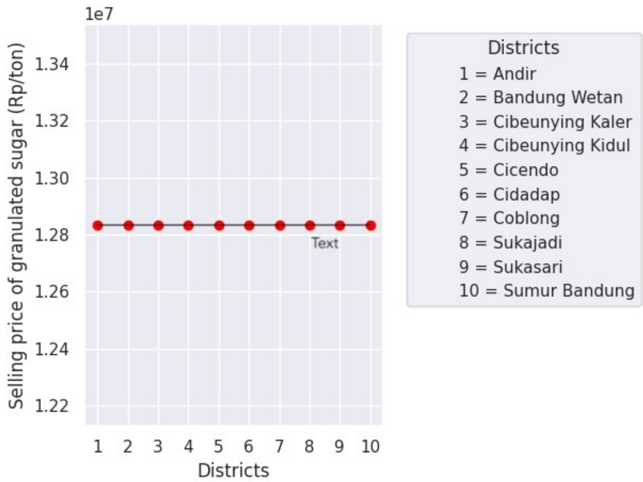

Fig. 2: Convex hulls of v_{ci}.

In this case study, the granulated sugar data capacities are assumed to lie within a polyhedral uncertainty set. The data can be seen in Table III. Thus, the determination of the convex hull for uncertain capacities of granulated sugar data parameters is based on the optimization model (60). Table V and Figure 3 present the polyhedral uncertainty set.

The next stage entailed conducting a numerical experiment using the RStudio software as a calculating tool to obtain the optimal solution. The BDA was applied following these calculation steps:

1) Finding the optimal solution for the discrete-valued decision variables y_{ckj}, w_{cji}, and x_j or the assumed v_1 in formulation Relaxed Master Problem. The optimal value of the objective function $f(v_1)$ was considered the lower bound (LB) or the lower limit.
2) Finding the optimal solution for the dual variable α, β, γ, and θ in Dual Subproblem formulation denoted as $S(\alpha, \beta, \gamma, \theta | v_1)$.. The optimal value of the objective function $S(\alpha, \beta, \gamma, \theta | v_1)$ is $f(\alpha, \beta, \gamma, \theta)$ used to calculate the upper bound (UB) or limit is determined in the

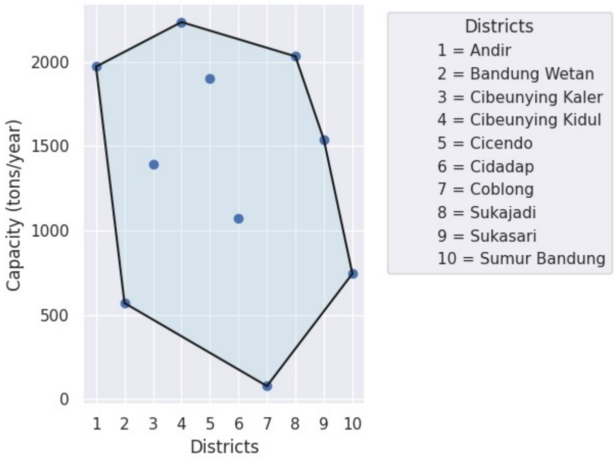

Fig. 3: Convex hulls of y_{ckj}

TABLE V: Determination of uncertainty parameters capacities of granulated sugar data

Decision Variable Optimal Solution y_{ckj}	Line equation	Form in matrix:
	$y-1,401x \leq -3,372$ $y-98.4x \leq -766.8$ $y+222x \leq 1,476$ $y+793x \leq 8,674$ $y+495x \leq 5,992$ $y+45x \leq 2,392$ $y-80.3x \leq 1,890.7$	$\begin{bmatrix} 1 & -1,401 \\ 1 & -98.4 \\ 1 & 222 \\ 1 & 793 \\ 1 & 495 \\ 1 & 45 \\ 1 & 80.3 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \leq \begin{bmatrix} -3,372 \\ -766.8 \\ 1,476 \\ 8,674 \\ 5,992 \\ 2,392 \\ 1,890.7 \end{bmatrix}$
		$D = \begin{bmatrix} 1 & -1,401 \\ 1 & -98.4 \\ 1 & 222 \\ 1 & 793 \\ 1 & 495 \\ 1 & 45 \\ 1 & 80.3 \end{bmatrix}, d = \begin{bmatrix} -3,372 \\ -766.8 \\ 1,476 \\ 8,674 \\ 5,992 \\ 2,392 \\ 1,890.7 \end{bmatrix}$

following formulation.

$$UB = \min\left(UB = \infty, f(v_1) + f(\alpha,\beta,\gamma,\theta)\right). \quad (72)$$

3) Perform a limit test to check whether $UB < LB + \epsilon|LB|, \forall \epsilon > 0$, using the following criteria.
   a) If True, then proceed to point four.
   b) If False, then the Benders Cut constraint function is added to the Full Master Problem formulation or $M(v_1, m)$, returns to point one, and the iteration continues.
4) Finding the optimal solution for the continuous-valued decision variable $\rho_h, \epsilon_y, Q_{xckj}$ and t was assumed as v_2 by substituting the optimal solution in point one into the formulation of the Inner Optimization Problem or $P(v_2, v_1)$. In addition, the iteration process was stopped.

