A Low Complexity Detection Algorithm for Generalized Spatial Modulation System

Xinhe Zhang, Wenbo Lv, and Haoran Tan

Abstract—As an extension of spatial modulation (SM) technology, generalized spatial modulation (GSM) is a promising technique for high data-rates systems. In GSM system, multiple antennas are activated in each time slot, which can greatly enhance the bandwidth efficiency. However, the usage of multiple active antennas for transmission also brings difficulties of the demodulation at the receiver. The computational complexity of the traditional maximum likelihood (ML) detector is closely related to the modulation order. Based on this point, we propose an ordered block hard-limited maximum likelihood (OB-HLML) algorithm for MQAM constellation, which is independent of modulation order. OB-HLML algorithm first sorts the possibility of transmit antenna combinations (TACs), and then detects the possible signal vectors of each ordered TAC by HLML detector. Simulation results show that the proposed OB-HLML algorithm can provide almost the same bit error rate (BER) performance as the ML algorithm with greatly reduced computational complexity.

Index Terms—Generalized spatial modulation (GSM), maximum likelihood (ML), bit error rate (BER), transmit antenna combination (TAC), computational complexity.

I. INTRODUCTION

In recent years, with the rapid development of wireless communication technology, multiple input multiple output (MIMO) technology is favored because it can provide higher spectrum efficiency and reliability through diversity and spatial multiplexing gain [1]-[2]. However, with the increase of activated transmit antennas in MIMO system, the inter-channel interference (ICI) will increase exponentially, which leads to the decrease of the reliability of information transmission.

In order to solve the above-mentioned problems, SM as a new MIMO antenna transmission technology has received wide attention as soon as it is proposed [3]-[5]. In the traditional SM technology, only one antenna in each time slot is activated to transmit data. Compared with the MIMO system, the SM technology has no advantage in transmission rate and throughput. To tackle the inherent problem of SM technology, GSM technology has been put forward.

GSM technology allows more than one antenna to be activated at each time slot, and the antenna combination is adopted to denote partial transmitted information bits. Compared with the traditional SM technology, GSM technology greatly improves the spectrum efficiency. In addition, the GSM system can transmit the same digital modulation symbol on the selected antenna combination, which not only eliminates ICI, but also introduces spatial diversity, thus improving the accuracy of signal detection. However, since the multiple activated transmit antennas will generate serious ICI, the signal detection at the receiver will become much more complicated than traditional SM systems. In order to reduce the detection complexity at the receiver, a variety of suboptimal detection algorithms have been put forward [6]-[11]. A low complexity sphere decoding (SD) detection algorithm was proposed in [6]. The detector calculates the equivalent Euclidean distance error between the signals received by each antenna combination at the receiver and the searched transmit symbol in turn, and sets a search radius to search the tree. Compared with the ML algorithm, SD algorithm effectively reduces the detection complexity. A signal detection algorithm based on zero-forcing (ZF) equalization was proposed in [7]. The first step is to detect the possible antenna combination, and the second step is to demodulate the modulation symbols transmitted by the antenna combination. The complexity of the algorithm is greatly reduced. In [8], a maximum ratio combining (MRC) algorithm was proposed to detect the antenna combination, and then the spatial diversity gain was put forward to estimate the modulation symbols on the antenna combination. However, the MRC detector is only suitable for constrained channels. In [9], an ordered block minimum mean square error (OB-MMSE) detection algorithm for GSM systems was proposed, which is also carried out in two steps. The first step is to sort the all antenna combinations, and the second step is to use the minimum mean square error (MMSE) detection algorithm to detect modulation symbols according to the ordered antenna combinations. A novel low complexity joint detection algorithm based on signal vector based detection and MMSE (SVD-MMSE) was proposed in [10]. Firstly, the algorithm uses the signal vector based detection (SVD) algorithm to detect the angle between the vector of the channel gain array vector corresponding to the antenna combination and the received signal, and sorts the antenna combination according
to the angle, and then uses the MMSE algorithm to detect the modulation symbol of the sorted antenna combination. Two improved SD algorithms for GSM systems were proposed in [11], which further reduce the complexity of SD algorithms.

