Real-time Multivariable Embedded Control of a Ball and Plate Prototype
Sebastian Cardona, Eduardo Giraldo

Abstract—In this work, a low-cost design and ball-and-plate implementation are proposed using an embedded system. The proposed approach allows the evaluation of multivariable controllers in a real-time environment by considering an Arduino Mega. The Ball-and-plate construction is performed on a 3D printer. In order to evaluate the system, three multivariable controllers are evaluated: a PID controller, a Sliding Mode Controller, and a Fuzzy controller. As a result, an efficient mechanical system to evaluate real-time multivariable controllers is obtained.

Index Terms—Real-time, Multivariable control, Ball-and-Plate.

I. INTRODUCTION

The design and evaluation of multivariable controllers over real systems is usually performed in two stages: a simulation stage and a prototype stage. The prototype stage allows verification of the simulation over a real environment where noise, analog-to-digital converters, and computational complexity of the algorithms are considered [1]. However, the computational complexity of the algorithms decreases the performance of the system. As a result, a higher delay in the processing stage [2]. Several adaptive controllers can be applied over multivariable systems, including ARMA or ARMAX structures [3] which allow real-time identification and control of systems where the model is unknown [4]. By considering a nonlinear controller with a variable structure, the design of the controller based on the error function can be improved [5], [6]. It is worth noting that the nonlinear controller requires for its design a detailed model of the system. On the other hand, the Fuzzy controller can be designed based on a knowledge of the behavior of the system [7]. It is worth mentioning that the tuning of controller parameters is performed in some cases using optimization [8], [9].

Project-based learning enhances the validation of fundamental concepts of control systems [10] and validation of controllers in real environments. In addition, for several years [11], Hardware-In-the-Loop (HIL) structures have also been used to validate concepts and rapid prototyping in a realistic simulated environment [12]. These validation methods allow controllers to consider real environments and actual conditions that involve noise, measurements, sensors and actuators dynamics, and delays. HIL validation has become a gold standard, especially for power systems [13], but also in automotive and aerospace areas [14], [15]. In [16], an implementation based on analog computation with operational amplifiers is presented to evaluate fractional-order controllers in real-time. On the other hand, the evaluation of the controller performance over small-scale prototypes is also an option. In [9], an assessment of a multivariable PID controller and a Fuzzy controller is presented for a two-input, two-output small-scale uncrewed aerial vehicle. In [17], an implementation of a Ball-and-Plate prototype is evaluated by using a real-time controller implemented in a computer and using cameras as a position sensor [18].

A low-cost design and implementation of a ball-and-plate is proposed by using an embedded system in this work. The proposed approach allows the evaluation of multivariable controllers in a real-time environment by considering an Arduino Mega and a resistor screen as a position sensor. The Ball-and-plate construction is based on a 3D printer. In order to evaluate the system, three multivariable controllers are evaluated: a PID controller, a Sliding Mode Controller, and a Fuzzy controller. As a result, an efficient mechanical system to evaluate real-time multivariable controllers is obtained. This paper is organized as follows: in section II is presented the mathematical model and the mechanical design with its prototype construction. In section III, is given the evaluation of the controllers over a simulated multivariate model. In addition, the review of the real prototype is also described for 40mm and 30mm ball diameters. And finally, in section IV the conclusions and final remarks are presented.

II. BALL AND PLATE DESIGN

A. Mathematical model

In Fig. 1 is presented the schematic model for an $i$-axis of the ball and plate, being $i = \{x, y\}$, $u_i(t)$ the input angle for each axis and $y_i(t)$ the transnational position for each axis.

![Fig. 1. Ball and plate schematic model for each $i$-axis](image)

The mathematical model can be obtained by applying a free diagram body and describing the dynamic of the system.

Manuscript received March 25, 2021; revised August 5, 2021. This work was carried out under the funding of the Universidad Tecnológica de Pereira, Vicerrectoría de Investigación, Innovación y Extensión. Research project: 6-20-7 “Estimación Dinámica de estados en sistemas multivariables acoplados a gran escala”.

Sebastian Cardona is a researcher at the Research Group in Automatic Control, Universidad Tecnológica de Pereira, Pereira, Colombia. E-mail: sebacardona@utp.edu.co.

Eduardo Giraldo is a Full Professor at the Department of Electrical Engineering, Universidad Tecnológica de Pereira, Pereira, Colombia. Research group in Automatic Control. E-mail: egiraldo@utp.edu.co.
by using the Newtons equation. In Fig. 2 is presented the free body diagram for the \( i \)-axis, where the fundamental forces are described. It is noticeable that in this ball and plate model, the input is considered as the angle \( u(t) \) applied to the plate.

\[
y_i(t) = mg \sin(u_i(t)) - B\dot{y}_i
\]

According to Fig. 2 the following equation is obtained

\[
m\ddot{y}_i = mg \sin(u_i(t)) - B\dot{y}_i
\]

and by considering an approximation around \( u_i = 0, y_i = 0 \) for both axis, the following linear approximated model is obtained

\[
m\ddot{y}_i \approx mgu_i(t) - B\dot{y}_i
\]

and its corresponding transfer function

\[
Y_i(s) = \frac{g}{s^2 + \frac{B}{m}s} U_i(s)
\]

By considering the structure the depicted in Fig- 1, a prototype can be designed and constructed in order to evaluate the performance of the system in closed-loop under several controllers.