IV. Discussion

The four points of BDA were applied to the constructed model, as stated in formulations (60) to (64) and (65) to (70) for the first and second objective functions using the same method. In the case of granulated sugar, as stated in Section III-C, Table V was used to determine the uncertain parameter for generating the polyhedral uncertainty set. The resulting calculation outputs were obtained in Table VI and Table VII. The optimal solution for the first objective function is presented in Table VI. Based on the Lexicographic Method in Section 2.4, the calculation process continued using the second objective function, delivering the final optimal solution in Table VII.

The final optimal solution of the ARC optimization model with polyhedral uncertainty sets was shown in Table VII. The result was obtained using the BDA for supply chain problems. Therefore, it was concluded that to maximize demand and minimize logistics costs, all districts in northern Bandung were selected as the optimal location for building LFH because the outputs were $x_1, x_2, ..., x_{10} = 1$. Furthermore, $w_{cji} = 1, \forall j, i = 10, c = 1$ the result showed that the optimal distribution of granulated sugar across all LFH, each with a capacity of 2,000 tons fulfilled the demand.

TABLE VI: First Objective Function Optimal Solution

Stage	Information	Result
1	Optimal solution $M(v_1, m=0)$	$x_1, x_2, ..., x_{10} = 1$ $w_{cji} = 0, \forall j, i = 10, c = 1$
	Lower bound calculation	$f(v_1) = LB = 0$
2	Optimal solution $S(\alpha,\beta,\gamma,\theta \mid v_1)$	$f(\alpha,\beta,\gamma,\theta) = -405,522,800$
	Upper bound calculation	$UB = \min(\infty, -405,522,800)$ $= -405,522,800$
3	Bound test	$UB < LB + \varepsilon \vert LB \vert, \forall \varepsilon > 0$ $-405,522,800 < 0 + \varepsilon \vert 0 \vert, \forall \varepsilon > 0$ (True)
4	$x_1, x_2, ..., x_{10} = 1$ solution $P(v_2, v_1)$	$t = 5,531,801$ (demand maximization optimal result) $w_{121}, w_{141}, w_{151} = 1$ $w_{161}, w_{171}, w_{191} = 1$ $w_{112}, w_{122}, w_{132}, w_{142} = 1$ $w_{162}, w_{182}, w_{192} = 1$ $w_{113}, w_{133}, w_{155} = 1$ $w_{163}, w_{173}, w_{183} = 1$ $w_{124}, w_{134}, w_{144} = 1$ $w_{154}, w_{174}, w_{194} = 1$

V. Conclusions

In conclusion, a new result is obtained as it is expected how the ARC model is solved using the BDA in addressing supply chain problems in sugar distribution. In this case, the uncertain data is on the selling price, which is assumed to be polyhedral uncertainty. The resulting model is a multi-objective optimization model with two objective functions, i.e., to maximize demand fulfillment and minimize logistics costs. The case of numerical experiment on sugar distribution was selected due to the significant influence as an agricultural processed product. The local solution obtained from solving the multi-objective integer ARC optimization model with a guaranteed polyhedral uncertainty set was globally optimal, based on the convexity analysis of the entire set of solutions,

TABLE VII: Optimal Solution of Second Objective Function (Final Solution)

Stage	Information	Result
1	Optimal solution $M(v_1, m=0)$	$x_1, x_2, ..., x_{10} = 1$ $w_{cji} = 0, \forall j, i = 10, c = 1$
	Lower bound calculation	$f(v_3) = LB' = 3 \times 10^9$
2	Optimal solution $S(\alpha, \beta, \gamma, \theta \| v_1)$	$f(\alpha, \beta, \gamma, \theta) = 0$
	Upper bound calculation	$UB = \min(UB = \infty, 0) = 0$
3	Bound test	$UB' < LB; +\varepsilon\|LB'\|, \forall\varepsilon > 0$ $0 < 3 \times 10^9 + \varepsilon\|3 \times 10^9\|,$ $\forall\varepsilon > 0$(True)
4	Optimal solution $P(v_2, v_1)$	$f(v_4, v_3) = 4,000,000$ (demand maximization optimal result) $x_1, x_2, ..., x_{10} = 1$ $w_{cji} = 1, \forall j, i = 10, c = 1$ $p_{cj} = 2,000, \forall j = 10, c = 1$

objective, and constraint functions. Numerical experiments were carried out for all districts in northern Bandung. The final optimal solution for addressing supply chain problems in sugar distribution is obtained.

For future research, solving the ARC with integer multi-objective using machine learning is recommended (see Lee *et al.* in [48]), also by using dynamic programming as mentioned by Shapiro [49].

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