In this paper, we propose a new low complexity detection algorithm. For the GSM system which transmits the same symbol, the received signal is preprocessed firstly by the pseudo-inverse of each column of the channel matrix, and then the weight factor is introduced to rank the possibility of the antenna combination. The modulation symbol is most likely to be transmitted by the first antenna combination, and then the HL-ML algorithm proposed in [12] is used to detect the transmitted symbols and the antenna indices. The contributions of this paper are as follows.

1) To reduce the computational complexity, the sorting strategy is used to order all possible antenna combination. In the next step, the transmitted symbol is detected according to the ordered antenna combination, and the optimal antenna combination and transmission symbols are determined in combination with the decision threshold.

2) In order to approach the optimal detection performance, the hard-limited maximum likelihood algorithm is used to estimate the transmitted symbols. The modulation symbol calculated by HL-ML algorithm is independent of the modulation order.

3) The simulation results show that the proposed OB-HLML algorithm is almost as the ML detector and the computational complexity is reduced greatly.

The rest of the paper is arranged as follows. We introduce the model of GSM system in Section II. In Section III, the specific implementation process of the proposed detection algorithm is presented. The experiment simulation results and computational complexity analysis of the proposed algorithm are presented in Section IV. Finally, we conclude the paper in Section V.

Notation: Boldface uppercase letters denote matrices, boldface lowercase letters denote vectors. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and conjugate transpose of a vector or a matrix, respectively. $\| \cdot \|_2$ is the two-norm of a vector. $\Re(\cdot)$ and $\Im(\cdot)$ are the real and imaginary parts of a complex-value variable. $\mathbb{C}$ represents the field of complex numbers. $(\cdot)^+$ denotes the pseudo-inverse of a vector or a matrix. $N - PAM = \{-N+1,-N+3,\ldots,-1,1,\ldots,N-3,N-1\}$, where $N$ is a power of two.


**II. SYSTEM MODEL**

Consider a GSM-MIMO system with $N_t$ transmitting antennas and $N_r$ receiving antennas. The system block diagram is shown in Fig. 1. In each time slot, only $N_r$ ($N_p < N_t$) transmitting antennas are activated to transmit the same modulation symbol, and the number of TACs used for information bit mapping is $N = 2^{N_t}$, where $N_t = \lceil \log_2 C_{N_t}^{N_p} \rceil$, $C_{N_t}^{N_p}$ represents a binomial coefficient and $\lceil x \rceil$ is the greatest integer smaller than $x$. The input binary bit stream is divided into two parts, the first part is used to map the TACs and the required bit length is $N_l$, and the second part is used to map the modulation symbol vector $s = [s_1, s_2, \ldots, s_{N_p}]^T$ and the required bit length is $l = \log_2 M$, where $s_1, s_2, \ldots, s_{N_p} \in S$ and $s_1, s_2, \ldots, s_{N_p}$ express the same modulation symbol. $M$ and $S$ represent the modulation order and the signal symbol set, respectively. It can be seen that the total length of information bits transmitted in each slot is $L = N_l + l$. The modulated symbol vector is transmitted through $N_r \times N_t$ dimensional channel gain matrix $H$. Each element of $H$ follows a complex Gaussian distribution with a variance of 1 and a mean of 0, and represents the channel gain coefficient between the transmitting antenna and the receiving antenna.

![Fig. 1. The block diagram of GSM system with $N_t$ transmitting antennas and $N_r$ receiving antennas.](image)

The model of the received signal can be expressed as follows

$$y = Hx + n,$$

where $x \in \mathbb{C}^N$ is the transmitted vector, $y \in \mathbb{C}^N$ is the received vector, and $n \in \mathbb{C}^N$ is the additive noise vector, in which each element is assumed to be an independent and identically distributed (i.i.d) zero mean complex Gaussian random variable with variance $\sigma^2$. The transmitted vector $x$ obtained after GSM mapping can be expressed as

$$x = [0, \ldots, s_1, 0, \ldots, s_2, 0, \ldots, s_{N_p}, 0, \ldots]^T,$$

(2)

Only the elements in the corresponding active antenna position in $x$ are non-zero. The GSM mapping scheme is listed in Table I under the condition of $N_r = 4$, $N_p = 2$, and BPSK modulation.