**B. Mechanical design**

The mechanical design of the system is performed by considering a structure where the inputs are the angles for each axis \( u_i(t) \). In Fig. 3 is presented the mechanical structure of the Ball and Beam prototype system, where the actuators are position servos with its corresponding embedded controllers, and the sensor is a resistive touchscreen.

The input and output transducers for the Ball and Plate are controlled by an Arduino Mega, which allows a low-cost implementation of the embedded multivariable controller in real-time. In Fig. 4 is presented the schematic diagram of the Ball and Plate prototype, which includes to position servos as actuators and a resistive touchscreen as a sensor.

By using a 3D printer, a real Ball and Pate model based on the mechanical system of Fig. 3 and the schematic diagram of Fig. 4 is presented. In Fig. 5 the 3D printed parts of the final prototype of the Ball and Plate system are presented.

And finally, in Fig. 6 the final assembled prototype of the Ball and Plate system is presented.
III. RESULTS

In order to evaluate the performance of the designed prototype, three multivariable controllers are implemented and evaluated over the embedded system: a PID controller, Sliding Mode controller and Fuzzy controller. Initially, the controllers are tuned by using a simulated model of (2). In Fig. 7 is presented the tracking performance of the PID controller for each axis by using a simulated Ball and Plate mathematical model.

In Fig. 8 is presented the corresponding control signal for each axis of the tracking performance of Fig. 7.

The design of the Fuzzy controller for each axis is based on an inference system with two inputs: error and error derivative with a Fuzzy surface defined as depicted in Fig. 11.

In Fig. 9 is presented the tracking performance of the Sliding Modes controller for each axis by using a simulated Ball and Plate mathematical model.

In Fig. 10 is presented the corresponding control signal for each axis of the tracking performance of Fig. 9.

The design of the Fuzzy controller for each axis is based on an inference system with two inputs: error and error derivative with a Fuzzy surface defined as depicted in Fig. 11.
Fuzzy controller for each axis by using a simulated Ball and Plate mathematical model.

In Fig. 13 is presented the corresponding control signal for each axis of the tracking performance of Fig. 12.

In order to evaluate the tracking performance of the aforementioned controllers, a comparison analysis by using the same references is considered. In this case, a square signal with variable amplitude is considered. The reference value for one axis is changing every ten seconds, while the other axis is changing every six seconds. In Fig. 14 is presented the comparison analysis.

The simulated system is also evaluated for impulse references. As presented in Fig. 15, the comparison of the aforementioned controllers is shown in Fig. 16.

An additional analysis for the simulated signals is performed for step references applied at each axis. The comparison of the aforementioned controllers is shown in Fig. 15. It can be seen that a similar response is achieved as presented in Fig. 14.

The designed controllers (PID, Sliding Modes and Fuzzy controller) are also validated over the constructed prototype. Two analysis are performed considering two different balls. The first test consider a ball of 30 mm diameter for initial conditions response. The initial conditions are 100mm for...
one axis ad 25mm for the second axis. In Fig. 17 is presented the tracking performance of the PID controller.

In Fig. 18 is presented the tracking performance of the Sliding Mode Controller controller.

In Fig. 19 is presented the tracking performance of the PD controller.

In Fig. 20 is presented the tracking performances for the three multivariable controllers (PID, Sliding Modes and Fuzzy controller) over the designed and constructed prototype of Fig. 6.

For the second test, a ball with 40mm diameter for initial conditions response is used. The initial conditions are 100mm for one axis ad 25mm for the second axis. In Fig. 21 is presented the tracking performance of the PID controller.

In Fig. 22 is presented the tracking performance of the SMC controller.

In Fig. 23 is presented the tracking performance of the Fuzzy controller.
In Fig. 24 is presented the tracking performances for the three multivariable controllers (PID, Sliding Modes Controller, and the Fuzzy controller) over the designed and constructed prototype of Fig. 6.

IV. CONCLUSIONS

A low-cost design and implementation of a ball-and-plate are proposed by using an embedded system in this work. The proposed approach evaluates multivariable controllers in a real-time environment by considering an Arduino Mega and a resistor screen as a position sensor. The Ball-and-plate construction is based on a 3D printer. Based on the results for tracking the performance of the PID controller, the Sliding Mode Controller, and the Fuzzy controller, it can be seen that an efficient mechanical system to evaluate real-time multivariable controllers is obtained. This conclusion is validated for simulation under step and impulse references and also for a square reference signal. The prototype system performance for each controller is also evaluated for impulse reference signals by using the Arduino Mega in a real-time environment.