<table>
<thead>
<tr>
<th>Bits</th>
<th>Set</th>
<th>BPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>(1,2)</td>
<td>-1</td>
</tr>
<tr>
<td>001</td>
<td>(1,2)</td>
<td>+1</td>
</tr>
<tr>
<td>010</td>
<td>(1,3)</td>
<td>-1</td>
</tr>
<tr>
<td>011</td>
<td>(1,3)</td>
<td>+1</td>
</tr>
<tr>
<td>100</td>
<td>(1,4)</td>
<td>-1</td>
</tr>
<tr>
<td>101</td>
<td>(1,4)</td>
<td>+1</td>
</tr>
<tr>
<td>110</td>
<td>(2,3)</td>
<td>-1</td>
</tr>
<tr>
<td>111</td>
<td>(2,3)</td>
<td>+1</td>
</tr>
</tbody>
</table>

Assuming that the $k$-th antenna combination is used to transmit modulated symbols, then $H$ can be equivalent to
the channel matrix $H_k$ of $N_c \times N_p$ dimension. The $H_k$ contains the $N_p$ columns in the channel matrix $H$ corresponding to the active antenna, thus the system model can be simplified as follows

$$ y = H_k s + n = \sum_{j=1}^{N_k} h_{kj}s_j + n, \quad (3) $$

where $s = [s_1, s_2, \ldots, s_{N_k}]^T$. Based on the simplified system model (3), the ML detection algorithm of GSM system can be written in the following form

$$ \hat{k} = \arg \min_{\hat{k} \in \{1, 2, \ldots, N_k\}} \| y - H_k \hat{s} \|, \quad (4) $$

where $I$ represents the set of active antenna combinations. If only considering the number of real multiplication and division operations, ML detection algorithm needs to do $6N_cN_p$ real multiplication and division operations, which will result in high computational complexity.

III. OB-HLML DETECTION ALGORITHM

In this section, an ordered block hard-limited maximum likelihood detection (OB-HLML) algorithm is proposed.

**Proposition 1:** If $S$ is a square or rectangular lattice constellation, the constellation can be viewed as the Cartesian product of two sets $S_1 = N_1 - PAM$ and $S_2 = N_2 - PAM$

$$ N_1 - PAM = \{-N_1 + 1, -N_1 + 3, \ldots, 1, -1, \ldots, N_1 - 3, N_1 - 1\}, \quad (5) $$

$$ N_2 - PAM = \{-N_2 + 1, -N_2 + 3, \ldots, 1, -1, \ldots, N_2 - 3, N_2 - 1\}, \quad (6) $$

where $N_1$ and $N_2$ are powers of two.

In the proposed OB-HLML algorithm, all possible TACs are sorted by calculating the product of the pseudo-inverse of each column of the channel matrix and the received vector $y$, then we can obtain $z = [z_1, z_2, \ldots, z_{N_k}]^T$ as follows

$$ z_k = (h_k)^T y, \quad (7) $$

where $(h_k)^T = \frac{h_{k,N_c}^T}{h_{k,N_c}}$ and $k = 1, 2, \ldots, N_k$.

To estimate the reliability of each TAC, a weighting factor $w_i$ is introduced to detect the reliability of each TAC. The weighting factor can be expressed as

$$ w_i = z_i^2 + z_{i+2}^2 + \cdots + z_{i+N}^2 = \sum_{n=i}^{i+N} z_n^2, \quad (8) $$

where $i \in \{1, 2, \ldots, N\}$ and $N$ is the number of TACs. After obtaining $w = [w_1, w_2, \ldots, w_N]$, then we obtain the ordered TACs by sorting the values of the weighting factor. It can be denoted as

$$ [k_1, k_2, \ldots, k_N] = \text{arg sort}(w), \quad (9) $$

where function sort( ) defines the descending order of the input vector elements, and $k_1$, $k_N$ are the indices of the maximum and minimum value in $w$, respectively. Thus, the ordered TACs with the weighting indices $[k_1, k_2, \ldots, k_N]$ can be obtained. For the ease of description, we define the new vector as

$$ H_{h_i} = h_{i,0} + h_{i,1} + \cdots + h_{i,v_{max}} = \sum_{j=0}^{v_{max}} h_{ij}, \quad (10) $$

where $j \in \{1, 2, \ldots, N\}$, $H_{h_i}$ denotes the sum of the column of channel gain matrix corresponding to the $I_{h_i}$. TAC after sorting.

The modulation symbol $\tilde{s}_j$ corresponding to the $I_{h_j}$ TAC can be expressed as

$$ \tilde{s}_j = \frac{(H_{h_j})^T y}{\| H_{h_j} \|^2}. \quad (11) $$

In the following, we quantize the modulation symbol $\tilde{s}_j$, which are expressed as

$$ q(s_j) = \min \left[ \max \left( 2 \times \text{round} \left( \frac{\text{Re}(\tilde{s}_j) + 1}{2} \right) - 1, -N_z + 1 \right), N_z - 1 \right], \quad (12) $$

$$ \text{Im}(s_j) = \min \left[ \max \left( 2 \times \text{round} \left( \frac{\text{Im}(\tilde{s}_j) + 1}{2} \right) - 1, -N_z + 1 \right), N_z - 1 \right], \quad (13) $$

where function round( ) indicates the operation of rounding a real number to the nearest integer, $s_j$ is the quantized symbol corresponding to the $j$-th TAC $I_{h_j}$.

The computational complexity of the detector is decreased by reducing the detected TACs. The detection process will terminate once the output satisfies

$$ d_j = \| y - H_{h_j} \tilde{s} \| < \varepsilon, \quad (14) $$

where $\varepsilon$ is a preset threshold for judging the reliability of the detected signal. Here, we follow the conclusion in [13], expressed as $\varepsilon = 2N_c\sigma^2$, where $\sigma^2$ is the noise variance.

If the $I_{h_j}$ output $(I_{h_j}, s_j)$ is satisfied in (13), the detector will generate the optimal TAC $\hat{I}$ and the estimated symbol vector $\hat{s}$, i.e., $\hat{I} = I_{h_j}$, $\hat{s} = s_j$. Otherwise, the detector will renew $j = j + 1$ and go on the next $(j < N)$ estimation. When $j = N$, the detector is equal to the optimal ML detection by choosing the optimal estimate $(\hat{I}, \hat{s})$ as

$$ m = \arg \min_j d_j, j \in \{1, 2, \ldots, N\}. \quad (15) $$

The proposed OB-HLML detection algorithm can be described in Table II.

IV. SIMULATION RESULTS AND COMPUTATIONAL COMPLEXITY ANALYSIS

In this section, the simulation results and the computational complexity analysis are given.

A. Simulation Results Analysis

In all our simulations, we assume that a frequency-flat
Considering a GSM system: (1) having \( N_r = 8 \), \( N_c = 4 \), \( N_p = 3 \) and employing 4-QAM signal sets; (2) having \( N_r = 8 \), \( N_c = 4 \), \( N_p = 2 \) and employing 8-QAM signal sets. Figs. 2-3 show the BER comparison of the proposed OB-HLML algorithm, ML algorithm and SVD-MMSE algorithm in above-mentioned two scenarios. It is obvious that the BER performance of OB-HLML algorithm is almost the same as ML algorithm, and it is obviously better than SVD-MMSE algorithm at high signal-to-noise ratio (SNR).

### Table II
**PROPOSED OB-HLML DETECTION ALGORITHM**

| 1: Input: \( y, H, N_r, N_c, N_p \) and the threshold \( \varepsilon = 2N_r \sigma^2 \); |
| 2: Calculate the pseudo inverse of channel matrix: |
| \( z = \begin{bmatrix} z_1, z_2, \ldots, z_N \end{bmatrix} \), \( z_i = (h_i) y, (h_i) y = h_i^H \); |
| 3: Calculate the weight: |
| \( w = \begin{bmatrix} w_1, w_2, \ldots, w_N \end{bmatrix} \), \( w_j = \sum_{i=1}^{N_r} z_i^H, j \in \{1, 2, \ldots, N\} \); |
| 4: Sorting the TACs: \( \{k_0, k_2, \ldots, k_N\} = \text{arg sort}(w) \); |
| 5: Initial: \( j = 1 \) |
| 6: while \( j \leq N \) calculate (8)-(13); |
| 7: if \( d_j < \varepsilon \) |
| \( \hat{I} = i_j, \hat{s} = s_j \), break; |
| 8: else |
| \( j = j + 1 \); |
| 9: end if |
| 10: end while |
| 11: if \( j > N \) |
| \( m = \text{arg min}_d j, j \in \{1, 2, \ldots, N\} \) |
| \( \hat{I} = I_m, \hat{s} = s_m \) |
| 12: end if |
| 13: Output the detected (\( \hat{I}, \hat{s} \)) for bit de-mapping. |

In order to further compare the computational complexity of the above-mentioned three algorithms, we adopt the upper limit of the computational complexity of the OB-HLML algorithm and the SVD-MMSE algorithm, let \( L = N \), where \( N \) is the number of TACs.

### Table III
**COMPARISON OF COMPUTATIONAL COMPLEXITY**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>( 6N_r N_c N_p M N )</td>
</tr>
<tr>
<td>SVD-MMSE</td>
<td>( 4 + (8N_r + 4)N_p + (16N_r + 6)L ) ((1 \leq L \leq N))</td>
</tr>
<tr>
<td>OB-HLML</td>
<td>( 4 + (6N_r + 2)N_p + 4N_r N_c + (12N_c + 6)N_p L ) ((1 \leq L \leq N))</td>
</tr>
</tbody>
</table>

Fig. 3. BER comparison of the proposed OB-HLML, ML and SVD-MMSE algorithms in a GSM system having \( N_r = 8 \), \( N_c = 4 \), \( N_p = 2 \), and employing 8-QAM signal sets in the presence of Rayleigh fading channel and perfect channel state information at receiver.

### B. Computational Complexity Analysis

In this subsection, the computational complexity of the proposed OB-HLML algorithm, ML algorithm and SVD-MMSE algorithm is presented. The computational complexity is determined by parameters \( N_r, N_c, N_p, N_p, L \) and \( M \). The number of operations of real multiplication and division is considered as computational complexity. The computational complexity of ML algorithm, OB-HLML algorithm and SVD-MMSE algorithm is shown in Table III.
Meanwhile, the computational complexity of OB-HLML algorithm is significantly lower than ML algorithm, and the computational complexity of the proposed OB-HLML detector is almost independent of the number of active antennas $\mathcal{N}$ in an GSM system with $\mathcal{N} = 8$, $\mathcal{N}_r = 8$, $\mathcal{N}_p = 3$.

It is clear from Figs. 4-5 that the proposed OB-HLML has the lowest computational complexity. The computational complexity of OB-HLML and SVD-MMSE algorithm is independent of $\mathcal{M}$. Meanwhile, the computational complexity of OB-HLML algorithm is significantly lower than that of ML algorithm. Specially, the computational complexity of OB-HLML algorithm is about 4% of ML algorithm when $\mathcal{M} = 64$ in the scenario of Fig. 4.

V. CONCLUSION

A novel OB-HLML detection algorithm independent of modulation order is proposed for GSM-MIMO system in this paper. All possible TAC sets are sorted firstly, and each ordered TAC is detected by HL-ML equalization with a termination threshold. Simulation results show that the BER performance of the proposed OB-HLML detector is almost the same as ML algorithm, and the computational complexity of OB-HLML algorithm is significantly lower than ML algorithm.

REFERENCES


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